MECHANICS' COMPANION.

Robt. E. Aitken
Phoenix Iron Works
Trenton N.J.

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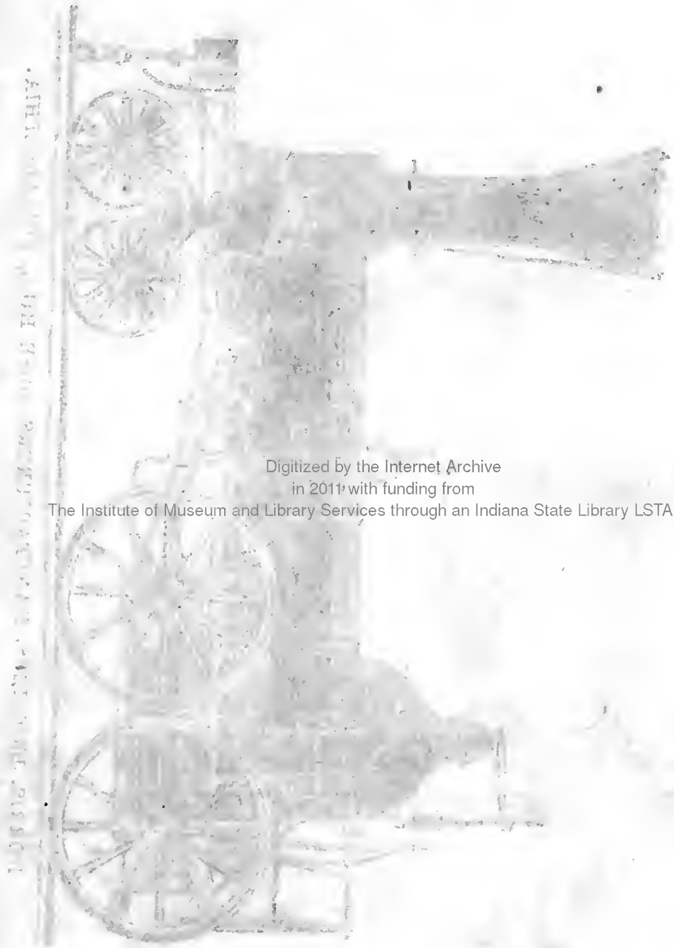

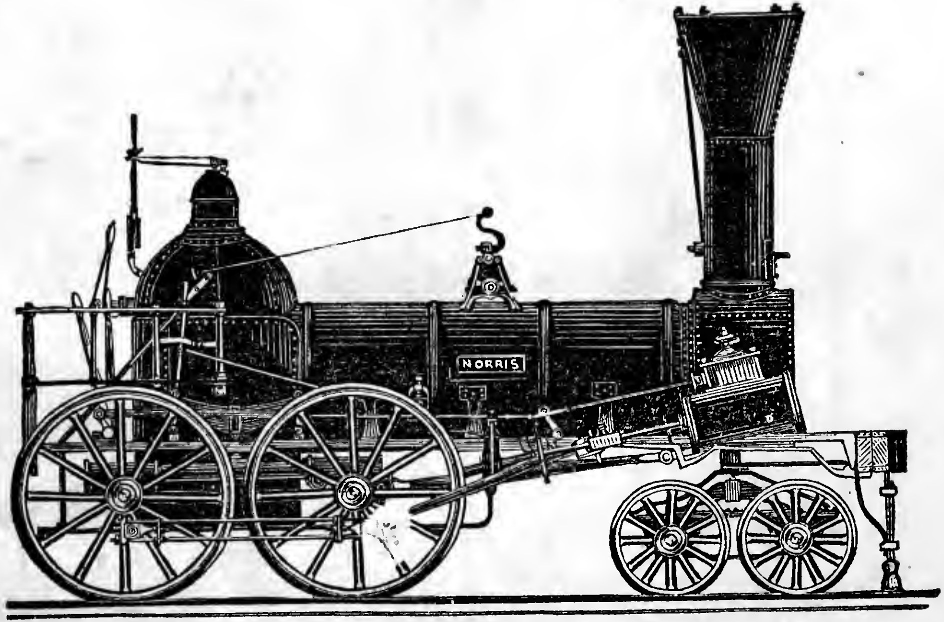

NORRIS, BROTHERS, MANUFACTURERS, BUSH HILL, PHILADELPHIA.

SCRIBNER'S

ENGINEERS' AND MECHANICS' COMPANION:

COMPRISING

UNITED STATES' WEIGHTS AND MEASURES

MENSURATION OF SUPERFICIES AND SOLIDS;

TABLES OF SQUARES AND CUBES, SQUARE AND CUBE ROOTS;
CIRCUMFERENCE AND AREAS OF CIRCLES.

THE MECHANICAL POWERS:

CENTERS OF GRAVITY, GRAVITATION OF BODIES, PENDULUMS, SPECIFIC GRAVITY OF BODIES, STRENGTH, WEIGHT AND CRUSH OF MATERIALS, WATER WHEELS, HYDROSTATICS, HYDRAULICS, STATICS, CENTERS OF PERCUSSION AND GYRATION, FRICTION, HEAT, TABLES OF THE WEIGHT OF METALS, PIPES, SCANTLING, AND INTEREST.

STEAM AND THE STEAM ENGINE.

FIFTH EDITION.—REVISED AND ENLARGED.

BY J. M. SCRIBNER, A.M.
AUTHOR OF ENGINEERS POCKET TABLE-BOOK.

NEW-YORK:
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PREFACE.

In presenting this work to the notice of the public, the author has merely to state, that it had its origin in a desire which he felt to prepare a concise and practical treatise, with a view to facilitate the daily calculations of Engineers, Mill-wrights, and Artisans in general; and by them it is to be hoped it will be duly appreciated, more especially when they consider that the subjects embraced in this small volume—with very few exceptions—have been diffused throughout a number of valuable and extensive works, which are too voluminous to be useful in the field or shop, where they are most needed; and, by reason of their cost, quite out of the reach of the operative mechanic, in general.

Besides this, the greater part of mechanical works have been swelled out to an immoderate degree, by scholastic theorems and theoretical problems, too profound and tedious for the practical man—a defect which it is desirous to obviate, and which, it is believed, has been obviated in this work, by commencing with a system of *decimals*, and carrying on the work, throughout, upon that system.

The Tables, which are numerous, will supersede the necessity of seeking for the various methods of calculations, in the hurry of business; and the accuracy of the results given, can be relied upon with the greatest certainty, as they have been thoroughly tested.

With regard to the subjects embraced in this work, comment is unnecessary, as every mechanic is well aware that one or the other of them is daily required. The utility of such a work as this, no one will question; and he who would not be a "*hewer of wood and drawer of water*" to the more intelligent and well-informed mechanic, must possess, to a considerable extent, the principles and rules embraced in this book; and more especially, if he would be master of his profession, and reputed as one whose skill and decisions can be implicitly relied upon.

The principal authors consulted in preparing it for the press, were Adcock, Templeton, Gregory, Grier, Brunton, "*Library of Useful Knowledge*," M. Morin, of the University of Paris, and "*Ordnance Manual;*" and to the labors of the authors of these valuable works, I freely acknowledge my indebtedness for very much of the valuable matter contained in the following pages. The article on Friction, from the pen of Professor Morin, under the sanction of the French Academy of Arts, will be found invaluable, as it will throw much light upon a subject intimately connected with mechanical arts, and about which little has heretofore been known with any degree of certainty.

In my own efforts, I have been materially assisted in the choice of subjects, and hints about their arrangement, by several distinguished mechanics and civil engineers.

The success which has attended the former edition of this work, is satisfactory proof that the author's labors have been duly appreciated by a discerning public; and this, it may be inferred, has been the case, the first edition having been disposed of in little more than a twelvemonth; and a second edition being called for, the author has been induced thoroughly to revise the whole throughout, and greatly to augment it with an addition of several very important and useful rules and tables, as well as other useful matter. Hence, it is anticipated that with these improvements, the work will be a still more useful companion for mechanics and practical men, in general.

To the mechanic I would say, earnestly endeavor to cultivate your mind, and store it with useful knowledge. Let your leisure hours be devoted to the cultivation of science, and a knowledge of the physical laws, without which, eminence in your profession can never be obtained. And should this little volume facilitate your progress in that praiseworthy employment, the desire of the author will be fully accomplished.

J. M. SCRIBNER.

Rochester, N. Y., September, 1845.

RECOMMENDATIONS.

HAVING examined "Scribner's Engineers' and Mechanics' Companion," we have much pleasure in saying that it comprises a great amount of information, both useful and necessary for Mechanics and Engineers. The Author has sought contributions for this work from numerous sources, and from several rare and valuable treatises, and we cheerfully recommend it as well adapted to the want it is intended to supply, viz., of a convenient and acceptable manual for practical men.

DANIEL MARSH,
WILLIAM WILEY, } *Civil Engineers.*

ROCHESTER, N. Y., Sept., 1845.

We cheerfully subscribe to the above recommendation of "Scribner's Engineers' and Mechanics' Companion."

MATTHEW M. HALL, *Civil Engineer.*
M. W. MASON, *Machinist.*
W. S. HUDSON, *Engine Builder.*
G. W. PERRY. *Machinist.*

Rob't R. Aitken
Trenton

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EXPLANATION OF CHARACTERS.

THERE are various characters or marks used in arithmetical computations, to denote several of the operations and propositions, the chief of which are as follows:

$=$ *Equal to.* The sign of equality; as 100 cents$=$\$1, signifies that 100 cents are equal to one dollar.

$-$ *Minus*, or *Less.* The sign of subtraction; as $8-2=6$; that is, 8, less 2, is equal to 6.

$+$ *Plus*, or *More.* The sign of addition; as $4+5=9$; that is, 4, added to 5, is equal to 9.

$\times$ *Multiplied by.* The sign of multiplication; as $6\times 6=36$; that is, 6, multiplied by 6, is equal to 36.

$\div$ *Divided by.* The sign of division; as $12\div 3=4$; that is, 12, divided by 3, is equal to 4.

: is to
: : so is
: to } The signs of proportion; as $2:4::8:16$; that is, as 2 is to 4, so is 8 to 16.

$7-2+5=10$. Shows that the difference between 7 and 2, added to 5, is equal to 10.

^{2} added to a number, signifies that the number is to be *squared;* thus: 6^2, means that 6 is to be multiplied by 6.

^{3} added to a number, signifies that the number is to be *cubed;* thus: $5^3=5\times 5\times 5=125$. The *index*, or *power*, is the number of times a number is to be multiplied by itself, and is shown by a small figure placed at the right of the number to be raised, and a little elevated.

The *bar* signifies that all the numbers under it are to be taken together; as $\overline{7+4}-3=8$; or, $5\times\overline{6+4}=50$. The parenthesis ( ) is sometimes used in place of the bar.

· A decimal point signifies, when prefixed to a number, that the number has a unit (1) for its denominator; as ·1 is $\frac{1}{10}$; ·125 is $\frac{125}{1000}$, &c.

A, A′, A″, A‴, signifies *A*, *A* prime, *A* seconds, *A* thirds, &c.

$\surd$ Prefixed to any number signifies that the square root of that number is required; as $\sqrt{9}=3$; or thus, $\sqrt{5^2-3^2}=4$, signifies that 3 squared, taken from 5 squared, and the square root extracted, $=4$.

$\sqrt[3]{\ }$ Prefixed to any number signifies that the cube root of that number is required; as $\sqrt[3]{64}=4$.

NOTE. The degree of temperature used in this treatise is Fahrenheit, which fixes upon 32° for the freezing point, and 212° for the boiling point.

SCRIBNER'S

ENGINEERS' AND MECHANICS' COMPANION.

UNITED STATES' WEIGHTS AND MEASURES.

MEASURE OF LENGTH.

3 barley corns	= 1 inch.	40 rods or 220 yds.	= 1 furlong.
12 inches	= 1 foot.	8 furlongs or 1760 yds.	= 1 mile.
3 feet	= 1 yard.	60 geo. miles	= 1 degree.
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ ft.	= 1 rod or pole.		

Ropes and Cables.

6 feet	= 1 fathom.
120 fathoms	= 1 cable's length.

SPECIAL MEASURE OF LENGTH.

Land Measure.

7·92	inches	= 1 link.
100	links or 22 yards	= 1 chain.
80	chains	= 1 mile.
69·121	miles	= 1 geographical degree.

Nautical Measure.

1 nautical mile	=	6082·66	feet.
3 miles	=	1	league.
20 leagues	=	1	degree.
360 degrees	=		the earth's circumference.

NOTE. Bowditch gives 6120 feet in a sea mile, which, if taken as the length, will make the divisions 51 feet and 5 1-10 feet for the knot and fathom.

Pendulums.

6 points	= 1 line.
12 lines	= 1 inch.

CLOTH MEASURE.

$2\frac{1}{4}$ inches	= 1 nail.	3 quarters	= 1 Flemish ell.
4 nails	= 1 quarter.	5 quarters	= 1 English ell.
4 quarters	= 1 yard.	6 quarters	= 1 French ell.

NOTE. The Standard Yard of the State of New York is a brass rod, which bears to a pendulum beating seconds in *vacuo*, in Columbia College, the relation of 1.000.000 to 1.086.141, at a temperature of 32° Fahrenheit; that of the English is 62°.

COMPARATIVE MEASURE OF LENGTH.

3 miles	=	1 league, *marked* lea.
$2\frac{3}{4}$ "	=	1 French league.
$3\frac{2}{3}$ "	=	1 Spanish league.
4 "	=	1 German mile.
$3\frac{1}{4}$ "	=	1 Dutch mile.
$\frac{11}{12}$ mile	=	1 Italian mile.
$\frac{3}{4}$ "	=	1 Russian verst.
$1\frac{3}{22}$ "	=	1 Scotch mile.
$1\frac{3}{11}$ "	=	1 Irish mile.

MEASURE OF SURFACE, OR SQUARE MEASURE.

144	square inches	=	1	square	foot.
9	" feet	=	1	"	yard.
$272\frac{1}{4}$	" feet	=	1	"	rod or pole.
$30\frac{1}{4}$	" yards	=	1	"	pole.
40	" rods	=	1	"	rood.
4	" roods	=	1	"	acre.
640	" acres	=	1	"	mile.

SPECIAL MEASURE OF SURFACE.

For Land.

62·7264	square inches	=	1 square link.
10.000	" links	=	1 " chain.
10	" chains	=	1 acre.

NOTE. By these tables, land measure and artificers' work are computed.

MEASURE OF SOLIDITY, OR CUBIC MEASURE.

1728	cubic inches - - - - -	=	1 cubic foot.
27	" feet - - - - - -	=	1 cubic yard.
50	" " of round timber	=	1 ton.
40	" " of hewn "	=	1 ton.
40	" " of shipping "	=	1 ton.
16	" " - - - - - - - -	=	1 cord foot.
8 cord feet or 128 cubic feet		=	1 cord of wood

NOTE. A cubic foot is equal to 2200 *cylindrical* inches, or 3300 spherical inches, or 6600 conical inches. A cubic foot of water equals $62\frac{1}{2}$ lbs.; a cylindrical foot equals 49·1; a cubic inch equals ·03617 lbs.; a cylindrical inch equals ·02642 lbs.

MEASURE OF CAPACITY, FOR LIQUIDS, FRUITS, &c.

Wine Measure.

4 gills	=	1 pint.
2 pints	=	1 quart.
4 quarts	=	1 gallon.
$31\frac{1}{2}$ gallons	=	1 barrel.
42 gallons	=	1 tierce.
63 "	=	1 hogshead.
2 hogsheads	=	1 pipe or butt.
2 pipes	=	1 tun.

MEASURE OF SOLIDITY.

(*Liquid Measure of the State of New York.*)

6·912 cubic inches	=	1 gill.
27·648 " "	=	1 pint.
55·296 cubic inches	=	1 quart.
221·184 " "	=	1 gallon.

NOTE. By the Revised Statutes of the State of New York, the *wine* gallon is the standard for all liquid measures, and the legislature have enacted that it shall contain 8 lbs. of pure water at its maximum density.

Dry Measure.

2 pints = 1 quart.	8 quarts = 1 peck.
4 quarts = 1 gallon.	4 pecks = 1 bushel.

MEASURE OF SOLIDITY.

8·64 cubic inches = 1 gill.	276·48 cubic inches = 1 gallon
34·56 " " = 1 pint.	552·96 " " = 1 peck.
69·12 " " = 1 quart.	2211·84 " " = 1 bushel.

NOTE. By the statute of the State of New York, the gallon of dry measure shall contain 10 pounds of pure water at its maximum density; the bushel, 80 pounds. These measures have this great advantage in ordinary use, that common pump or spring water, fresh drawn, is sufficiently near the standard density to be employed in regulating them in all cases where scientific accuracy is not required.

IMPERIAL (*British*) MEASURE.

1 quarter of wheat = 8 bushels.
231 cubic inches = 1 Winchester or U. S. gallon.
282 " " = 1 " ale gallon.
2150·42 " " = 1 " or U. S. bushel.
277·274 " " = 1 Imperial gallon, for dry, beer, and wine
2218·192 " " = 1 " bushel.

Winchester	gallon, ×	·9575	=	N. York	gallon.
"	bushel, ×	1·028561	=	"	bushel.
Imperial	gallon, ×	·7977	=	"	gallon.
"	bushel, ×	·99714	=	"	bushel.

NOTE. The Winchester bushel (so called because the standard measures were kept at Winchester) is 18½ inches diameter, and 8 inches deep, and contains 2150·42 cubic inches; but the number is susposed to vary in different states. In Connecticut it is 2198, which is 47·58 cubic inches larger than the Winchester bushel. The statute bushel of the State of New York contains 2211·84 cubic inches, or 80 pounds of pure water at its maximum density, as shown in the table above. Consequently, in determining the capacity of any vessel, in *gallons* or *bushels*, reference must be had to the standards of the state where these tables may be referred to.

MISCELLANEOUS MEASURE.

For Various Purposes.

1 chaldron = 36 bushels, or 57·25 cubic feet.
1 perch of stone = 16½ cubic feet.
1 hand = 4 inches.
1 span = 9 inches.

1 cubic foot of anthracite coal,	weighs from	50 to 55 lbs.
1 " " bituminous coal,	" "	42 " 55 "
1 " " Cumberland coal,	"	53 "
1 " " charcoal, (hard wood)	"	18·5 "
1 " " " (pine)	"	18 "

NOTE. Coals are usually purchased at a conventional rate of 28 bushels (5 pecks) to a ton.

COKE.—This is a fossil coal, charred, or deprived of its bitumen, sulphur, or other extraneous or volatile matter, by fire.

MEASURES OF WEIGHT.—TROY.

By which Gold, Silver, and Precious Stones are weighed.

24 grains = 1 pennyweight.
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

Diamond.

16 parts = 1 grain = 0·8 Troy grain.
4 grains = 1 carat = 3·2 " "

NOTE. An ounce of gold is divided into 24 equal parts, called carats, and an ounce of silver into 20 parts, called pennyweights; therefore, to distinguish fineness of metals, such gold as will abide the fire, without loss, is accounted 24 carats fine; if it lose 2 carats in trial, it is called 22 carats fine, &c. A pound of silver which loses nothing in trial is 12 ounces fine; but if it lose 3 pennyweights, it is 11 ounces 17 pennyweights fine, &c.

AVOIRDUPOIS, OR THE GENERAL MEASURES OF WEIGHT.

$27\frac{11}{32}$ Troy grains = 1 dram.
16 drams = 1 ounce.
16 ounces = 1 pound.
28 pounds = 1 quarter.
4 quarters = 1 cwt.
20 cwt. = 1 ton.

STANDARD POUND OF NEW YORK.

NOTE. Of the two denominations of weight, the troy and avoirdupois, the latter is alone retained, and its unit is defined by declaring that a cubic foot of pure water, at its maximum density, weighs $62\frac{1}{2}$ such pounds, or 1000 ounces, using brass weights, at the mean pressure of the atmosphere at the level of the sea.

1 lb. avoir. = 14 oz. 11 pwt. 16 gr. troy.
1 oz. " = 18 " $5\frac{1}{2}$ " "
1 dr. " = 1 " $3\frac{11}{32}$ " "
7000 troy grains = 1 lb. avoirdupois.
5760 " " = 1 lb. troy.
175 " pounds = 144 lbs. avoirdupois.
175 " ounces = 192 oz. "
$437\frac{1}{2}$ " grains = 1 oz. "
1 " pound = ·8228 lb. "

NOTE. The standard avoirdupois pound of the State of New York is the weight of 27·7015 cubic inches of distilled water, weighed in air, at a maximum density, (39° 83) the barometer being 30 inches.

THE RELATIVE VALUE OF TROY AND AVOIRDUPOIS WEIGHTS.

Troy lb.	1	2	3	4	5	6	7	8	9
Avoir. lb.	0·823	1·646	2·469	3·291	4·114	4·937	5·760	6·583	7·406
Avoir. lb.	1	2	3	4	5	6	7	8	9
Troy lb.	1·215	2·431	3·646	4·861	6·076	7·292	8·507	9·722	10·937

• By comparing the number of grains in the avoirdupois and troy pound and ounce, respectively, it appears that the troy pound is less than the avoirdupois in the proportion of 14 to 17.0138; but the troy ounce is greater than the avoirdupois in the proportion of 79 to 72. Hence, the following approximating rules for changing avoirdupois weight to troy, and troy to avoirdupois, will often be found very convenient and useful.

Avoirdupois	lbs. ×	1·21527	=	troy lbs.
"	ounces ×	·9115	=	" ounces.
Troy	lbs. ×	·823	=	avoir. lbs.
"	ounces ×	1·1	=	" ounces.
"	grains ×	·03657	=	" drams.

APOTHECARIES' WEIGHT.

20 grains = 1 scruple.	8 drams = 1 ounce.
3 scruples = 1 dram.	12 ounces = 1 pound.

NOTE. The apothecaries' pound and ounce, and pound and ounce troy, are the same, only differently divided and subdivided. All weights now used by apothecaries, above grains, are avoirdupois.

TABLE OF PARTICULAR THINGS.

12 particular things	= 1 dozen.
12 dozen	= 1 gross.
12 gross	= 1 great gross.
20 particular things	= 1 score.
24 sheets of paper	= 1 quire.
20 quires	= 1 ream = 480 sheets.

SIZES OF DRAWING PAPER.

Wove Antique,	52 by 31 in.	Elephant,	27¾ by 23¼ in.
Uncle Sam,	48 by 120 "	Imperial,	29 by 21¼
Double Elephant,	40 by 26 "	Super-royal	27 by 19 "
Emperor,	40 by 60 "	Royal,	24 by 19 "
Atlas,	32 by 26 "	Medium,	22 by 18 "
Colombier,	33¾ by 23 "	Demy,	19 by 15½

RELATIVE MINT VALUE OF FOREIGN GOLD COINS.

By Law of Congress, August, 1834.

	COINS.	WEIGHT. pwt.	gr.	VALUE.
BRAZIL,	1 Johannes,	18		$17·068
"	1 Dobraon,	34	12	32·714
"	1 Dobra,	18	06	17·305
"	1 Moidore,	6	22	6·560
"	1 Crusado,	0	16¼	·638
ENGLAND,	1 Guinea,	5	9¼	5·116
"	1 Sovereign,	5	3¼	4·875
FRANCE,	1 Double Louis, (1786)	10	11	9·694
"	1 Double Napoleon,	8	7	7·713
COLUMBIA,	1 Doubloon,	17	8½	15·538
MEXICO,	1 Doubloon,	17	8½	15·538
PORTUGAL,	1 Doubloon,	34	12	32·714
"	1 Dobra,	18	6	17·305
"	1 Johannes,	18		17·068
"	1 Moidore,	6	22	6·560
"	1 Milrea,		19¾	·780
SPAIN,	1 Doubloon, (1772)	17	8½	16·030
"	1 Doubloon, (1801)	17	9	15·538
"	1 Pistole,	4	3½	3·883

23·2 grains of pure gold - - =	$1·00.
United States' Eagle, till 1834, =	10·668.

MINT VALUE OF FOREIGN COINS.

	COINS.		VALUE.
ENGLAND,	1 Shilling, - - - - -	=	$0·244
FRANCE,	5 Francs, - - - - - -	=	0·935
"	1 Sous, - - - - - -	=	0·0093
AUSTRIA,	1 Crown, or rix dollar,	=	0·97
"	1 Ducat, - - - - - -	=	2·22
PRUSSIA,	1 Ducat, - - - - - -	=	2·202
RUSSIA,	1 Ducat = 10 rubles,	=	7·724
"	1 Ruble, - - - - - -	=	0·748
SWEDEN,	1 Ducat, - - - - - -	=	2·19
"	1 Rix dollar, - - - -	=	1·08

AMERICAN STANDARD OF MONEY.

Gold.

	WEIGHT.	
	pwt.	gr.
Eagle, valued at $10, - - - -	11	6.
Half eagle, valued at $5, - -	5	15.
Quarter eagle, valued at $2.50,	2	19½.

Silver.

	WEIGHT.	
	pwt.	gr.
Dollar, - - - - - - - -	17	7
Half dollar, - - - - -	8	16
Quarter dollar, - - - -	4	4
French crown, at $1.18,	18	17

TABLE,

Showing the relative value of French and English Weights and Measures.

DECIMAL SYSTEM.

MEASURES OF LENGTH.

FRENCH.		ENGLISH.
Millimetre, - - - - - -	=	0·03937 inches.
Centimetre, - - - - - -	=	0·39371 inches.
Decimetre, - - - - - -	=	3·93710 inches.
Metre, - - - - - - - -	=	39·37100 inches.
Decametre, - - - - - -	=	32·80916 feet.
Hectometre, - - - - - -	=	328·09167 feet.
Kilometre, - - - - - -	=	1093·6389 yards.
Myriametre - - - - - -	=	10936·38900 yards, or 6 miles, furlong 28 poles, 2⅓ yards.

SUPERFICIAL MEASURES.

FRENCH.		ENGLISH.
Milliare - - - - - - -	=	·1196 square yards.
Centiare, - - - - - - - -	=	1·1960 square yards.
Are, (a square decametre) -	=	119·6046 square yards.
Decare, - - - - - - - -	=	1196·0460 square yards.
Hectare, - - - - - - - -	=	11960·4604 square yards, or 2 acres, 1 rood, 35 perches.

SOLID MEASURES.

Millistere, - - - - - - -	=	·035317 cubic feet.
Centistere, - - - - - - -	=	·35317 cubic feet.
Decistre, - - - - - - - -	=	3·5317 cubic feet.
Stere, (a cubic metre) - -	=	35·3171 cubic feet.
Decastere, - - - - - - -	=	353·1714 cubic feet.

WEIGHTS.

Milligramme, - - - - - -	=	0·0154 grains.
Centigramme, - - - - -	=	0·1544 grains.
Decigramme, - - - - - -	=	1·5444 grains.
Gramme, - - - - - - - -	=	15·4440 grains.
Decagramme, - - - - -	=	154·4402 grains, or 5·64 drams avoirdupois.
Hectogramme, - - - - -	=	3·2154 oz. troy, or 3·527 oz. avoirdupois.
Kilogramme, - - - - - -	=	2 lb. 8 oz. 3 pwt. 2 gr. troy, or 2 lbs. 3 oz. 4·428 drams avoirdupois.
Myriagramme - - - - -	=	26·795 lbs. troy, or 22·0485 lbs. avoirdupois.
Quintal, - - - - - - - -	=	1 cwt. 3 qrs. 25 lbs., nearly.
Millier, or Bar, - - - - -	=	9 tons, 16 cwt. 3 qrs. 12 lbs.

MEASURES OF CAPACITY.

Millitre, - - - - - - - -	=	0·06103 cubic inches.
Centilitre, - - - - - - - -	=	0·61028 cubic inches.
Decilitre, - - - - - - - -	=	6·10280 cubic inches.
Litre, (a cubic decimetre) -	=	61·02802 cubic inches, or 2·1135 wine pints.
Decalitre, - - - - - - - -	=	610·28028 cubic inches, or 2·642 wine gallons.
Hectolitre, - - - - - - -	=	3·5317 cubic ft., or 2·838 Winchester bushels.
Kilolitre, - - - - - - - -	=	35·3171 cubic feet, or 1 tun, 12 wine gallons.
Myrialitre, - - - - - - -	=	353·17146 cubic feet.

USUAL SYSTEM.

MEASURES OF LENGTH.

USUAL.	METRICAL.	ENGLISH.
Ligne, - =	2·31 millimetres, - - =	0·091 inches.
Pouce, - =	2·77 centimetres, - - =	1·090 inches.
Pied, - - =	3·33 decimetres, - - =	13·110 inches.
Aune, - - =	12 decimetres, - - =	3 feet, 11·24 inches.
Toise, - - =	2 metres, - - - =	6 feet, 6·74 inches.

MEASURE OF CAPACITY.

Boisseau, = 12·5 litres, - - - - = 2·837 gallons.

WEIGHTS.

Grain, - - =	5·425 centigrammes, =	0·837 grains.
Gros, - - =	3·906 grammes, - - =	60·285 grains.
Once, - - =	31·25 grammes, - - =	482·312 grains, or 1 oz., 1·628 drams avoir.
Livre, - - =	500 grammes, - - =	1 lb. 4 oz. 1 pwt. 13 gr. troy, or 1 lb. 1 oz. $10\frac{1}{5}$ gr. avoirdupois.

DIVISION OF THE CIRCLE.

100 seconds = 1 minute.
100 minutes = 1 degree.
100 degrees = 1 quadrant.
4 quadrants = 1 circle.

THE ENGLISH DIVISION.

60 seconds = 1 minute
60 minutes = 1 degree.
360 degrees = 1 circle.

NOTE. In the new French system, the values of the base of each measure—viz.: *Metre, Litre, Stere, Are,* and *Gramme*—are increased or decreased by the following words prefixed to them. Thus:

Milli,	expresses the	1000th part.			
Centi,	" "	100th	"		
Deci,	" "	10th	"		
Deca,	"	10 times the value.			
Hecato,	"	100	"	"	"
Chilio,	"	1000	"	"	"
Myrio,	"	10,000	"	"	"

SYNOPSIS OF ARITHMETIC.

The following *synopsis* of several of the rules of arithmetic, often referred to in elementary books on mechanical science, are here inserted for the convenience of reference. These rules and examples are given merely to *refresh the memory*, it being taken for granted that the reader has already acquainted himself with the principles of common arithmetic. They will, however, be found serviceable, both as a convenience of reference, and to give some insight to the subjects on which they treat.

DECIMAL FRACTIONS.

A *decimal fraction* derives its name from the Latin *decem*, "ten," which denotes the nature of its numbers, representing the parts of an integral quantity, divided into a tenfold proportion. It has for its denominator a UNIT, or whole thing, as a gallon, a pound, a yard, &c., and is supposed to be divided into ten equal parts, called tenths; those tenths into ten equal parts, called hundredths, and so on, without end.

The denominator of a decimal being always known to consist of a *unit*, with as many ciphers annexed as the numerator has places, is never expressed, being understood to be 10, 100, 1000, &c., according as the numerator consists of 1, 2, 3, or more figures. Thus: $\frac{2}{10}$ $\frac{24}{100}$ $\frac{125}{1000}$ &c., the numerators only are written with a dot or comma before them, thus ·2 ·24 ·125.

The use of the dot (·) is to separate the decimal from the whole numbers.

The first figure on the right of the decimal point is in the place of *tenths*, the second in the place of *hundredths*, the third in the place of *thousandths*, &c., always decreasing from the left towards the right in a tenfold ratio, as in the following

TABLE.

	&c. &c
5	Tens of Millions.
5	Millions.
5	Hundreds of Thousands.
5	Tens of Thousands.
5	Thousands.
5	Hundreds.
5	Tens.
5	Units.
.	Decimal point.
5	Tenths.
5	Hundredths.
5	Thousandths.
5	Ten Thousandths.
5	Hundred Thousandths.
5	Millionths.
5	Ten Millionths
	&c. &c.

Ascending. Descending.

A cipher placed on the left hand of a decimal, *decreases* its value in a tenfold ratio by removing it farther from the decimal point. But annexing a cipher to any decimal, does not alter its value at all. Thus, 0·4 is ten times the value of 0·04, and a hundred times 0·004. But 0·7=0·70=0·700=0·7000, &c., as above remarked.

0·2	is read		two-tenths.
0·25	"	"	twenty-five hundredths.
0·375	"	"	three hundred and seventy-five thousandth.
0·1876	"	"	one thousand eight hundred and seventy-six ten thousandth, and so on.

Mixed numbers consist of a whole number and a decimal; as 4·25 and 3·875.

ADDITION OF DECIMALS.

Rule.—Arrange the numbers so that the decimal points shall be directly over each other, and then add as in whole numbers, and place the *decimal* point directly below all the other points.

Example.—Add the following *units* and *fractions:*

Hundreds. Tens. UNITS. Tenths Hundredths. THOUSANDTHS. Ten Thousandths. Hundred Thousandths. MILLIONTHS. Ten Millionths.

FRACTIONS.				
$\frac{5}{10}$	is the	same as	·5 -	read 5 Tenths.
$\frac{7}{100}$	"	"	·07 -	read 7 Hundredths.
$\frac{30}{1000}$	"	"	·030 -	read 30 Thousandths.
$\frac{1248}{10000}$	"	"	·1248 -	read 1248 Ten Thousandths.
$8\frac{6}{10}$	"	"	8·6	read 8 and 6 Tenths.
$7\frac{8}{1000000}$		"	7·000008	read 7 and 8 Millionths.
$84\frac{25}{100}$		"	84·25	read 84 and 25 Hundredths.
$5\frac{6}{10000000}$		"	5·0000006 -	read 5 and 6 Ten Millionths.
480			480·	read 480.
			585·5748086·	read 585 and 5748086 ten Millionths.

SUBTRACTION OF DECIMALS.

Rule.—Place the numbers directly under each other, according to their several values, as in addition; then subtract as in whole numbers, and point off the decimals, as in the last rule.

Example.—Subtract 7·75 from 15·125.

15·125
7·75
7·375 remainder.

MULTIPLICATION OF DECIMALS.

Rule.—Place the factors under each other, and multiply them together as in whole numbers; then point off as many figures from the right hand of the product as there are decimals places in both factors, observing, if there be not enough, to annex as many ciphers to the left hand of the product as will supply the deficiency.

Example.—Multiply 3·625 by 2·75

$$3{\cdot}625 \times 2{\cdot}75 = 9{\cdot}96875. \text{ Ans.}$$

DIVISION OF DECIMALS.

Rule.—Prepare the decimal as directed for multiplication; divide as in whole numbers; cut off as many figures for decimals in the quotient as the number of decimals in the dividend exceeds the number in the divisor; and if the places in the quotient be not so many as the rule requires, supply the deficiency by annexing ciphers to the left hand of the quotient.

Example 1.—Divide 173·5425 by 3·75.

3·75)173·5425(46·27.
1500
2354
2250
1042
750
2925
2625
300

Example 2.—Divide 63·50 by 4·25.

4·25)63·50(14·94
425
2100
1700
4000
3825
1750
1700

A TABLE

Of the fractional parts of an inch, when divided into thirty-two parts: likewise, a foot of twelve inches, reduced to decimals.

Parts.	Decimals.	Parts.	Decimals.	Parts of a foot.	Decimals.
$\frac{1}{32}$	= ·03125	$\frac{1}{2}$ & $\frac{1}{32}$	= ·53125	11	= ·9166
$\frac{1}{16}$	= ·0625	$\frac{1}{2}$ & $\frac{1}{16}$	= ·5625	10	= ·8333
$\frac{3}{32}$	= ·09375	$\frac{1}{2}$ & $\frac{3}{32}$	= ·59375	9	= ·75
$\frac{1}{8}$	= ·125	$\frac{5}{8}$	= ·625	8	= ·6666
$\frac{1}{8}$ & $\frac{1}{32}$	= ·15625	$\frac{5}{8}$ & $\frac{1}{32}$	= ·65625	7	= ·5833
$\frac{1}{8}$ & $\frac{1}{16}$	= ·1875	$\frac{5}{8}$ & $\frac{1}{16}$	= ·6875	6	= ·5
$\frac{1}{8}$ & $\frac{3}{32}$	= ·21875	$\frac{5}{8}$ & $\frac{3}{32}$	= ·71875	5	= ·4166
$\frac{1}{4}$	= ·25	$\frac{3}{4}$	= ·75	4	= ·3333
$\frac{1}{4}$ & $\frac{1}{32}$	= ·28125	$\frac{3}{4}$ & $\frac{1}{32}$	= ·78125	3	= ·25
$\frac{5}{16}$	= ·3125	$\frac{3}{4}$ & $\frac{1}{16}$	= ·8125	2	= ·1666
$\frac{3}{8}$	= ·375	$\frac{3}{4}$ & $\frac{3}{32}$	= ·84375	1	= ·0833
$\frac{3}{8}$ & $\frac{1}{32}$	= ·40625	$\frac{7}{8}$	= ·875	$\frac{7}{8}$	= ·07291
$\frac{3}{8}$ & $\frac{1}{16}$	= ·4375	$\frac{7}{8}$ & $\frac{1}{32}$	= ·90625	$\frac{3}{4}$	= ·0625
$\frac{3}{8}$ & $\frac{3}{32}$	= ·46875	$\frac{7}{8}$ & $\frac{1}{16}$	= ·9375	$\frac{5}{8}$	= ·0528
$\frac{1}{2}$	= ·5	$\frac{7}{8}$ & $\frac{3}{32}$	= ·96875	$\frac{1}{2}$	= ·04166
				$\frac{3}{8}$	= ·03125
				$\frac{1}{4}$	= ·02083
				$\frac{1}{8}$	= ·01031

The great utility of the above table will appear evident by means of the following example:

Suppose a plate of copper or brass to be $30\frac{1}{4}$ inches long, $8\frac{5}{8}$ inches broad, and $\frac{3}{8}$ and $\frac{1}{16}$ of an inch in thickness, what is its contents in cubic inches?

$$30{\cdot}25\times8{\cdot}625=260{\cdot}90625\times{\cdot}4375=114{\cdot}146 \text{ cubic inches.}$$

DUODECIMALS.

In decimals, it has already been shown that the figures decrease from the right towards the left, in a tenfold *ratio*.

In *duodecimals*, where the dimensions are taken in *feet*, *inches*, and twelfths of an inch, this decrement follows in a *twelvefold ratio*.

The foot is divided into 12 equal parts, called primes, marked (′); each prime into 12 parts, called seconds, marked (″); and each second into 12 parts, called thirds, marked (‴), &c.

That is, $1' = \frac{1}{12}$ of 1 foot.

$1'' = \frac{1}{12}$ of $\frac{1}{12}$; $= \frac{1}{144}$ of 1 foot.

$1''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12} = \frac{1}{1728}$ of 1 foot, &c.

Feet	× feet	give	feet.
"	× primes	give	primes.
"	× seconds	"	seconds.
Primes	× primes	"	seconds.
Primes	× seconds	"	thirds.
Seconds	× seconds	"	fourths.
Seconds	× thirds	"	fifths, &c.

That is, the product will always be of the denomination indicated by the sum of the accents. Thus, $5'' \times 6''' = 30'''''$.

ADDITION AND SUBTRACTION OF DUODECIMALS.

Rule.—Proceed precisely as in addition and subtraction of other denominate numbers, only *remembering that* 12 *of any denomination make one of the next greater denomination.*

Examples.

```
18f. 4' 9"
16f. 8' 3"               From 21f. 9'  7"
24f. 5' 2"               Take 17f. 3'  9"
----------                    -----------
59f. 6' 2"                     4f. 5' 10"
```

MULTIPLICATION OF DUODECIMALS.

Duodecimals are chiefly used in measuring *surfaces* and *solids*

Rule.—Set down the dimensions to be multiplied, together, one under the other, so that feet may stand under feet, inches under inches, &c. Then begin at the *right* hand, and multiply the several terms of the multiplicand, by the several terms of the multiplier, successively, and set the result of each of the partial products immediately under its multiplier, remembering to carry one for every 12, both in multiplying and adding. The sum of the partial products will be the answer sought.

Examples.

```
  17f. 9'                                  13f.  7'
   5f. 4'                                   3f.  5'
 ---------                               ----------
  5f. 11' 0"                              5f. 7' 11"
 88f.  9'                                40f. 9'
 -----------                             -----------
 94f.  8' 0"=94f.+8/12 of 1 foot.        46f. 4' 11"

   9f. 10'                                  3f. 11'
   7f.  6'                                  9f.  5'
  ---------                              -----------
 4f. 11' 0"                             1f.  7'  7"
68f. 10'                               35f.  3'
-----------                            -----------
73f.  9' 0"                            36f. 10'  7"
```

VULGAR FRACTIONS.

A fraction is a broken number, or one or more parts of a *unit.* Thus: 12 inches are 1 foot. Here 1 *foot* is the unit of measure,

and 12 inches are its parts. 4 inches therefore are one-*third* of a foot, for 4 is the third of 12.

A vulgar fraction consists of *two numbers or terms*, one written above the other, with a line between them.

The number above the line is called the *numerator*, because it shows the number of parts taken.

The number below the line is called the *denominator*, because it shows how many parts the unit is divided into.

Thus $\frac{3}{4}$ is a vulgar fraction, whose numerator is 3 and denominator 4. So also $\frac{5}{8}$ expresses the division of 5 by 8. In the same way $\frac{1}{5}$ indicates that 1 is divided into five equal parts, and is read, *one-fifth of one*.

A proper fraction is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An improper fraction is one whose numerator is equal to or greater than its denominator; as $\frac{4}{3}$, and $\frac{5}{5}$.

A mixed fraction is a compound of a whole number and a fraction, as $5\frac{2}{3}$.

A compound fraction is the fraction of a fraction, connected by the word *of*; as $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{6}{9}$, &c.

A fraction denotes division, and its value is equal to the quotient arising from the division of the numerator by the denominator.

Thus $\frac{16}{4}=4$; $\frac{10}{3}=3\frac{1}{3}$; $\frac{1}{8}=\cdot125$; $\frac{1}{4}=\cdot25$; $\frac{5}{8}=\cdot625$, &c.

REDUCTION OF VULGAR FRACTIONS.

To reduce Fractions to their lowest Terms.

Rule.—Divide both terms by any number that will divide them without a remainder, and the quotient again as before, and so on till no number greater than 1 will divide them.

Examples.—$\frac{544}{1088}\div544=\frac{1}{2}$, and $\frac{72}{288}\div12=\frac{6}{24}=\frac{1}{4}$.

To reduce a Whole Number to an Improper Fraction, having a given Denominator.

Rule.—Multiply the whole number by the given denominator, and place the product over the said denominator.

Example.—Reduce 7 to a fraction whose denominator shall be 11.

$7\times11=\frac{77}{11}$. Answer.

To reduce a Mixed Number to an Improper fraction.

Rule.—Multiply the whole number by the denominator of the fraction; and to the product add the numerator, and place the sum above the denominator.

Example.—Reduce $17\frac{3}{7}$ and $8\frac{5}{8}$ to improper fractions.

$17\frac{3}{7}=\frac{122}{7}$; and $8\frac{5}{8}=\frac{69}{8}$. Answers.

To reduce Compound Fractions to an Equivalent Simple One.

Rule.—Multiply all the numerators together for a numerator, and all the denominators for a denominator.

NOTE.—All the terms that are *common* to the numerator and denominator may be canceled.

Reduce $\frac{1}{2}$ of $\frac{6}{8}$ of $\frac{2}{3}$ of $\frac{3}{6}$ to a simple fraction.

$\frac{1}{2}$ of $\frac{6}{8}$ of $\frac{2}{3}$ of $\frac{3}{6}=\frac{36}{288}=\frac{1}{8}$. Ans.

By canceling the terms that are common to both numerator and denominator, and multiplying together the remaining terms, the answer, $\frac{1}{8}$, will be more easily obtained.

To reduce Fractions of different Denominations to Equivalent Ones, having a Common Denominator.

Rule.—Multiply each numerator by all the denominators, except *its own*, for the new numerators, and all the denominators together for a common denominator.

Example.—Reduce $\frac{2}{3}$ $\frac{1}{2}$ $\frac{3}{4}$ to a common denominator.

$\frac{16}{24}$ $\frac{12}{24}$ $\frac{18}{24}$ $=$ $\frac{46}{24}$ $=$ $1\frac{11}{12}$. Ans.

NOTE.—Before reducing fractions to a common denominator, all mixed numbers, or compound numbers, must first be reduced to a common denominator.

To reduce Complex Fractions to Simple Ones.

Rule.—Reduce the fractions to simple ones; then multiply the numerator of each by the denominator of the other.

Example.—Reduce to a simple fraction $2\frac{7}{8}\div 1\frac{3}{4}$

Thus, $2\frac{7}{8}=\frac{23}{8}$

and $1\frac{3}{4}=\frac{7}{4}$. Then $\frac{23}{8}\times\frac{4}{7}=\frac{92}{56}=\frac{23}{14}$. Answer.

ADDITION OF VULGAR FRACTIONS.

Rule.—Reduce compound fractions to single ones; mixed numbers to improper fractions; and all of them to their least common denominator: then the sum of the numerators, written over the common denominator, will be the sum of the fractions required.

Examples.

1. Add $5\frac{1}{2}$ $\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{7}{8}$ together.

$5\frac{1}{2}$ $=$ $\frac{11}{2}$ and $\frac{2}{3}$ of $\frac{7}{8}$ $=$ $\frac{14}{24}$

Then $\frac{11}{2}$ $\frac{3}{4}$ $\frac{14}{24}$, reduced to their least common denominator, will become $\frac{132}{24}$ $\frac{18}{24}$ $\frac{14}{24}$.

Then $132+18+14=\frac{164}{24}=6\frac{20}{24}$ or $6\frac{5}{6}$. Answer.

SUBTRACTION OF VULGAR FRACTIONS.

Rule.—Prepare the fractions as in Addition; and the difference of the numerators, written above the common denominator, will give the difference of the fraction required.

Examples.—1. From $\frac{3}{4}$ take $\frac{2}{3}$ of $\frac{7}{8}$

$\frac{2}{3}$ of $\frac{7}{8}=\frac{14}{24}=\frac{7}{12}$. Then $\frac{3}{4}$ and $\frac{7}{12}=\frac{9}{12}$ and $\frac{7}{12}$.
Therefore $\frac{9}{12}-\frac{7}{12}=\frac{2}{12}=\frac{1}{6}$. Answer.

2. From $\frac{25}{30}$ take $\frac{4}{7}$. Answer, $\frac{11}{42}$.

MULTIPLICATION OF VULGAR FRACTIONS.

Rule.—Prepare the fractions, as previously required; then multiply all the numerators together for a new numerator, and all the denominators for a new denominator.

Examples.—1. Multiply $\frac{3}{4}$ of 7 by $\frac{4}{5}$

$\frac{3}{4}$ of $7=\frac{21}{4}$ and $\frac{21}{4}\times\frac{4}{5}=\frac{84}{20}=4\frac{1}{5}$. Ans.

2. What is the product of 5 and $\frac{2}{3}$ of 6?

$5\times\frac{12}{3}=\frac{60}{3}=20$. Ans.

DIVISION OF VULGAR FRACTIONS.

Rule.—Prepare the fractions as before; then invert the divisor, and proceed exactly as in multiplication:—The products will be the quotient required.

NOTE.—If either the dividend or divisor is a whole number, it may be converted into an improper fraction, having 1 for its denominator.

Examples.—1. Divide $\frac{5}{7}$ by $\frac{3}{4}$. Thus, $4\times5\div3\times7=\frac{20}{21}$. Ans.

2. Divide $\frac{17}{21}$ by $\frac{3}{5}$. Answer, $1\frac{22}{63}$.

RULE OF THREE.

The RULE OF THREE teaches how to find a fourth proportional term to three given numbers.

The rule of three is either *direct* or *inverse*.

When *more* requires *more*, or *less* requires *less*, it is *direct*. Thus, if 5 barrels of beef cost $30, what will 12 barrels cost? Or, if 30 cubic inches of cast iron weigh 8 lbs., what will 378 cubic inches weigh?

The *proportion* in both of the above cases is *direct*, and the statement must be

As 5 : 30 : : 12 : 4th term =72. Ans.
30 : 8 : : 378 : " =$100\frac{4}{5}$ lbs. Ans.

When more requires *less*, or less requires *more*, the rule is *inverse*. Thus, if 3 men do a certain piece of work in 5 days, in how many days will 4 men do the like quantity? Or, if 12 men build a certain quantity of wall in 28 days, in how many days will 8 men perform the same work?

Here the *proportion* is inverse, and the statement must be

As: 4 : 5 :: 3 : 4th term $=3\frac{3}{4}$. Ans.
8 : 28 :: 12 : " $=42$. Ans.

The product of the *second* and *third* terms, divided by the first, always gives the *fourth* term.

Three numbers are necessary for a statement; and two of these must contain the *supposition*, and the third the demand.

Rule.—Of the three given numbers, place that for the third term which is of the same kind with the answer sought.

Then consider, from the nature of the question, whether the answer will be greater or less than this term. If the answer is to be greater, place the greater of the two numbers for the second term, and the less number for the first term; but if it is to be less, place the less of the two remaining numbers for the second term, and the greater for the first; and in either case multiply the second and third terms together, and divide the product by the first for the answer, which will always be of the same denomination as the third term.

NOTE.—If the first and second terms contain different denominations, they must both be reduced to the same denomination; and compound numbers to INTEGERS of the lowest denomination contained in it.

Example 1.—If 40 tons of iron cost \$450, what will 130 tons cost?

TONS. DOLLS. TONS.
40 : 450 :: 130
130
13500
450
4|0) 5850|0
1462·5 dollars. Ans.

Example 2.—If ten cubic feet of marble weigh 1700 lbs., what will $27\frac{1}{2}$ cubic feet weigh? Ans. 4675.

INVOLUTION.

Involution is the product arising from multiplying one number into itself a certain number of times. The products obtained are called *powers*. The number is called the *root* or first a power. When number is multiplied by itself, the result is called the *second power* or *square* of that number. Thus, $3\times3=9$, the number 9 is the *square* or *second power* of 3. When multiplied twice, *the cube;* three times, the *biquadrate*, &c. To denote that a number is to be raised to a power, a small figure is placed above, and a little to the right of the number whose power is to be found. This small figure is called the *index* or *exponent*.

5 is the root, or first power.
$5^2=5\times5=25$ is the square, or 2d power.
$5^3=5\times5\times5=125$ is the cube, or 3d power.
$5^4=5\times5\times5\times5=625$ is the biquadrate, or 4th power, &c.

For finding the *square* or *cube* of any number less than 600, use the Tables of Squares and Cubes. These are *perfectly* accurate, and will be found exceedingly convenient, especially in hasty calculations.

EVOLUTION.

Evolution is the opposite of *involution*, already explained; that is, it is the finding of the *root* from having the power given. Thus, the *second* or *square* root of 64 is 8, because the square of 8 is 64, and the *cube* root of 64 is 4, because $4\times4\times4=64$, &c.

The sign $\sqrt{}$ placed before a number indicates that the square root of that number is to be found. The same symbol or character expresses any other root by placing the index above it.

Thus $\sqrt{25}=5$ and $5+7=\sqrt{144}$
" $\sqrt[3]{27}=3$ " $3+1=\sqrt[3]{64}$

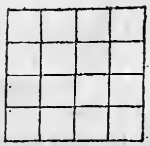
Rule.—1. Separate the given number into periods of two figures each, by placing a point over the right-hand figure, and then over every second figure, counting to the left. These *periods* show the number of figures the *root* will consist of.

2. Find the greatest square number in the left-hand period, and place the root of it at the right hand of the given number, after the manner of a quotient in division. Subtract the square of this root from the said period, and to the remainder bring down the next period for a new dividend.

3. Double the quotient already found, and place it at the left of the dividend for a *divisor*. See how many times this divisor is contained in the *new* dividend, (*except the right-hand figure*,) and place the result in the root for the second figure of it, and likewise on the right hand of the *divisor*. Multiply the divisor by the *last* quotient figure, and subtract the product from the dividend, and to the remainder join the next period for a new dividend.

4. Double the quotient already found for a *partial divisor*, and from these find the next figure in the root, as last directed, and continue the operation in the same manner until all the periods have been annexed.

Examples.



Example 1.—What is the length of each side of a square tract of land, containing 16 square miles; that is, what is the square root of 16?

$\sqrt{16}=4$ miles. Ans.

NOTE. Four square miles may be represented by any four of the small squares in the above figure, while 4 miles square comprises the whole figure.

2. Required the square root of 42025.

```
      . . .
      42025(205
      4
     ------
  405)2025
      2025
      ----
         0
```

3. Required the square root of 470596.

```
                     . . .
                     470596(686
                     36
                    ----
1st divisor      128)1105
                     1024
                    ------
2d divisor      1366)8196
                     8196
                     ----
```

For the extraction of the cube root, *see Tables of Square and Cube Roots.*

ARITHMETICAL PROGRESSION.

This subject is often referred to in elementary books on mechanical science ; and for this reason, we shall draw the attention of the reader for a little while to it.

ARITHMETICAL PROGRESSION is a series of numbers which succeed each other regularly, *increasing* or *diminishing* by a constant number or *common difference :*

As 1, 3, 5, 7, 9, &c. } increasing series.
15, 12, 9, 6, 3, &c. } decreasing series.

The numbers which form the series are called *terms.* The first and the last term are called the *extremes*, and the others are called the *means.*

In arithmetical progression, there are five things to be considered, viz. :

1, The first term.
2, The last term.
3, The common difference.
4, The number of terms.
5, The sum of all the terms.

These quantities are so related to each other, that when any three of them are given, the remaining two can be found.

Given the First Term, the Common Difference, and the Number of Terms, to find the Last Term.

Rule.—Multiply the number of terms, *less one*, by the common difference, and to the product add the first term.

Example.—What is the 20th term of the arithmetical progression, whose first term is one, the common difference $\frac{1}{2}$?

$$20-1=19 \text{ and } 19\times\tfrac{1}{2}=9\tfrac{1}{2}\text{; and } 9\tfrac{1}{2}+1=10\tfrac{1}{2}. \text{ Ans.}$$

Given the Number of Terms and the Extremes, to find the Common Difference.

Rule.—Divide the difference of the extremes by one less than the number of terms.

Example.—The extremes are 3 and 29, and the number of terms 14, required the common difference.

$$29-3=26\text{; and } 26\div13=2. \text{ Ans.}$$

Given the Common Difference and the Extremes, to find the Number of Terms.

Rule.—Divide the difference of the extremes by the common difference, and to the quotient add *one.*

Example.—The first term of an arithmetical progression is 11, the last term 88, and the common difference 7. What is the number of terms?

$$88-11=77\text{; and } 77\div7=11\text{; } 11+1=12. \text{ Ans.}$$

Given the Extremes and the Number of Terms, to find the Sum of all the Terms

Rule.—Multiply half the sum of the extremes by the number of terms.

Example.—How many times does the hammer of a clock strike in 12 hours?

$$1+12=13= \text{the sum of the extremes.}$$
$$\text{Then } 12\times(13\div2)=78. \text{ Ans.}$$

The following formula will be found much shorter, and very convenient. Let a be the first term; d the common difference; n the number of terms; l the nth term; and s the sum of nth terms.

Thus: $l=a+d(n-1)$; $s=\frac{n}{2}(a+b)$.

GEOMETRICAL PROGRESSION.

Geometrical Progression is any series of numbers which succeed each other regularly by a constant *multiplier*, or *decrease* by a constant *divisor*. The constant multiplier or divisor is called the ratio.

As, 1, 3, 9, 27, 81, &c., is an ascending geometrical progression, whose *ratio* is 3; and 15, $7\frac{1}{2}$, $3\frac{3}{4}$, is a descending geometrical progression, whose ratio is $\frac{1}{2}$.

In geometrical progression, as in arithmetical progression, when any three of the following parts are given, the remaining two can be found, viz.: The *first* term, the *last* term, the *number* of terms, the ratio, and the *sum* of *all the terms.*

Given the Ratio, the Number of Terms, and the First Term, to find the Last Term.

Rule.—1. Write some of the leading powers of the *ratio*, and over them place their several *indices*, beginning with a cipher.

2. Add together the most convenient indices to make an index less by 1 than the number of terms sought.

3. Multiply together the powers or terms of the series standing under those indices; and their product, multiplied by the first term, will be the answer sought.

Example.—The first term of a geometrical series is 4, the ratio 3 and the number of terms 11. What is the last term?

$$\left.\begin{matrix} 1, & 2, & 3, & 4, & 5. \\ 3, & 9, & 27, & 81, & 243. \end{matrix}\right\}$$

The indices $5+3+2=10$.

Then, $9\times27\times243=59049$, which, multiplied by the first term $4=236196$, the last term required.

Given the Ratio, the First Term, and the Number of Terms, to find the Sum of all the Terms.

Rule.—Raise the ratio to a power whose index is equal to the number of terms, and from this power subtract 1; divide the remainder by the ratio, less 1, and multiply the quotient by the first term for the answer.

Example.—The first term of a geometrical progression is 2, the ratio 3, and the number of terms 6. What is the sum of all the terms?

$$3^6=729 \text{ and } -1=728$$
$$728\div(3-1)\times2=728. \text{ Ans.}$$

PERMUTATION.

Permutation is the method for ascertaining how many different ways any given number of persons or things may be varied in their positions.

Rule.—Multiply all the terms of the natural series continually together, and the last product will be the number of changes required

Example.—How many changes may be made by 8 scholars in seating themselves differently at their recitations?

1 2 3 4 5 6 7 8,

and $1\times2\times3\times4\times5\times6\times7\times8=40320$ times. Ans.

MENSURATION.

MENSURATION OF SUPERFICIES.

SECTION I.

Mensuration is that branch of mathematics by which we ascertain the contents or superficial areas, and the extension, solidities, and capacities of bodies.

The *area*, or superficial contents of any figure, is the measure of its surface, or the space contained within the bounds of that surface, without any regard to thickness.

In calculating the area, or the contents of any plane figure, some particular portion of surface is fixed upon as the *measuring unit*, with which the figure is to be compared.

This is commonly a *square*, the side of which is the unit of length, being an *inch*, or a *foot*, or a *yard*, or any other fixed quantity, according to the measure peculiar to different artists; and the area or contents of any figure is computed by the number of those squares contained in that figure.

For the same reason, determining the quantity of surface in a figure is called *squaring it;* that is, determining the square or number of squares to which it is equal.

In order to form correct estimates of the extent of surfaces and solids, various rules have been adopted, most of which, the most valuable and useful in practice, will be found accompanying their respective problems in the following treatise, and with which the mechanic may speedily perform all the calculations that ordinarily occur in the practical details of his business.

DEFINITIONS.

The following definitions, which are similar in substance to those found in Euclid, are here inserted for the convenience of reference.

I. *Four-sided* figures are variously named, according to their relative position and length of their sides.

1. A *line* is length, without breadth or thickness.

2. *Parallel lines* are always at the same perpendicular distance and they never meet, though ever so far produced.

3 An *angle* is the inclination or opening of two lines, having different directions, and meeting in a point.

4. A *parallelogram* has its opposite sides parallel and equal.

5. A *rectangle*, or *right parallelogram*, has its opposite sides equal, and all its angles right angles.

6. A *square* is a figure whose sides are of equal length, and all its angles right angles.

7. A *rhomboid* has its opposite sides equal, and its angles oblique

8. A *rhombus* is an equilateral rhomboid, having all its sides equal, but its angles oblique.

9. A *trapezoid* is a quadrilateral figure, having only two of its sides parallel.

10. A *trapezium* is an irregular figure, of four unequal sides and angles.

II. When figures have more than four sides, they are classed under the head of *Polygons*.

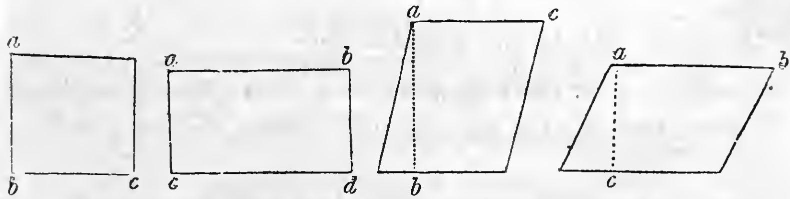
These again are either regular or irregular, according as their sides and angles are equal or unequal, and they are named from their number of sides or angles. Thus, a regular polygon has all its sides and angles equal.

A pentagon	has	five	sides
A hexagon	"	six	"
A heptagon	"	seven	"
An octagon	"	eight	"
A nonagon	"	nine	"
A decagon	"	ten	"
An undecagon	"	eleven	"
A dodecagon	"	twelve	"

III. A figure of three sides and angles is called a *triangle*, and receives particular denominations from the relations of its sides and angles.

1. An equilateral triangle is that whose three sides are equal.
2 The *height* of a triangle is the length of a perpendicular drawn from one of the angles to the opposite side.
3. An *isosceles triangle* is that which has only two sides equal.
4 The *height* of a four-sided figure is the perpendicular distance between two of its parallel sides.

OF FOUR-SIDED FIGURES.



PROBLEM I.

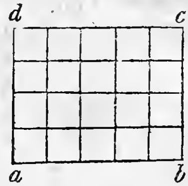
To find the Area of a Four-sided Figure, whether it be a parallelogram, square, rhombus, or rhomboid.

Rule.—Multiply the length by the breadth or perpendicular height, and the product will be the area.

Example.—What is the area of a parallelogram, *a b c d*, whose length, *c d*, is 12 feet 3 inches, and whose breadth, *a c*, is 8 feet 6 inches?

BY DECIMALS.	BY DUODECIMALS.
Feet.	Feet.
12·25	12·3′
8·50	8·6′
61250	6· 1′ 6″
9800	98· 0′
104·1250 feet. Ans.	104· 1′ 6″. Ans.

NOTE. The fundamental problem, in the mensuration of superficies, is the very simple one of determining the area of a *right parallelogram.* The contents of other figures may readily be obtained by finding parallelograms which are equal to them



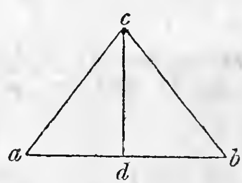
Take any parallelogram, *a b c d*, and divide each of its sides, respectively, into as many equal parts as are expressed by the number of times they contain the linear measuring unit, and let all the opposite points of division be connected by right lines. Then it is evident that these lines divide the parallelogram into a number of squares, each equal to the superficial measuring unit, and that the number of these squares, or the area of the figure, is equal to the number of linear measuring units in the length, repeated as often as there are linear measuring units in the breadth or height; that is, equal to the length multiplied by the height, *which is the rule.*

OF TRIANGLES.

PROBLEM II.

To find the Area of a Triangle.

Rule.—Multiply the length of one of the sides by the perpendicular falling upon it, and half the product will be the area. Or multiply half the side by the perpendicular.



Example.—What is the area of a triangle whose base, *a b*, is 18 feet 4 inches, and height, *c d*, 11 feet 10 inches?

$18{\cdot}4 \times 11{\cdot}10 \div 2 = 108$ feet $5\frac{2}{3}$ inches.

Example 2.—How many square rods of land are there in a lot which is laid out in a right-angled triangle, the base measuring 19 rods, and the perpendicular breadth 15 rods? Ans. 142·5.

CASE II.

To find the Area of a Triangle from the length of its sides.

Rule.—1. Add together the lengths of the three sides, and take half their sum.

2. From this half sum subtract each side separately.

3. Multiply together the half sum and each of the three remain-

ders, and extract the square root of the product; the quotient will be the required area of the triangle.

Example.—If the sides of a triangle are 134,108 and 80 rods, what is the area?

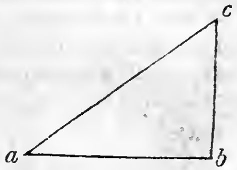
134	161	161	161
108	134	108	80
80	27 1st rem.	53 2d rem.	81 3d rem.
$\overline{322} \div 2 = 161$ half sum.			

Then, to obtain the products, we have $161 \times 27 \times 53 \times 81 = 18661671$: from which we find area $= \sqrt{18661671} = 4319$ square rods.

To find the Hypotenuse of a Right-angled Triangle, when the base and perpendicular are known.

1. Square each of the sides separately.
2. Add together these squares.
3. Extract the square root of the sum, which will be the hypotenuse.

Example.—The wall of a building, *b c*, on the bank of a river, *a b*, is 120 feet high, and the breadth of the river 210 feet: what is the length of a line, *a c*, which will reach from the top of the wall to the opposite bank of the river?

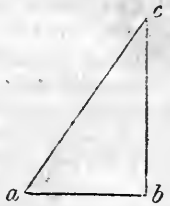


$$\overline{120}^2 \times \overline{210}^2 = 58500 \text{ and } \sqrt{58500} = 241{\cdot}86 \text{ ft. Ans.}$$

To find one of the Legs when the Hypotenuse and the other Leg are known.

Rule.—Substract the square of the leg whose length is known, from the square of the hypotenuse, and the square root of their difference will be the answer.

Example.—The hypotenuse, *a c*, of a triangle is 53 yards, and the perpendicular, *b c*, 45 yards: what is the length of the base, *a b*?



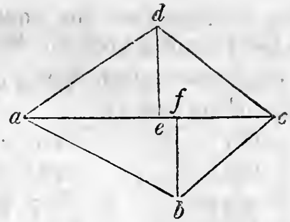
$$\overline{53}^2 - \overline{45}^2 = 784, \text{ and } \sqrt{783} = 28 \text{ yds.} \quad \text{Ans. 28 yds.}$$

OF TRAPEZIUMS AND TRAPEZOIDS.

PROBLEM III.

To find the Area of a Trapezium.

Rule.—Divide the trapezium into triangles by drawing diagonals; and the sum of the areas of these triangles will be the area of the trapezium.



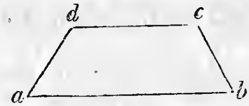
Example.—What is the area of a trapezium whose diagonal, *a c*, is 42 feet, and the two perpendiculars, *d e* and *b f*, 18 and 16 feet?

$$\left.\begin{array}{l}42\times 9=378\\ 42\times 8=336\end{array}\right\}=714\ sq.\ ft.\ \text{Ans.}$$

PROBLEM IV.

To find the Area of a Trapezoid.

Rule.—Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product will be the area.



Example 1.—Required the area of the trapezoid, *a b c d*, having given *a b* =321·51 feet, *d c*=214·24 feet, and whose height is 171·16 feet.

We first find the sum of the sides, and then multiply it by the perpendicular height; after which, we divide the product by 2 for the area.

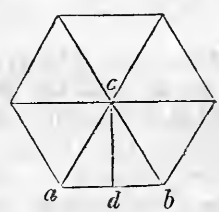
321·51+214·24=535·75=the sum of the parallel sides.
Then,
535·75×171·16=91698·97.
And,
91698·97÷2=45849·485. Ans

OF REGULAR POLYGONS.

PROBLEM V.

To find the Area of a Regular Polygon, or any regular figure.

Rule 1.—Multiply one of its sides into half its perpendicular distance from the centre, and this product into the number of sides.



It is evident, on inspection, that a regular polygon contains as many equal triangles as the figure has sides.

Thus, the adjoining hexagon has six triangles, each equal to *a b c*. Now, the area of *a b c* is equal to the product of the side *a b* into $\frac{1}{2}$ of *c d*. The area of the whole, therefore, is equal to this product multiplied into the *number* of sides.

Example.—1. Required the area of a regular hexagon, each of whose sides, *a b*, &c., is 45 feet, and the perpendicular, *c d*, 24 feet.

We first multiply one side by $\frac{1}{2}$ of the perpendicular, *c d*, and that product by the number of sides: this gives the area.

48×12×6=3240 *ft.* Ans

To facilitate the measurement of polygons, the following table is constructed, showing the multipliers of the ten regular polygons, when the sides of each are equal to 1:

No. of sides.	Name of Polygon.	Angle.	Angle of Polygon	Area, or Multipliers.	A	B	C
3	Triangle,	120°	60°	0·433012	2·	1·732	·5773
4	Square,	90	90	1·	1·41	1·414	·7071
5	Pentagon,	72	108	1·720477	1·238	1·175	·8506
6	Hexagon,	60	120	2·598076	1·156	=Radius.	Lgth of side.
7	Heptagon,	$51\frac{3}{7}$	$128\frac{4}{7}$	3·633912	1·11	·8677	1·152
8	Octagon,	45	135	4·828427	1·08	·7653	1·3065
9	Nonagon,	40	140	6·181824	1·06	·6840	1·4619
10	Decagon,	36	144	7·694208	1·05	·6180	1·6180
11	Undecagon,	$32\frac{8}{11}$	$147\frac{3}{4}$	9·365640	1·04	·5634	1·7747
12	Dodecogon,	30	150	11·196152	1·037	·5176	1·9318

Now, since the areas of similar polygons are to each other as the squares of their homologous sides, if the square of a side of a polygon be multiplied by the multiplier of the like figure, the product will be the area sought. And hence we have,

$$1^2 : \text{tabular area} : : \text{any side squared} : \text{area}.$$

To find the Area of a Regular Polygon, when the side only is given.

Rule.—Multiply the square of the side by the multiplier opposite the name of the polygon in the above table, and the product will be the area.

Example.—What is the area of a regular decagon whose side is 87 feet?

$$87^2 \times 7·694208 = 58237·46. \text{ Ans.}$$

ADDITIONAL USE OF THE ABOVE TABLE.

The third and fourth columns of the table will greatly facilitate the construction of those figures with the aid of the sector. Thus, if it is required to describe an *octagon*, opposite to it, in the third column, is 45 ; then with the chord of 60 on the sector as radius, describe a circle, taking the length 45 on the same line of the sector : mark this distance off on the circumference, which, being repeated around the circle, will give the points of the side.

The fourth column gives the angle which any two adjoining sides of the respective figures make with each other.

Take the length of a perpendicular drawn from the centre of one of the sides of a polygon, and multiply this by the numbers in column A ; the product will be the radius of the circle that contains the figure.

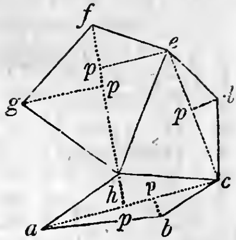
The radius of a circle, multiplied by the number in column B, will give the length of the side of the corresponding figure which that circle will contain. The length of the side of a polygon, multiplied by the corresponding number in the column C, will give the radius of the circumscribing circle.

OF IRREGULAR BODIES.

To find the Area of an Irregular Polygon.

Rule.—Draw diagonals to divide the figure into trapeziums and triangles ; find the area of each separately, and the sum of the whole will give the area required.

What is the area of the adjoining polygon, $a\ b\ c\ d\ e\ f\ g\ h$?



Let $a\,c=20$ rods.
" $b\,p=4$ "
" $a\,c=20$ "
" $h\,p=6$ "
" $c\,e=25$ "
" $d\,p=3$ "
" $f\,h=28$ "
" $g\,p=7$ "
" $f\,h=28$ "
" $e\,p=8$ "
" $h\,c\,e=25$ " each. 618·8 sq. rods. Ans.

NOTE. The triangle, $h\ c\ e$, is solved by Problem II., Case II.

PROBLEM VI.

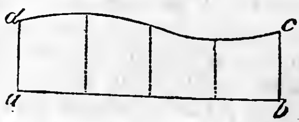
To find the Area of a long Irregular Figure, bounded on one side by a straight line.

Rule.—1. Measure the breadth in several places, and at equal distances from each other.

2. Add together all the different breadths, and half the sum of the two extremes.

3. Multiply this sum by the base line, and divide the product by the number of equal parts of the base.

Example.—1. The breadths of an irregular figure, $a\ b\ c\ d$, at five equidistant places, being 8·2, 7·4, 9·2, 10·2, 8·6, and the whole length 39, required the area.




```
     8·2
     8·6
 2)16·8=sum of extremes.
     8·4=mean of extremes.
     7·4
     9·2
    10·2
    35·2 sum.
```

```
  35·2=sum.
    39
  3168
 1056
4)13728
  3432. Ans.
```

2. The length of an irregular figure being 84, and the breadths at six equidistant places, 17·4, 20·6, 14·2, 16·5, 20·1, 24·4, what is the area? 1550·64. Ans.

NOTE. If the perpendiculars or breadths be not at equal distances, add them together, and divide their sum by the number of them, for the mean breadth; then multiply the mean breadth by the length, and the product will be the whole area not far from the truth.

SECTION II.

OF THE CIRCLE AND ITS PARTS.

DEFINITIONS.

1. A *circle* is a plane figure, bounded by a curve line, called the circumference, every part of which is equally distant from a certain point within, called the center.

2. A *diameter* of a circle is a straight line, passing through the center, and terminating at the circumference.

3. A *radius* or *semi-diameter* is a straight line, extending from the center to the circumference.

4. A *semi-circle* is one half of the circumference.

5. A *quadrant* is one quarter of the circumference.

6. An *arc* is any portion of the circumference.

7. A *chord* is a straight line, which joins the two extremes of an arc.

8. A *circular segment* is the space contained between an arc and its chord. The chord is sometimes called the *base* of the segment. The *height* of the segment is the perpendicular from the middle of the base to the arc.

9. A *circular sector* is the space contained between an arc and the two radii, drawn from the extremes of the arc.

10. A *circular zone* is the space contained between two parallel chords which form its bases.

11. A *circular ring* is the space between the circumferences of two concentric circles.

12. A *lune* or *crescent* is the space between two circular arcs, which intersect each other.

13. An *ellipse* or *oval* is a curve line, which returns into itself like a circle, but has two diameters of unequal length, the longest of which is called the transverse, and the shortest the conjugate axis.

PROBLEM I.

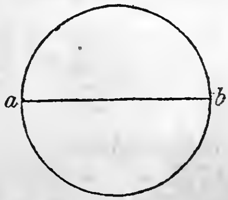
To find the Circumference of a Circle when the Diameter is given.

Rule.—Multiply the diameter by 3·1416, and the product will be the circumference. Or, multiply the diameter by 22, and divide the product by 7. Or, multiply the diameter by 355, and divide the product by 113.

NOTE.—The latter rule is a little more accurate than any other expressed in small numbers.

Example.—1. What is the circumference of a circle whose diameter, a b, is 40 feet?

$40 \times 3{\cdot}1416 = 125{\cdot}66$. Ans.



Example.—2. Required the circumference of a circle whose diameter is $73\frac{3}{4}$.

Ans. 231·6922.

NOTE. See Table of Circumferences of Circles.

PROBLEM II.

To find the Diameter of a Circle when the Circumference is given.

Rule.—Divide the circumference by 3·1416, and the quotient will be the diameter. Or, multiply the circumference by 7, and divide the product by 22.

Example.—The circumference of a circle is 69·115 yards: what is the diameter?

$$69{\cdot}115 \div 3{\cdot}1416 = 22 \text{ yards.}$$

The same result may be obtained more conveniently, by exchanging the *divisor*, 3·1416, for a *multiplier*, which will give the same answer, for, in the proportion 3·1416 : 1 : : Circ. : Diam., the fourth term may be directly found by dividing the second by the first, and multiplying the quotient into the third. Thus, 1÷3·1416 =0·31831. Therefore, if the circumference of any circle be *multiplied* by the decimal ·31831, the product will be the diameter.

In many cases there will be a decided saving of labor by exchanging the *divisor* for a *multiplier*, as will be seen in the following example:

Example.—What is the diameter of a circle whose circumference is 50?

$$50 \times {\cdot}31831 = 15{\cdot}91550.$$

Note.—As multiplication is more easily performed than division, this last method is decidedly the more preferable.

PROBLEM III.

To find the Area of a Circle when the Diameter and Circumference are both known.

Rule.—Multiply the square of the diameter by ·7854. Or, the square of the circumference by ·07958. Or, multiply the circumference by the diameter, and divide the product by 4; in either case the product will be the area.

Example.—1. Required the number of square inches in a piston whose diameter is $12\frac{1}{2}$ inches.

$\overline{12\frac{1}{2}}^2 = 12{\cdot}5 \times 12{\cdot}5 = 156{\cdot}25$, and $156{\cdot}25 \times {\cdot}7854 = 122{\cdot}71$ sq. in. Ans.

2. The piston of the railroad engine Boston is 15 inches diameter: how many square inches does it contain? 176·71. Ans.

Note.—The reason of this rule will appear by considering that if the circumference of a circle be 1, the diameter will=0·31831 (Prob. II.), and $\frac{1}{4}$ of this diameter into the circumference is 0·7958=area. (See Table of Areas of Circles.)

PROBLEM IV.

I. *To find the Length of an Arc of a Circle, when either the number of degrees which it contains, or the radius, chord, and height are given.*

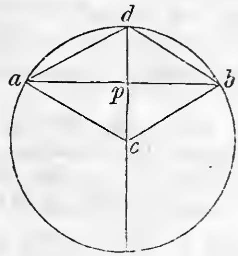
Rule.—Multiply the number of degrees in the arc by the decimal ·01745, and that product by the radius of the circle. Or, from 8 times the chord of half the arc, subtract the chord of the whole

arc, and $\frac{1}{3}$ of the remainder will be the length of the arc, nearly. Or, as 3 is to the number of degrees in the arc, so is ·05236 times the radius to its length.

Example.—1. What is the length of an arc of 40 degrees, in a circle whose radius, $a\,c$, is 12 feet?

$$·0745\times 40\times 12=8·376=\text{length of the arc.}$$

2. What is the length of an arc whose chord, $a\,b$, is 120, and whose height, $p\,d$, is 45?



$120\div 2=60=\frac{1}{2}$ chord of the arc.

And $\overline{60}^2=3600$

" $\overline{45}^2=2025$

$\overline{5625}$=sum of the squares.

Then $\sqrt{5625}=75$=chord of $\frac{1}{2}$ the arc.

And $75\times 8—120\div 3=160$. Ans.

Note. The chord of half the arc is equal to the square root of the sum of the squares of the height and half the chord of the whole arc.

II. *When the Chord of the Arc and the Chord of half the Arc are given.*

Rule.—From the square of the chord of half the arc subtract the square of half the chord of the entire arc; the remainder will be the square of the versed sine. Then proceed as before.

Note. The square root of the sum of the squares of the versed sine or height, and half the chord of the entire arc is equal to the chord of half the arc.

III. *When the Diameter and the Versed Sine of half the Arc are given.*

Rule.—From 60 times the diameter subtract 27 times the versed sine, and *reserve* the number. Multiply the diameter by the versed sine, and the square root of the product will be the *chord* of half the arc. Multiply twice the chord of half the arc by 10 times the versed sine, divide the product by the *reserved number*, and add the quotient to twice the chord of half the arc; the sum will be the length of the arc, very nearly.

Table *of the relative proportions of the Circle, its Equal and Inscribed Squares.*

1.	The diameter of a circle	× ·8862	} = side of an equal square.	
2.	" circumference "	× ·2821	}	
3.	" diameter "	× ·7071	} = side of an inscribed sq.	
4.	" circumference "	× ·2251	}	
5.	" arc "	× ·6366	= contents of inscribed sq.	
6.	" side of inscribed square	× 1·4142	= diam. circumscrib'g cir.	
7.	" side of inscribed square	× 4·443	= circum. circumscrib'g cir.	
8.	" side of a square	× 1·128	= diam. of an equal circle.	
9.	" side of a square	× 3·545	= circum. of an equal sq.	

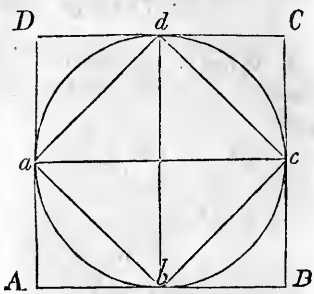
PROBLEM V.

To find the Side of a Square inscribed in a Circle, from its Circumference o Diameter.

Rule.—Multiply the diameter by ·7071= the side of the inscribed square. Or, multiply the circumference by ·2251 = side of the inscribed square.

Example.—1. The circumference of a circle is 68 inches: what is the side of the inscribed square?

$68 \times \cdot 2251 = 15\cdot 30$ inches. Ans.



2. The diameter of a tree is $37\frac{1}{2}$ inches at the small end: what is the measure of the side of the greatest square which can be sawed from it?

$37\cdot 5 \times \cdot 7071 = 26\cdot 51$ inches. Ans.

NOTE. The *area* of a circle is to the area of the *circumscribed square* as ·7854 is to 1, and to that of the *inscribed square* as ·7854 is to $\frac{1}{2}$. If the reader will examine the above figure, he will see that the square, *A B C D*, which is circumscribed about the circle, is equal to the square of the diameter of the circle, since the diameter, *a c*, equals the side *A B*, and *A B* squared gives the area of the square *A B C D*; also, that the inscribed square, *a b c d*, is just $\frac{1}{2}$ of the circumscribed square. Since each of the triangles into which the inscribed square is divided is precisely half of each of the four squares, into which the circumscribed square, *A B C D*, is divided. That is, the inscribed square contains only 4 right-angled triangles; while the circumscribed square contains 8. Consequently, the scuare described within a circle is precisely half of the square described without it.

PROBLEM VI.

To find the Area of a Sector of a Circle.

Rule.—1. Find the length of the arc by problem vii.

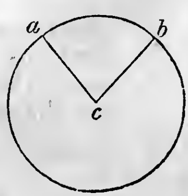
2. Multiply the length of the arc thus found, by half the length of the radius, and the product will be the area.

Or, as 360 degrees is to the number of degrees in the arc of the sector, so is the area of the cirle to the area of the sector.

NOTE. If the diameter of radius is not given, add the square of half the chord of the arc to the square of the versed sine of half the arc, and divide the sum by the versed sine; the quotient will be the diameter.

It is manifest that the area of the sector has the same ratio to the area of the circle which the number of *degrees* in the arc has to the number of degrees in the whole circumference; and the rule for finding the area of the sector, is the same as that for finding the area of the whole circle.

Example.—What is the area of a sector of a circle, *a c b*, in which the radius, *a c*, is 25 and the arc of 26 degrees?



By problem vii. Rule 3.

As, $3 : 26 :: 25 \times \cdot 05236 : 11\cdot 344$; and $11\cdot 344 \times 12\frac{1}{2} = 141\cdot 8$. Ans.

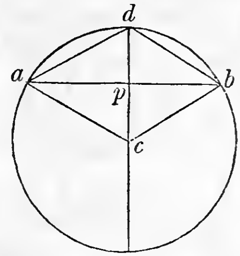
PROBLEM VII.

To find the Area of the Segment of a Circle.

Rule.—1. To the chord of the whole arc add $\frac{4}{3}$ of the chord of half the arc.

2. Then multiply the sum by the versed sine, or height of the segment, and $\frac{4}{10}$ of the product will be the area of the segment, very nearly.

3. Divide the height or versed sine by the diameter of the circle, and find the quotient in the column of versed sines. (See table.) Then take out the corresponding area in the next column on the right hand, and multiply it by the square of the diameter for the answer.



Example.—1. Required the area of a circular segment whose chord, $a\,b$,$=24$, and whose radius, $c\,a$,$=20$ feet?

$\overline{c\,a}^2-\overline{a\,p}^2=\overline{c\,p}^2=\sqrt{400-144}=16=c\,p.$
$c\,d-c\,p=d\,p=20-16=4=$ height of segment.
$\overline{a\,p}^2+\overline{p\,d}^2=\overline{a\,d}^2=\sqrt{144+16}=12{\cdot}64911=$ chord $a\,d$.

24 =the chord of the segment.
12·64911=chord of $\frac{1}{2}$ the segment.
4·21637=$\frac{1}{3}$ of the chord of $\frac{1}{2}$ the arc.
40·86548
4=the height of the segment.
$163{\cdot}46192\times4\div10=65{\cdot}384768$=area of the segment. Ans.

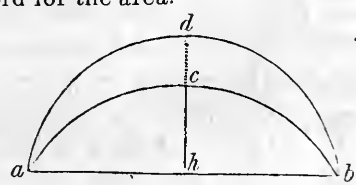
(See Table of Areas of the Segments of Circles.)

OF LUNES.

PROBLEM VIII.

To find the Area of a Lune or Crescent.

Rule.—Find the difference of the two segments which are between the arcs of the crescent and its chord for the area.



Example.—The chord of two segments, $a\,b$, is 72, and the height of the greater segment, $h\,d$, is 30, and of the lesser, $h\,c$, 20: what is the area of the crescent?

$\overline{30}^2+\overline{36}^2=2196$ and $\sqrt{2196}=46{\cdot}8$=chord of half the arc.

And $46{\cdot}8\times\frac{4}{3}=62{\cdot}4$: Then, $62{\cdot}4+72\times30\times\frac{4}{10}=1612{\cdot}8$=area of segment, $a\,b\,d$.

Again, $\overline{20}^2+\overline{36}^2=1696$ and $\sqrt{1696}=41{\cdot}2$=chord of $\frac{1}{2}$ arc.

Then, $41{\cdot}2\times\frac{4}{3}=50{\cdot}8$, and $50{\cdot}8+72\times20\times\frac{4}{10}=982{\cdot}4$=area of segment, $a\,b\,c$.

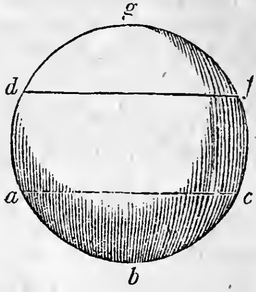
The difference of these areas is 630·4=the area of the lune or crescent.

NOTE. If upon the three sides of a right-angled triangle, as diameters, semicircles be described, two lunes will be formed, whose united areas will be equal to the area of the triangle.

PROBLEM IX.

To find the Area of a Circular Zone.

Rule.—From the area of the whole circle, subtract the areas of the two segments on the sides of the zone.



If from the whole circle there be taken the two segments, *a b c* and *d f g*, there will remain the circular zone, *a b f d.*

Example.—1. What is the area of the zone, *a b f d*, if *a b* is 7·75, *d f* 6·93, and the diameter of the circle 8?

50·26=area of the whole circle.
17·23=area of the segment, *a b c*.
9·82=area of the segment, *d f g*.
27·05

And, 50·26−27·05=23·21=area of the zone, *a b f d.*

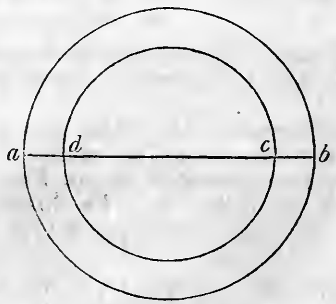
PROBLEM X.

To find the Area of a Ring included between the Circumferences of two Concentric Circles.

Rule.—1. Square the diameter of each circle, and subtract the square of the less from that of the greater.

2. Multiply the difference of the squares by the decimal ·7854, and the product will be the area.

Or, multiply the product of the *sum* and *difference* of the two diameters by ·7854.



Example.—If the diameter of the outer circle, *a b*, be 221, and the inner circle, *d c*, 106, what is the area of the ring?

First, $\overline{221}^2\times{}$·7854= 38359·72
And, $\overline{106}^2\times{}$·7854= 8824·75.
Ans. 29534·97.

NOTE. The area of each of these circles is equal to the square of the diameter multiplied by ·7854. (Prob. 3.) And the difference of these squares is equal to the product of the *sum* and *difference* of the diameters. Therefore, the area of the ring is equal to the product of the sum and difference of the two diameters, multiplied by ·7854.

OF ELLIPSES.

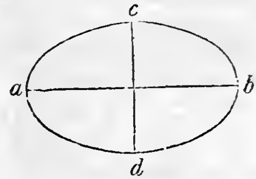
PROBLEM XI.

To find the Area of an Ellipse.

Rule.—Multiply the longer axis by the shorter, and the product, multiplied by the decimal ·7854, will be the area required.

NOTE. A common and more scientific name for the longer axis of an ellipse, is the *transverse*, or *major*, and for the shorter, the *conjugate* or *minor*.

Example.—1. What is the area of an ellipse whose longer axis, $a\ b$, is 70 feet, and whose shorter, $d\ c$, is 50 feet?



$a\ b \times d\ e = 70 \times 50 = 3500.$
Then, $3500 \times {}^{\cdot}7854 = 2748{\cdot}9 =$ area.

2. What is the area of an ellipse whose axes are 16 and 12?
150·79. Ans.

PROBLEM XII.

To find the Circumference of an Ellipse.

Rule.—Square the two axes, and multiply the square root of half their sum by 3·14159; the product will be the circumference, nearly.

Example.—What is the circumference of an ellipse whose transverse and conjugate axes are 16 and 18 feet?

$\overline{16}^2 + \overline{18}^2 = 580 =$ sum of the squares of the axes.
And, $290 =$ half sum.
Then, $290 \times 3{\cdot}14159 = 911{\cdot}064 =$ circumference.

PROBLEM XIII.

To find the Area of an Elliptic Segment, cut off by a line perpendicular to either Axis.

Rule.—Find the area of a corresponding circular segment, having the same height and the same vertical axis or diameter. Then say, as the vertical axis is to the other axis, parallel to the segment's base, so is the area of the circular segment before found, to the area of the elliptic segment sought.

Example.—The height of an elliptic segment is 10, and the axes 25 and 35 respectively: what is the area?

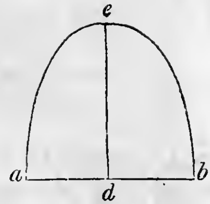
$10 \div 35 = {}^{\cdot}2857$ tabular versed sine, and segment $= {}^{\cdot}185153 \times 35 \times 25 = 162{\cdot}0088.$ Ans.

PROBLEM XIV.

To find the Area of a Parabola.

Rule.—Multiply the base by the height, and two-thirds of the product will be the area.

Example.—What is the area of a parabola whose base, $a\ b$, is 26 inches, and height, $d\ c$, 18 inches?



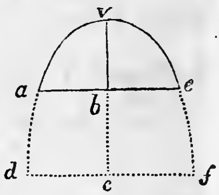
$26 \times 18 = 468 =$ product of base and height.
$468 \times \frac{2}{3} = 312 =$ area in square inches.
Then $312 \div 144 = 2\frac{1}{6}$ square feet. Ans.

PROBLEM XV.

To find the Area of a Frustrum of a Parabola, cut off by a line drawn parallel to the base.

Rule.—Multiply the difference of the cubes of the two ends of the frustrum by twice its altitude, and divide the product by three times the difference of their squares.

Example.—What is the area of a frustrum of a parabola whose height, $c\ b$, is 12 feet, and its upper end, $a\ e$, 12 feet, and its base, $d\ f$, 20 feet?



$\overline{20}^2 = 400$ $\qquad$ $\overline{20}^3 = 8000$
$\overline{12}^2 = 144$ $\qquad$ $\overline{12}^3 = 1728$
$256 =$ diff. of their squares $\qquad$ 6272
3 $\qquad$ $24 =$ twice the height.
768 $\qquad$ 25088
12544
$150528 \div 768 = 196$ ft. Ans

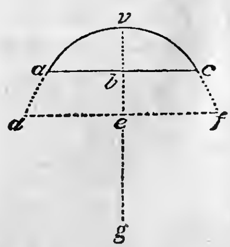
OF HYPERBOLAS.

PROBLEM XVI.

To find the Area of a Hyperbola.

Rule.—To five-sevenths of the abscissa, $v\ e$, add the transverse diameter; multiply the sum by the abscissa, and extract the square root of the product. Then, multiply the transverse diameter, $v\ g$, by the abscissa, $v\ e$, and extract the square root of that product. Then, to 21 times the first root, add 4 times the second root; multiply the sum by double the product of the conjugate and abscissa, and divide by 75 times the transverse; this will give the area, nearly.

Example.—What is the area of a Hyperbola, $d\ f\ v$, whose transverse diameter, $v\ g$, is 80, and conjugate, $d\ f$, 50, and whose abscissa, $v\ e$, is 45?



$\frac{5}{7}$ of $45 = 32{\cdot}14$ and $\sqrt{32{\cdot}14 + 80 \times 45} = 71{\cdot}03$
$\sqrt{80 \times 45} = 60$
$71{\cdot}03 \times 21 = 1491{\cdot}63$
$60 \quad \times \ 4 = \ 240$
$1731{\cdot}63$
$1731{\cdot}63 \times (50 \times 45 \times 2) \div (80 \times 75) = 1298{\cdot}72.$ Ans.

TABLE

OF THE AREAS OF THE SEGMENTS OF A CIRCLE,

Whose diameter is Unity, and supposed to be divided into 1000 equal parts.

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·001	·00004	·039	·01014	·077	·02782	·115	·05016	·153	·07602
·002	·00011	·040	·01053	·078	·02835	·116	·05080	·154	·07674
·003	·00021	·041	·01093	·079	·02889	·117	·05144	·155	·07746
·004	·00033	·042	·01133	·080	·02943	·118	·05209	·156	·07819
·005	·00047	·043	·01173	·081	·02997	·119	·05273	·157	·07892
·006	·00061	·044	·01214	·082	·03052	·120	·05338	·158	·07964
·007	·00077	·045	·01255	·083	·03107	·121	·05403	·159	·08038
·008	·00095	·046	·01297	·084	·03162	·122	·05468	·160	.08111
·009	·00113	·047	·01339	·085	·03218	·123	·05534	·161	.08184
·010	·00132	·048	·01381	·086	·03274	·124	·05600	·162	.08258
·011	·00153	·049	·01424	·087	·03330	·125	·05666	·163	.08332
·012	·00174	·050	·01468	·088	·03387	·126	·05732	·164	.08405
·013	·00196	·051	·01511	·089	·03444	·127	·05799	·165	.08480
·014	·00219	·052	·01556	·090	·03501	·128	·05865	·166	.08554
·015	·00243	·053	·01600	·091	·03558	·129	·05932	·167	.08628
·016	·00268	·054	·01645	·092	·03616	·130	·05999	·168	.08703
·017	·00294	·055	·01691	·093	·03674	·131	·06067	·169	.08778
·018	·00320	·056	·01736	·094	·03732	·132	·06134	·170	.08853
·019	·00347	·057	·01783	·095	·03790	·133	·06202	·171	.08928
·020	·00374	·058	·01829	·096	·03849	·134	·06270	·172	.09004
·021	·00403	·059	·01876	·097	·03908	·135	·06338	·173	.09079
·022	·00432	·060	·01923	·098	·03968	·136	·06407	·174	.09155
·023	·00461	·061	·01971	·099	·04027	·137	·06476	·175	.09231
·024	·00492	·062	·02019	·100	·04087	·138	·06544	·176	.09307
·025	·00523	·063	·02068	·101	·04147	·139	·06614	·177	.09383
·026	·00554	·064	·02116	·102	·04208	·140	·06683	·178	.09460
·027	·00586	·065	·02165	·103	·04268	·141	·06752	·179	.09536
·028	·00619	·066	·02215	·104	·04329	·142	·06822	·180	.09613
·029	·00652	·067	·02265	·105	·04390	·143	·06892	·181	.09690
·030	·00686	·068	·02315	·106	·04452	·144	·06962	·182	.09767
·031	·00720	·069	·02365	·107	·04513	·145	·07032	·183	.09844
·032	·00755	·070	·02416	·108	·04575	·146	·07103	·184	.09922
·033	·00791	·071	·02468	·109	·04638	·147	·07174	·185	.09999
·034	·00827	·072	·02519	·110	·04700	·148	·07245	·186	·10077
·035	·00863	·073	·02571	·111	·04763	·149	·07316	·187	·10155
·036	·00900	·074	·02623	·112	·04826	·150	·07387	·188	·10233
·037	·00938	·075	·02676	·113	·04889	·151	·07458	·189	·10311
·038	·00976	·076	·02728	·114	.04952	·152	·07530	190	·10390

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·191	·10468	·240	·14494	·289	·18814	·338	·23358	·387	·28066
·192	·10547	·241	·14579	·290	·18904	·339	·23452	·388	·28164
·193	·10626	·242	·14665	·291	·18995	·340	·23547	·389	·28261
·194	·10705	·243	·14751	·292	·19086	·341	·23642	·390	·28359
·195	·10784	·244	·14837	·293	·19177	·342	·23736	·391	·28456
·196	·10863	·245	·14923	·294	·19268	·343	·23831	·392	·28554
·197	·10943	·246	·15009	·295	·19359	·344	·23926	·393	·28652
·198	·11022	·247	·15095	·296	·19450	·345	·24021	·394	·28749
·199	·11102	·248	·15181	·297	·19542	·346	·24116	·395	·28847
·200	·11182	·249	·15268	·298	·19633	·347	·24212	·396	·28945
·201	·11262	·250	·15354	·299	·19725	·348	·24307	·397	·29043
·202	·11342	·251	·15441	·300	·19816	·349	·24402	·398	·29141
·203	·11423	·252	·15528	·301	·19908	·350	·24498	·399	·29239
·204	·11503	·253	·15614	·302	·20000	·351	·24593	·400	·29336
·205	·11584	·254	·15701	·303	·20092	·352	·24688	·401	·29434
·206	·11665	·255	·15789	·304	·20184	·353	·24784	·402	·29533
·207	·11746	·256	·15876	·305	·20276	·354	·24880	403	·29631
·208	·11827	·257	·15963	·306	·20368	·355	·24975	·404	·29729
·209	·11908	·258	·16051	·307	·20460	·356	·25071	·405	·29827
·210	·11989	·259	·16138	·308	·20552	·357	·25167	·406	·29925
·211	·12071	·260	·16226	·309	·20645	·358	·25263	·407	·30023
·212	·12152	·261	·16314	·310	·20737	·359	·25359	·408	·30122
·213	·12234	·262	·16401	·311	·20830	·360	·25455	·409	·30220
·214	·12316	·263	·16489	·312	·20922	·361	·25551	·410	·30318
·215	·12398	·264	·16578	·313	·21015	·362	·25647	·411	·30417
·216	·12481	·265	·16666	·314	·21108	·363	·25743	·412	·30515
·217	·12563	·266	·16754	·315	·21201	·364	·25839	·413	·30614
·218	·12645	·267	·16843	·316	·21294	·365	·25935	·414	·30712
·219	·12728	·268	·16931	·317	·21387	·366	·26032	·415	·30811
·220	·12811	·269	·17020	·318	·21480	·367	·26128	·416	·30909
·221	·12894	·270	·17108	·319	·21573	·368	·26224	·417	·31008
·222	·12977	·271	·17197	·320	·21666	·369	·26321	·418	·31106
·223	·13060	·272	·17286	·321	·21759	·370	·26417	·419	·31205
·224	·13143	·273	·17375	·322	·21853	·371	·26514	·420	·31304
·225	·13227	·274	·17464	·323	·21946	·372	·26611	·421	·31402
·226	·13310	·275	·17554	·324	·22040	·373	·26707	·422	·31501
·227	·13394	·276	·17643	·325	·22134	·374	·26804	·423	·31600
·228	·13478	·277	·17733	·326	·22227	·375	·26901	·424	·31699
·229	·13562	·278	·17822	·327	·22321	·376	·26998	·425	·31798
·230	·13646	·279	·17912	·328	·22415	·377	·27095	·426	·31897
·231	·13730	·280	·18001	·329	·22509	·378	·27192	·427	·31995
·232	·13815	·281	·18091	·330	·22603	·379	·27289	·428	·32094
·233	·13899	·282	·18181	·331	·22697	·380	·27386	·429	·32193
·234	·13984	·283	·18271	·332	·22791	·381	·27483	·430	·32292
·235	·14068	·284	·18361	·333	·22885	·382	·27580	·431	·32391
·236	·14153	·285	·18452	·334	·22980	·383	·27677	·432	·32490
·237	·14238	·286	·18542	·335	·23074	·384	·27774	·433	·32590
·238	·14323	·287	·18632	·336	·23168	·385	·27872	·434	·32689
·239	·14409	.288	·18723	·337	·23263	·386	·27969	·435	·32788

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·436	·32887	·449	·34178	·462	·35473	·475	·36770	·488	·38070
·437	·32986	·450	·34278	·463	·35573	·476	·36870	·489	·38169
·438	·33085	·451	·34377	·464	·35673	·477	·36970	·490	·38269
·439	·33185	·452	·34477	·465	·35772	·478	·37070	·491	·38369
·440	·33284	·453	·34576	·466	·35872	·479	·37170	·492	·38469
·441	·33383	·454	·34676	·467	·35972	·480	·37270	·493	·38569
·442	·33482	·455	·34775	·468	·36072	·481	·37370	·494	·38669
·443	·33582	·456	·34875	·469	·36171	·482	·37470	·495	·38769
·444	·33681	·457	·34975	·470	·36271	·483	·37570	·496	·38869
·445	·33781	·458	·35074	·471	·36371	·484	·37670	·497	·38969
·446	·33880	·459	·35174	·472	·36471	·485	·37770	·498	·39069
·447	·33979	·460	·35274	·473	·36571	·486	·37870	·499	·39169
·448	·34079	·461	·35373	·474	·36671	·487	·37970	·500	·39269

USE OF THE ABOVE TABLE.

To find the Area of a Segment of a Circle.

Rule.—Divide the height, or versed sine, by the diameter of the circle, and find the quotient in the column of versed sines.

Then take out the corresponding area, in the next column on the right hand, and multiply it by the square of the diameter; this will give the area of the segment.

Example.—Required the area of a segment of a circle, whose height is $3\frac{1}{4}$ feet, and the diameter of the circle 50 feet.

$$3\tfrac{1}{4}=3{\cdot}25\,; \text{ and } 3{\cdot}25\div 50={\cdot}065.$$

$\cdot065$, as per table, $=\cdot021659$; and $\cdot021659\times 50^2=54\cdot147500$, the area required.

Approximating Rule to find the Area of a Segment of a Circle.

Rule.—Multiply the chord of the segment by the versed sine, divide the product by 3, and multiply the remainder by 2.

Cube the height, or versed sine, find how often twice the length of the chord is contained in it, and add the quotient to the former product; this will give the area of the segment, very nearly.

Example.—Required the area of the segment of a circle, the chord being 12, and the versed sine 2.

$$12\times 2=24\,;\ 24\div 3=8\,;\ \text{and } 8\times 2=16.$$

$$2^3\div 24={\cdot}3333.$$

Hence $16+\cdot3333=16\cdot3333$, the area of the segment, very nearly.

TABLE

OF THE AREAS OF THE ZONES OF A CIRCLE.

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·001	·00100	·044	·04394	·087	·08655	·130	·12852	·173	·16948
·002	·00200	·045	·04494	·088	·08754	·131	·12948	·174	·17042
·003	·00300	·046	·04593	·089	·08852	·132	·13045	·175	·17135
·004	·00400	·047	·04693	·090	·08951	·133	·13141	·176	·17229
·005	·00500	·048	·04792	·091	·09049	·134	·13237	·177	·17323
·006	·00600	·049	·04892	·092	·09147	·135	·13334	·178	·17416
·007	·00700	·050	·04991	·093	·09246	·136	·13430	·179	·17510
·008	·00800	·051	·05091	·094	·09344	·137	·13526	·180	·17603
·009	·00900	·052	·05190	·095	·09442	·138	·13622	·181	·17696
·010	·01000	·053	·05290	·096	·09540	·139	·13718	·182	·17789
·011	·01100	·054	·05389	·097	·09638	·140	·13814	·183	·17882
·012	·01199	·055	·05489	·098	·09736	·141	·13910	·184	·17975
·013	·01299	·056	·05588	·099	·09835	·142	·14006	·185	·18068
·014	·01399	·057	·05687	·100	·09933	·143	·14102	·186	·18161
·015	·01499	·058	·05787	·101	·10030	·144	·14198	·187	·18254
·016	·01599	·059	·05886	·102	·10128	·145	·14294	·188	·18347
·017	·01699	·060	·05985	·103	·10226	·146	·14389	·189	·18439
·018	·01799	·061	·06084	·104	·10324	·147	·14485	·190	·18532
·019	·01899	·062	·06184	·105	·10422	·148	·14581	·191	·18624
·020	·01999	·063	·06283	·106	·10520	·149	·14676	·192	·18717
·021	·02099	·064	·06382	·107	·10617	·150	·14771	·193	·18809
·022	·02199	·065	·06481	·108	·10715	·151	·14867	·194	·18901
·023	·02299	·066	·06580	·109	·10813	·152	·14962	·195	·18993
·024	·02399	·067	·06679	·110	·10910	·153	·15057	·196	·19085
·025	·02499	·068	·06779	·111	·11008	·154	·15153	·197	·19177
·026	·02598	·069	·06878	·112	·11105	·155	·15248	·198	·19269
·027	·02698	·070	·06977	·113	·11203	·156	·15343	·199	·19361
·028	·02798	·071	·07076	·114	·11300	·157	·15438	·200	·19453
·029	·02898	·072	·07175	·115	·11397	·158	·15533	·201	·19544
·030	·02998	·073	·07274	·116	·11495	·159	·15627	·202	·19636
·031	·03098	·074	·07372	·117	·11592	·160	·15722	·203	·19727
·032	·03197	·075	·07471	·118	·11689	·161	·15817	·204	·19819
·033	·03297	·076	·07570	·119	·11786	·162	·15911	·205	·19910
·034	·03397	·077	·07669	·120	·11883	·163	·16006	·206	·20001
·035	·03497	·078	·07768	·121	·11980	·164	·16101	·207	·20092
·036	·03596	·079	·07867	·122	·12077	·165	·16195	·208	·20183
·037	·03696	·080	·07965	·123	·12174	·166	·16289	·209	·20274
·038	·03796	·081	·08064	·124	·12271	·167	·16384	·210	·20365
039	·03896	·082	·08163	·125	·12368	·168	·16478	·211	·20455
·040	·03995	·083	·08261	·126	·12465	·169	·16572	·212	·20546
·041	·04095	·084	·08360	·127	·12562	·170	·16666	·213	·20637
·042	·04195	·085	·08458	·128	·12658	·171	·16760	·214	·20727
·043	·04294	·086	·08557	·129	12755	·172	·16854	·215	·20817

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·216	·20908	·265	·25201	·314	·29192	·363	·32793	·412	·35882
·217	·20998	·266	·25285	·315	·29270	·364	·32862	·413	·35939
·218	·21088	·267	·25370	·316	·29347	·365	·32931	·414	·35995
·219	·21178	·268	·25454	·317	·29425	·366	·32999	·415	·36051
·220	·21268	·269	·25539	·318	·29502	·367	·33067	·416	·36107
·221	·21357	·270	·25623	·319	·29579	·368	·33135	·417	·36162
·222	·21447	·271	·25707	·320	·29656	·369	·33202	·418	·36217
·223	·21536	·272	·25791	·321	·29733	·370	·33270	·419	·36272
·224	·21626	·273	·25875	·322	·29809	·371	·33337	·420	·36326
·225	·21715	·274	·25959	·323	·29886	·372	·33404	·421	·36380
·226	·21805	·275	·26042	·324	·29962	·373	·33470	·422	·36434
·227	·21894	·276	·26126	·325	·30038	·374	·33537	·423	·36487
·228	·21983	·277	·26209	·326	·30114	·375	·33603	·424	·36541
·229	·22072	·278	·26292	·327	·30190	·376	·33669	·425	·36593
·230	·22161	·279	·26375	·328	·30265	·377	·33735	·426	·36646
·231	·22249	·280	·26458	·329	·30341	·378	·33801	·427	·36698
·232	·22335	·281	·26541	·330	·30416	·379	·33866	·428	·36750
·233	·22426	·282	·26624	·331	·30491	·380	·33931	·429	·36801
·234	·22515	·283	·26706	·332	·30566	·381	·33996	·430	·36853
·235	·22603	·284	·26788	·333	·30641	·382	·34060	·431	·36904
·236	·22691	·285	·26871	·334	·30715	·383	·34125	·432	·36954
·237	·22780	·286	·26953	·335	·30789	·384	·34189	·433	·37004
·238	·22868	·287	·27035	·336	·30864	·385	·34253	·434	·37054
·239	·22955	·288	·27117	·337	·30937	·386	·34317	·435	·37104
·240	·23043	·289	·27198	·338	·31011	·387	·34380	·436	·37153
·241	·23131	·290	·27280	·339	·31085	·388	·34443	·437	·37201
·242	·23218	·291	·27361	·340	·31158	·389	·34506	·438	·37250
·243	·23306	·292	·27442	·341	·31231	·390	·34569	·439	·37298
·244	·23393	·293	·27523	·342	·31305	·391	·34631	·440	·37346
·245	·23480	·294	·27604	·343	·31377	·392	·34694	·441	·37393
·246	·23568	·295	·27685	·344	·31450	·393	·34756	·442	·37440
·247	·23655	·296	·27766	·345	·31523	·394	·34817	·443	·37486
·248	·23741	·297	·27846	·346	·31595	·395	·34879	·444	·37533
·249	·23828	·298	·27927	·347	·31667	·396	·34940	·445	·37578
·250	·23915	·299	·28007	·348	·31739	·397	·35001	·446	·37624
·251	·24001	·300	·28087	·349	·31811	·398	·35061	·447	·37669
·252	·24088	·301	·28167	·350	·31882	·399	·35122	·448	·37713
·253	·24174	·302	·28247	·351	·31953	·400	·35182	·449	·37758
·254	·24260	·303	·28326	·352	·32024	·401	·35242	·450	·37801
·255	·24346	·304	·28406	·353	·32095	·402	·35301	·451	·37845
·256	·24432	·305	·28485	·354	·32166	·403	·35361	·452	·37888
·257	·24518	·306	·28564	·355	·32237	·404	·35420	·453	·37930
·258	·24604	·307	·28643	·356	·32307	·405	·35479	·454	·37972
·259	·24690	·308	·28722	·357	·32377	·406	·35537	·455	·38014
·260	·24775	·309	·28801	·358	·32447	·407	·35595	·456	·38055
·261	·24860	·310	·28879	·359	·32517	·408	·35653	·457	·38096
·262	·24946	·311	·28958	·360	·32586	·409	·35711	·458	·38136
·263	·25021	·312	·29036	·361	·32655	·410	·35768	·459	·38176
·264	·25116	·313	·29114	·362	·32725	·411	·35825	·460	·38216

Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.	Versed Sine.	Area of Segment.
·461	·38255	·469	·38549	·477	·38808	·485	·39026	·493	·39120
·462	38293	·470	·38583	·478	·38837	·486	·39050	·494	·39208
·463	·38331	·471	·38617	·479	·38866	·487	·39073	·495	·39222
·464	·38369	·472	·38650	·480	·38895	·488	·39095	·496	·39236
·465	·38406	·473	·38683	·481	·38922	·489	·39116	·497	·39248
·466	·38442	·474	·38715	·482	·38949	·490	·39137	·498	·39258
·467	·38478	·475	·38746	·483	·38975	·491	·39156	·499	·39265
·468	·38514	·476	·38777	·484	·39001	·492	·39174	·500	·39269

USE OF THE ABOVE TABLE.

To find the Area of a Circular Zone.

Rule 1.—When the zone is less than a semicircle, divide the height by the longest chord, and seek the quotient in the column of versed sines. Take out the corresponding area, in the next column on the right hand, and multiply it by the square of the longest chord; the product will be the area of the zone.

Example.—Required the area of a zone, whose longest chord is 50, and height 15.

$15 \div 50 = \cdot300$; and $\cdot300$, as per table, $= \cdot28087$.

Hence, $\cdot28087 \times 50^2 = 702\cdot19$, the area of the zone.

Rule 2.—*When the zone is greater than a semicircle*, take the height on each side of the diameter of the circle, and find, by Rule 1, their respective areas; the areas of these two portions, added together, will be the area of the zone.

Example.—Required the area of a zone, the diameter of the circle being 50, and the height of the zone on each side of the line which passes through the diameter of the circle 20 and 15, respectively.

$20 \div 50 = \cdot400$; $\cdot400$, as per table, $= \cdot35182$; and $\cdot35182 \times 50^2 = 879\cdot56$.

$15 \div 50 = \cdot300$; $\cdot300$, as per table, $= \cdot28087$; and $\cdot28087 \times 50^2 = 702\cdot19$.

Hence, $879\cdot56 + 702\cdot19 = 1581\cdot75$.

TABLE

Of the Proportions of the Lengths of Circular Arcs.

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·100	1·0265	·144	1·0544	·188	1·0917	·232	1·1379	·276	1·1921
·101	1·0270	·145	1·0552	·189	1·0927	·233	1·1390	·277	1·1934
·102	1·0275	·146	1·0559	·190	1·0936	·234	1·1402	·278	1·1948
·103	1·0281	·147	1·0567	·191	1·0946	·235	1·1414	·279	1·1961
·104	1·0286	·148	1·0574	·192	1·0956	·236	1·1425	·280	1·1974
·105	1·0291	·149	1·0582	·193	1·0965	·237	1·1436	·281	1·1989
·106	1·0297	·150	1·0590	·194	1·0975	·238	1·1448	·282	1·2001
·107	1·0303	·151	1·0597	·195	1·0985	·239	1·1460	·283	1·2015
·108	1·0308	·152	1·0605	·196	1·0995	·240	1·1471	·284	1·2028
·109	1·0314	·153	1·0613	·197	1·1005	·241	1·1483	·285	1·2042
·110	1·0320	·154	1·0621	·198	1·1015	·242	1·1495	·286	1·2056
·111	1·0325	·155	1·0629	·199	1·1025	·243	1·1507	·287	1·2070
·112	1·0331	·156	1·0637	·200	1·1035	·244	1·1519	·288	1·2083
·113	1·0337	·157	1·0645	·201	1·1045	·245	1·1531	·289	1·2097
·114	1·0343	·158	1·0653	·202	1·1055	·246	1·1543	·290	1·2120
·115	1·0349	·159	1·0661	·203	1·1065	·247	1·1555	·291	1·2124
·116	1·0355	·160	1·0669	·204	1·1075	·248	1·1567	·292	1·2138
·117	1·0361	·161	1·0678	·205	1·1085	·249	1·1579	·293	1·2152
·118	1·0367	·162	1·0686	·206	1·1096	·250	1·1591	·294	1·2166
·119	1·0373	·163	1·0694	·207	1·1006	·251	1·1603	·295	1·2179
·120	1·0380	·164	1·0703	·208	1·1117	·252	1·1616	·296	1·2193
·121	1·0386	·165	1·0711	·209	1·1127	·253	1·1628	·297	1·2206
·122	1·0392	·166	1·0719	·210	1·1137	·254	1·1640	·298	1·2220
·123	1·0399	·167	1·0728	·211	1·1148	·255	1·1653	·299	1·2235
·124	1·0405	·168	1·0737	·212	1·1158	·256	1·1665	·300	1·2250
·125	1·0412	·169	1·0745	·213	1·1169	·257	1·1677	·301	1·2264
·126	1·0418	·170	1·0754	·214	1·1180	·258	1·1690	·302	1·2278
·127	1·0425	·171	1·0762	·215	1·1190	·259	1·1702	·303	1·2292
·128	1·0431	·172	1·0771	·216	1·1201	·260	1·1715	·304	1·2306
·129	1·0438	·173	1·0780	·217	1·1212	·261	1·1728	·305	1·2321
·130	1·0445	·174	1·0789	·218	1·1223	·262	1·1740	·306	1·2335
·131	1·0452	·175	1·0798	·219	1·1233	·263	1·1753	·307	1·2349
·132	1·0458	·176	1·0807	·220	1·1245	·264	1·1766	·308	1·2364
·133	1·0465	·177	1·0816	·221	1·1256	·265	1·1778	·309	1·2378
·134	1·0472	·178	1·0825	·222	1·1266	·266	1·1791	·310	1·2393
·135	1·0479	·179	1·0834	·223	1·1277	·267	1·1804	·311	1·2407
·136	1·0486	·180	1·0843	·224	1·1289	·268	1·1816	·312	1·2422
·137	1·0493	·181	1·0852	·225	1·1300	·269	1·1829	·313	1·2436
·138	1·0500	·182	1·0861	·226	1·1311	·270	1·1843	·314	1·2451
·139	1·0508	·183	1·0870	·227	1·1322	·271	1·1856	·315	1·2465
·140	1·0515	·184	1·0880	·228	1·1333	·272	1·1869	·316	1·2480
·141	1·0522	·185	1·0889	·229	1·1344	·273	1·1882	·317	1·2495
·142	1·0529	·186	1·0898	·230	1·1356	·274	1·1897	·318	1·2510
·143	1·0537	·187	1·0908	·231	1·1367	·275	1·1908	·319	1·2524

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·320	1·2539	·357	1·3112	·393	1·3711	.429	1·4349	·465	1·5022
·321	1·2554	·358	1·3128	·394	1·3728	.430	1 4367	·466	1·5042
·322	1·2569	·359	1·3144	·395	1·3746	.431	1·4386	·467	1·5061
·323	1·2584	·360	1·3160	·396	1·3763	.432	1·4404	·468	1·5080
·324	1·2599	·361	1·3176	·397	1·3780	.433	1·4422	·469	1·5099
·325	1·2614	·362	1·3192	·398	1·3797	.434	1·4441	·470	1·5119
·326	1·2629	·363	1·3209	·399	1·3815	.435	1·4459	·471	1·5138
·327	1·2644	·364	1·3225	·400	1·3832	.436	1·4477	·472	1·5157
·328	1·2659	·365	1·3241	·401	1·3850	.437	1·4496	·473	1·5176
·329	1·2674	·366	1·3258	·402	1·3867	.438	1·4514	·474	1·5196
·330	1·2689	·367	1·3274	·403	1·3885	.439	1·4533	·475	1·5215
·331	1·2704	·368	1·3291	·404	1·3902	.440	1·4551	·476	1·5235
·332	1·2720	·369	1·3307	·405	1·3920	.441	1·4570	·477	1·5254
·333	1·2735	·370	1·3323	·406	1·3937	.442	1·4588	·478	1·5274
·334	1·2750	·371	1·3340	·407	1·3955	.443	1·4607	·479	1·5293
·335	1·2766	·372	1·3356	·408	1·3972	.444	1·4626	·480	1·5313
·336	1·2781	·373	1·3373	·409	1·3990	.445	1·4644	·481	1·5332
·337	1·2786	·374	1·3390	·410	1·4008	.446	1·4663	·482	1·5352
·338	1·2812	·375	1·3406	·411	1·4025	.447	1·4682	·483	1·5371
·339	1·2827	·376	1·3423	·412	1·4043	.448	1·4700	·484	1·5391
·340	1·2843	·377	1·3440	·413	1·4061	.449	1·4719	·485	1·5411
·341	1·2858	·378	1·3456	·414	1·4079	.450	1·4738	·486	1·5430
·342	1·2874	·379	1·3473	·415	1·4097	.451	1·4757	·487	1·5450
·343	1·2890	·380	1·3490	·416	1·4115	.452	1·4775	·488	1·5470
·344	1·2905	·381	1·3507	·417	1·4132	.453	1·4794	·489	1·5489
·345	1·2921	·382	1·3524	·418	1·4150	.454	1·4813	·490	1·5509
·346	1·2937	·383	1·3541	·419	1·4168	.455	1·4832	·491	1 5529
·347	1·2952	·384	1·3558	·420	1·4186	.456	1·4851	·492	1·5549
·348	1·2968	·385	1·3574	·421	1·4204	.457	1·4870	·493	1·5569
·349	1·2984	·386	1·3591	·422	1·4222	.458	1·4889	·494	1·5585
·350	1·3000	·387	1·3608	·423	1·4240	.459	1·4908	·495	1·5608
·351	1·3016	·388	1·3625	·424	1·4258	.460	1·4927	·496	1·5628
·352	1·3032	·389	1·3643	·425	1·4276	.461	1·4946	·497	1·5648
·353	1·3047	·390	1·3660	·426	1·4295	.462	1·4965	·498	1·5668
·354	1·3063	·391	1·3677	·427	1·4313	.463	1·4984	·499	1·5688
·355	1·3079	·392	1·3694	·428	1·4331	.464	1·5003	·500	1·5708
·356	1·3095								

To find the Length of an Arc of a Circle by the foregoing Table.

Rule.—Divide the height by the base, and the quotient will be the height of an arc, of which the base is unity. Seek in the table for a number corresponding to the quotient, and take the length of that height from the next right-hand column. Multiply the number, thus found, by the base of the arc, and the product will be the length of the arc or curve required.

Example.—The profiles of the intradoses of the arches of a bridge are each a semi-ellipse; the span of the middle arch is 150 feet, and the height 38 feet: required the length of the curve.

$38 \div 150 = \cdot253$, and $\cdot253$, as per table $= 1\cdot1628$.

Hence, $1\cdot1628 \times 150 = 174\cdot4200$, the length required.

TABLE

Of the Proportions of the Lengths of Semi-elliptic Arcs.

Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.	Height of Arc.	Length of Arc.
·100	1·0416	·265	1·2306	·450	1·4931	·635	1·7850	·820	2·0971
·101	1·0426	·270	1·2371	·455	1·5008	·640	1·7931	·825	2·1060
·102	1·0436	·275	1·2436	·460	1·5084	·645	1·8013	·830	2·1148
·103	1·0446	·280	1·2501	·465	1·5161	·650	1·8094	·835	2·1237
·104	1·0456	·285	1·2567	·470	1·5238	·655	1·8176	·840	2·1326
·105	1·0466	·290	1·2634	·475	1·5316	·660	1·8258	·845	2·1416
·110	1·0516	·295	1·2700	·480	1·5394	·665	1·8340	·850	2·1505
·115	1·0567	·300	1·2767	·485	1·5472	·670	1·8423	·855	2·1595
·120	1·0618	·305	1·2834	·490	1·5550	·675	1·8505	·860	2·1685
·125	1·0669	·310	1·2901	·495	1·5629	·680	1·8587	·865	3·1775
·130	1·0720	·315	1·2960	·500	1·5709	·685	1·8670	·870	2·1866
·135	1·0773	·320	1·3038	·505	1·5785	·690	1·8753	·875	2·1956
·140	1·0825	·325	1·3106	·510	1·5863	·695	1·8836	·880	2·2047
·145	1·0879	·330	1·3175	·515	1·5941	·700	1·8919	·885	2·2139
·150	1·0933	·335	1·3244	·520	1·6019	·705	1·9002	·890	2·2230
·155	1·0989	·340	1·3313	·525	1·6097	·710	1·9085	·895	2·2322
·160	1·1045	·345	1·3383	·530	1·6175	·715	1·9169	·900	2·2414
·165	1·1106	·350	1·3454	·535	1·6253	·720	1·9253	·905	2·2506
·170	1·1157	·355	1·3525	·540	1·6331	·725	1·9337	·910	2·2597
·175	1·1213	·360	1·3597	·245	1·6409	·730	1·9422	·915	2·2689
·180	1·1270	·365	1·3669	·550	1·6488	·735	1·9506	·920	2·2780
·185	1·1327	·370	1·3741	·555	1·6567	·740	1·9599	·925	2·2872
·190	1·1384	·375	1·3815	·560	1·6646	·745	1·9675	·930	2·2964
·195	1·1442	·380	1·3888	·565	1·6725	·750	1·9760	·935	2·3056
·200	1·1501	·385	1·3961	·570	1·6804	·755	1·9845	·940	2·3148
·205	1·1560	·390	1·4034	·575	1·6883	·760	1·9931	·945	2·3241
·210	1·1620	·395	1·4107	·580	1·6963	·765	2·0016	·950	2·3335
·215	1·1680	·400	1·4180	·585	1·7042	·770	2·0102	·955	2·3429
·220	1·1741	·405	1·4253	·590	1·7123	·775	2·0187	·960	2·3524
·225	1·1802	·410	1·4327	·595	1·7203	·780	2·0273	·965	2·3619
·230	1·1864	·415	1·4402	·600	1·7283	·785	2·0360	·970	2·3714
·235	1·1926	·420	1·4476	·605	1·7364	·790	2·0446	·975	2·3810
·240	1·1989	·425	1·4552	·610	1·7444	·795	2·0533	·980	2·3906
·245	1·2051	·430	1·4627	·615	1·7525	·800	2·0620	·985	2·4002
·250	1·2114	·435	1·4702	·620	1·7606	·805	2·0708	·930	2·4098
·255	1·2177	·440	1·4778	·625	1·7687	·810	2·0795	·995	2·4194
·260	1·2241	·445	1·4854	·630	1·7768	·815	2·0883	·1000	2·4291

To find the Length of the Curve of a Right Semi-ellipse.

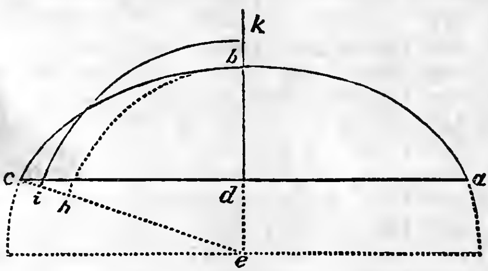
Rule.—The rule for circular arcs in the preceding table is equally applicable here.

The two last tables are not entirely confined to works which may be carried into practice, but are useful in estimating, to a very minute degree of accuracy, the quantity of work which is to be executed from drawings to a scale.

As the tables, however, do not afford the means of finding the lengths of the curves of elliptic arcs, which are less than half of the entire figure, the following geometrical method is given to supply the defect.

To find the Length of an Elliptic Curve, which is less than half the figure.

Let the curve, of which the length is required to be found, be *a b c*.



Produce the versed sine, *b d*, to meet the centre of the curve in *e*. Draw the right line, *c e*, and from the centre, *e*, with the distance, *e b*, describe an arc, *b h*. Bisect *c h* in *i*, and from the centre, *e*, with the radius, *e i*, describe the arc, *i k*, meeting *e b* produced to *k*; then, *i k* is half the arc *a b c*.

NOTE. * When the quotient is not given in the column of heights, divide the difference between the two nearest heights by ·5; multiply the quotient by the excess of the height given, and the height in the table first above it, and add this sum to the tabular area of the least height.

Thus, if the height is 118,

·115, per table, = 1·0567
·120, " " = 1·0618

·0051 ÷ ·5 = ·00102 × (118 − 115) = ·00306, which, added to 1·0567, = 1·05976, the length for 118.

* Haswell.

SECTION III.

OF SOLIDS BOUNDED BY PLANE SURFACES.

The mensuration of solids is divided into two parts.
I. The mensuration of the surfaces of solids.
II. The mensuration of their solidities.

The *measure* of any solid body is the whole capacity or contents of that body, when considered under the triple dimensions of length, breadth, and thickness. A *cube*, whose side is one inch, one foot, or one yard, &c., is called the *measuring unit;* and the contents or solidity of any figure is computed by the number of those cubes contained in that figure.

DEFINITIONS.

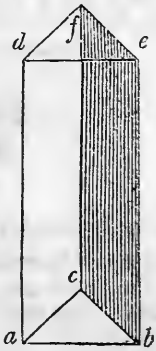
1. A *cube* is a right prism, bounded by six equal square faces, of which any two, opposite to each other, are parallel.
2. A *parallelopiped* is a prism bounded by six quadrilateral planes, every opposite two of which are equal and parallel.
3. A *prism* is a solid, whose ends are parallel, similar, and equal, and the sides connecting these are parallelograms.
4. A *pyramid* is a solid, whose base is any plane figure, and whose sides are triangles, having all their vertices meeting together in a point above the base, called the *vertex* of the pyramid.
5. A *frustrum* or *trunk* of a pyramid is a portion of the solid that remains after any part has been cut off parallel to the base.
6. A *wedge* is a solid of five sides, two of which are rhomboidal, and meet in an edge, a rectangular base, and two triangular ends.
7. A *prismoid* is a solid, whose ends or bases are parallel, but not similar, and whose sides are quadrilateral.

OF CUBES AND PARALLELOPIPEDS.

PROBLEM I.

To find the Lateral Surface of a Prism.

Rule.—Multiply the perimeter of the base into the altitude, and the product will be the convex, or lateral surface. When the *entire* surface of the prism is required, add to the convex surface the area of the bases.



Example.—Required the lateral surface of a prism whose base is a regular hexagon, and whose sides are each 2 feet 3 inches, the height being 11 feet?

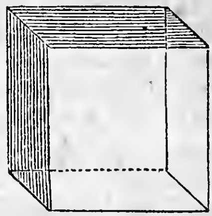
2 ft. 3 in. $=27$ in. and $27\times6=$perimeter of the base.
11 ft. $=132$ inches$=$height.
Then, $132\times162=21384$ square inches.
$21384\div144=148{\cdot}50$ sq. ft. Ans.

PROBLEM II.

To find the Solidity of a Cube or Right Prism.

Rule.—Multiply the area of the base by the perpendicular height, and the product will be the solid contents.

NOTE. The capacity of a vessel, in gallons or bushels, of any given dimensions, may be readily ascertained by calculating its contents in *inches*, and then dividing the contents by the number of cubic inches in one gallon or bushel.



Examples.—1. Required the number of *ale* gallons there are in a *cistern* which is 6 feet 8 inches deep, and whose base is 5 feet 4 inches square?

6 ft. 8 in. = 80 in. | Then, $\overline{64}^2 = 4096$, and
5 ft. 4 in. = 64 in. | $4096 \times 80 = 327680 =$ solidity in inches.

And $327680 \div 282 = 1162$ gal.

2. What is the solidity of a *prism* of granite, 9 feet 2 inches long, and 16 by 12 inches side dimension, and what will be its weight, reckoning 169 lbs. to the cubic foot?

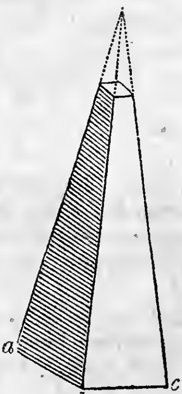
9 ft. 2 in. = 110 in. = length. | $192 \times 110 = 21120$ = solidity in in.
$16 \times 12 = 192$ in. = area of base. | $21120 \div 1728 = 12{\cdot}22$ cubic ft. Ans.

$12{\cdot}22 \times 169 = 2065$ lbs. Ans.

OF PYRAMIDS.

PROBLEM III.

To find the Lateral Surface of a Regular Pyramid.



Rule.—Multiply the perimeter of the base by the slant height, and half the product will be the surface. If the whole surface be required, add to this the area of the base.

Example.—What is the lateral surface of a regular triangular pyramid, *a b c*, whose slant height, *d a*, is 20 feet, and the sides of whose base are each 8 feet?

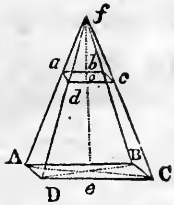
$$\begin{array}{r l} 8 \times 3 = 24 & = \text{perimeter of the base.} \\ \underline{20} & = \text{slant height.} \\ 2)\overline{480} & \\ \overline{240} & = \text{lateral surface.} \end{array}$$

PROBLEM IV.

To find the Lateral Surface of the Frustrum of a Regular Pyramid.

Rule.—Multiply the perimeters of the two ends by the slant height of the frustrum, and half the product will be the surface required. To this add the surface of the two ends when the entire surface is required.

Example.—What is the lateral surface of the frustrum of a regular octagonal pyramid, *A B C D*, whose slant height, *a* A, is 42 feet, and the sides of the lower base, *D C*, 5 feet each, and of the upper base, *a b*, 3 feet each?



First, $5\times 8=40=$perimeter of lower base.
$3\times 8=24=$ " upper "
$\overline{64}=$sum of the two ends.

Then, $64\times 42\div 2=1344=$area of lateral surface.

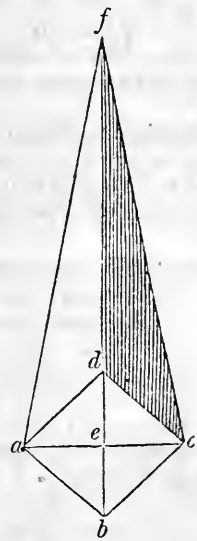
PROBLEM V.

To find the Solidity of a Pyramid.

Rule.—Find the area of the base, and multiply that area by $\frac{1}{3}$ of the height.

Note. This rule follows from that of the *prism*, because any pyramid is $\frac{1}{3}$ of a prism of the same base and altitude. It is manifest, therefore, that the solidity of a pyramid, whether right or oblique, is equal to the product of the area of the base into $\frac{1}{3}$ of the perpendicular height.

Example.—What is the solidity of a square pyramid, *a b c d*, the sides of whose base are each 30 feet, and its perpendicular height, *e f*, 25 feet?



First, $30\times 30=900=$area of the base.
$25\div 3=\quad 8\frac{1}{3}$
7200
300
$\overline{7500}=$solidity.

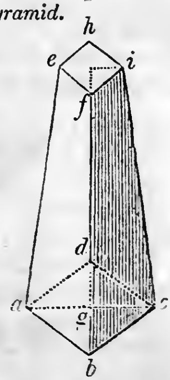
PROBLEM VI.

To find the Solidity of the Frustrum of a Pyramid.

Rule.—To the areas of the two ends of the frustrum, add the square root of their product; and this sum, multiplied by $\frac{1}{3}$ of the perpendicular height, will give the solid contents.

Note This rule holds equally true to a pyramid of any form. For the solidities of pyramids are equal when they have equal heights and bases, whatever be the figure of their bases.

Example.—What is the cubic or solid contents of the frustrum of a marble pyramid, whose lower base, *a b c d*, is 20 inches square, and upper base, *e f*, 14 inches, and whose height, *h g*, is 8 feet 4 inches? And what will be its weight, reckoning 169 lbs. to the cubic foot?



$\overline{20}^2=400$=area of lower base. 8 ft. 4=100

$\overline{14}^2=196$= " upper " $100\div3=33\frac{1}{3}=\frac{1}{3}$ of height.

596=sum of areas. Then, $\sqrt{400\times196}=280$.

And, $\overline{596+280}\times33\frac{1}{3}=29200$.

$29200\div1728=16{\cdot}9$ cubic feet. Ans.

To find the weight, $16{\cdot}9\times169=2856$ lbs. Ans.

NOTE. By this rule, marble cutters can easily determine the solidity and weight of any piece of marble, such as shafts of monuments, slabs, &c., by reference to the Table of Specific Gravities, for a multiplier for the weight of a cubic foot or inch.

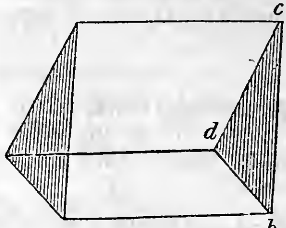
OF WEDGES AND PRISMOIDS.

PROBLEM VII.

To find the Solidity of a Wedge.

Rule.—To the length of the edge of the wedge add twice the length of the base.

Then multiply this sum by the height of the wedge and the breadth of the base, and $\frac{1}{6}$ of the product will be the solid contents.



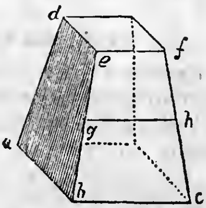
Example.—Required the solidity of a wedge whose base, *a b*, is 27 feet, *b d*, 8 feet, and whose edge, *c b*, is 36 feet, and the perpendicular height 22 feet?

First, 36=length of edge.
54=twice the lgth. of the base.
$\overline{90}\times22\times8\div6=2660$ cubic ft.

PROBLEM VIII.

To find the Solidity of a Rectangular Prismoid.

Rule.—To the sum of the areas of the two ends, *a b c*, *d e f*, add four times the area of a section, *g h*, parallel to and equally distant from the parallel ends, and this sum, multiplied by $\frac{1}{6}$ of the height, will give the solidity.



Example.—What is the solidity of a rectangular prismoid, *a b c d*, the length and breadth of one end being 14 by 12 inches, and the other 6 by 4 inches, and the perpendicular 30 feet 6 inches?

First, $14\times12=168$=area of lower base.
$6\times4=24$= " upper "
192

$\overline{14+6}\div2=10$ } length and breadth
$\overline{12+4}\div2=8$ } of middle section.
80
4
320=area of 4 times middle section.

Then,
192
320
512
$61=\frac{1}{6}$ height.
512
3072
31232

And $31232\div1728=18{\cdot}074$ cubic ft. Ans.

SECTION IV.

OF THE CYLINDER, CONE, AND SPHERE.

DEFINITIONS.

1. A *cylinder* is a solid, having equal and parallel circles for its ends, and is described by the revolution of a rectangle about one of its sides.

2. A *cone* is a solid body, of a true taper from the base to a point, which is called the vertex, and has a circle for its base.

3. A *frustrum* of a cone is what remains after a portion is cut off by a plane, parallel to the base.

4. A *conoid* is a solid, generated by the revolving of a parabola or hyperbola around its axis.

5. A *spheroid* is a solid, generated by the revolution of an ellipse about either of its axes.

6. A *sphere* is a solid, terminated by a curved surface, all the points of which are equally distant from a point within, called the center. A sphere may be described by the revolution of a semicircle about a diameter.

7. A *radius* of a sphere is a line drawn from the center to any part of the surface; as,

8. The *diameter* of a sphere is a line drawn through the center, and terminated at both ends by the surface. All diameters of a sphere are equal to each other, and each is double the radius.

9. A *segment* of a sphere is a portion of the sphere cut off by any plane. This plane is called the *base* of the segment. The *height* of a segment is the distance from the middle of its base to the convex surface.

10. A *zone* is a portion of the surface of a sphere, included between two parallel planes, which form its bases. If the bases are equally distant from the center, it is called the *middle zone*. The *height* of a zone is the perpendicular distance between the two planes which form its bases.

11. A *cylindrical ring* is a solid, formed by bending a cylinder, as a cylindrical bar of iron, until the two ends meet each other.

12. A *parabola* is a section of a cone when cut by a plane parallel to its sides.

13. A *hyperbola* is the section of a cone when cut by a plane, making a greater angle with the base than the side of a cone makes.

14. The *transverse axis* is the longest straight line that can be drawn in an ellipse.

15. The *conjugate axis* is a line drawn through the center, at right angles to the transverse axis.

16. An *abscissa* is a part of any diameter contained between its vertex and an ordinate.

17. The *focus* is the point in the axis where the ordinate is equal to half the perimeter.

PROBLEM I.

To find the Convex Surface of a Cylinder.

Rule.—Multiply the circumference of the base by the length of the cylinder, and the product will be the convex surface required: To this add the areas of the two ends when the entire surface is required.

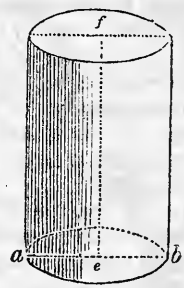
Example.—What is the convex surface of a right cylinder, whose length is 23 feet, and the diameter of its base 3 feet?

$$3 \times 3{\cdot}14159 = 9{\cdot}42477$$
Then, $9{\cdot}42477 \times 23 = 216{\cdot}76971 =$ surface.

PBOBLEM II.

To find the Solidity of a Cylinder.

Rule.—Multiply the area of the base by the height, and the product will give the solid contents.



Examples.—1. What is the solidity of a cylinder, the diameter, *a b*, of whose base is 16 feet, and its height, *e f*, 28 feet?

First, find the area of the base by $\overline{16}^2 = 256$.
Then, $256 \times {\cdot}7854 = 201{\cdot}0624 =$ area of the base.
Then, $201{\cdot}0624 \times 28 = 5629{\cdot}7472 =$ solid contents.

2. The Winchester bushel is a hollow cylinder, $18\frac{1}{2}$ inches in diameter and 8 inches deep: what is its capacity?

First, the area of the base $= \overline{18{\cdot}5}^2 \times {\cdot}7854 = 268{\cdot}8025$.
Then, $268{\cdot}8025 \times 8 = 2150{\cdot}42 =$ capacity in cubic inches.

NOTE. By this rule, every sealer of weights and measures may determine the exact capacity of any *measure* submitted to his inspection. And so any one may test the accuracy of any measure, whether dry or liquid, by reducing its capacity to cubic inches, and dividing by the number of cubic inches contained in such measure. The divisor for any measure may be found in the Table of Weights and Measures, page 13.

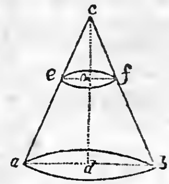
3. How many gallons of oil will a can of a cylindrical form hold, whose diameter is $28\frac{5}{8}$ inches, and whose height is 4 feet 3 inches.

Area of the base by Prob. iii., Sec. ii., or by the Tables of Areas of Circles, $= 643{\cdot}54$; and $643{\cdot}54 \times 51 \div 221{\cdot}184 = 148{\cdot}39$ gallons.
1 gallon $= 221{\cdot}184$ cubic inches.

PROBLEM III.

To find the Convex Surface of a Cone.

Rule.—Multiply the perimeter of the base by the slant height, and $\frac{1}{2}$ the product will be the surface; to which add the area of the base when the entire surface is required.



Example.—The diameter of the base of a right cone, *a b*, is 3 feet, and the slant height, *c a*, is 15 feet: what is the convex surface?

First, $3\times3{\cdot}14159=9{\cdot}42477=$ circum. of base.
Then, $9{\cdot}42477\times15\div2=70{\cdot}686$ sq. ft.

PROBLEM IV

To find the Solidity of a Cone.

Rule.—Multiply the area of the base by $\frac{1}{3}$ of the height, and the product will be the solidity

Example.—What is the solidity of a right cone, whose perpendicular height, *c d*, is $10\frac{1}{2}$ feet, and the circumference of the base is 9 feet?

We here multiply the area of the base by $\frac{1}{3}$ of the height, and the product is the solidity.

First, $9^2=81$, and $10\frac{1}{2}\div3=3\frac{1}{2}=\frac{1}{3}$ height.
Now, $81\times{\cdot}7854=63{\cdot}6174$, area of base.
Then, $63{\cdot}6174\times3\frac{1}{2}=222{\cdot}6609$. Ans.

PROBLEM V.

To find the Surface of a Frustrum of a Cone.

Rule.—Add together the circumferences of the two ends, and multiply the sum by $\frac{1}{2}$ the slant of the frustrum; the product will be the convex surface: to which add the areas of the two bases when the entire surface is required.

NOTE. This rule is precisely the same as that for a *frustrum* of a pyramid, and if a cone be considered as a pyramid of an infinite number of sides, it is equally applicable to the measurement of the *frustrum* of a cone.

Example.—What is the convex surface of the frustrum of a cone, the circumference of the greater base, *a b*, being 30 feet, and of the smaller, *e f*, 10 feet, the slant height, *c a*, being 20 feet?

$30+10=40=$ circum. of two ends. $10\frac{1}{2}=$ slant height.
$40\times10=400=$ convex surface.

PROBLEM VI.

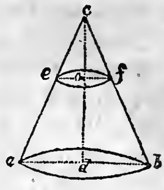
To find the Solidity of the Frustrum of a Cone.

Rule.—Add to the areas of the two ends of the frustrum the *square*

root of their product. Then multiply this sum by $\frac{1}{3}$ of the perpendicular height, and the product will be the solidity.

NOTE. If a *cone* and a *pyramid* have equal bases and altitudes, they are equal in their solidity. Consequently, the rule already given for the *frustrum* of a *pyramid* is equally applicable to the frustrum of a cone.

Example.—How many gallons of ale are contained in a cistern in the form of a conic frustrum, *a b e f*, if the larger diameter, *a b*, be 9 feet, and the smaller diameter, *e f*, 7 feet, and the depth, *c o d*, 9 feet?



$$\left.\begin{array}{l}\overline{9}^2=81\\ 7^2=49\end{array}\right. \text{ and } \left\{\begin{array}{l}81\times\cdot7854=63\cdot61=\text{area of lower base.}\\ 49\times\cdot7854=38\cdot48=\text{ “ upper “}\\ \qquad\qquad\qquad\overline{102\cdot09}\end{array}\right.$$

Then, $63\cdot61\times38\cdot48=2447\cdot71$ | $102\cdot09+49\cdot46=151\cdot55.$

$\sqrt{2447\cdot71}=49\cdot46$ | $151\cdot55\times3=454\cdot65$ cubic feet.

$454\cdot65\times1728=785635$ cubic inches.

$785635\div282=2785$ gals. Ans.

OF SPHERES.

PROBLEM VII.

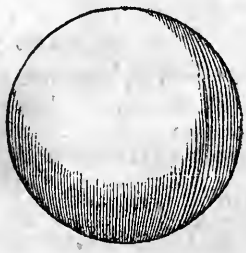
To find the Surface of a Sphere or Globe.

Rule.—Multiply the diameter of the sphere by its circumference, and the product will be the surface. Or, multiply the square of the diameter by 3·14159.

Example.—What is the surface of a sphere whose diameter is 7 feet?

First, $7\times3\cdot14159=21\cdot99113$ = circumference.

Then, $21\cdot99113\times7=153\cdot93791$ sq. ft. = surface.

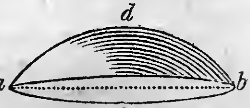


PROBLEM VIII.

To find the Convex Surface of a Spherical Zone or Segment.

Rule.—Multiply the height of the zone or segment by the whole circumference of the sphere of which it is a part, and the product will be the convex surface.

Example.—If the axis of a sphere, be 42 inches, what is the convex surface of a segment or zone, *a b d*, whose height, *c d*, is 9 inches?



First, $42 \times 3{\cdot}14159 = 131{\cdot}9468$ = circumference.

9 = height.

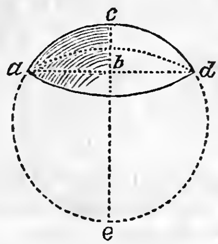
$1187{\cdot}5212$ = surface in square inches.

PROBLEM IX.

To find the Solidity of a Sphere or Globe.

Rule.—Multiply the cube of the diameter, *c e*, by the decimal ·5236. Or multiply the square of the diameter by the circumference, and $\frac{1}{6}$ of the product will be the contents.

Example.—What is the solidity of a globe whose diameter, *c e*, is 12 inches?



$\overline{12}^2 \times 3{\cdot}14159 = 452.38996$ = surface of the sphere.

Then, $452{\cdot}38996 \times 12 \div 6 = 904{\cdot}78$ = solidity.

Or thus: $\overline{12}^3 = 1728$ = cube of the diameter.

And $1728 \times {\cdot}5236 = 904{\cdot}78$ = solid contents.

PROBLEM X.

To find the Solidity of a Spherical Segment.

Rule.—To three times the square of the radius, *a b*, of its base, add the square of its height, *b c*; then multiply the sum by the height, and the product by ·5236, for the contents.

Example.—What is the solidity of the segment, *a d c*, (of the sphere *e c*,) whose height, *b c*, is 8 feet, and the diameter of whose base, *a d*, is 14 feet?

$7^2 = 49 \times 3 = 147$

$8^2 = \quad 64$

$211 \times 8 = 1688 \times {\cdot}5236 = 883{\cdot}836$. Ans.

NOTE. The solidity of a spherical segment is frequently required when the radius of its base is not given; but if the *diameter* of the sphere and the height of the segment be known, the solidity may be easily found by the following

Rule.—From three times the diameter of the sphere, subtract twice the height of the segment; then multiply the remainder by the square of the height, and the product by the decimal ·5236.

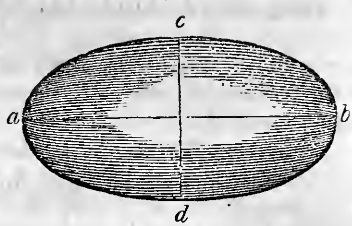
OF SPHEROIDS.

PROBLEM XI.

To find the Solidity of a Spheroid.

Rule.—Multiply the square of the revolving axis by the fixed axis; and the product, multiplied by ·5236, will give the solidity.

Example.—What is the solidity of an oblong spheroid, whose longer axis, *a b*, is 30, and the shorter, *c d*, 20, the revolving axis being *c d*?



$\overline{20}^2 \times 30 = 12000.$

Then, $12000 \times \cdot 5236 = 6283 \cdot 2.$

NOTE. If the generating ellipse revolves about its major axis, the spheroid is *prolate* or oblong; if about its minor axis, the spheroid is *oblate.*

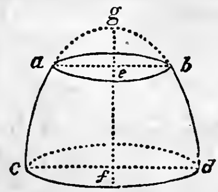
OF PARABOLIC CONOIDS AND SPINDLES.

PROBLEM XII.

To find the Solidity of a Parabolic Conoid.

Rule.—Multiply the square of the diameter of the base by the altitude, and the product by ·3927 (which is ½ of 7854), and it will give the contents.

Example.—What is the solidity of a parabolic conoid, whose height, *f g*, is 60, and the diameter, *c d*, of its base 100 inches?



$\overline{100}^2 = 10000$

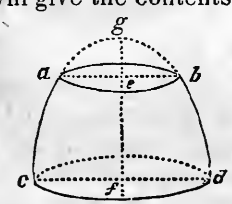
And $10000 \times 60 \times \cdot 3927 = 235620$. Ans.

PROBLEM XIII.

To find the Solidity of a Frustrum of a Paraboloid

Rule.—Multiply the sum of the squares of the diameters of the two ends, *a b* and *c d*, by the height of the frustrum, *e f*, and the product by ·3927 (which is ½ of ·7854), and it will give the contents.

Example.—What is the solidity of the *frustrum* of a paraboloid, *a b c d*, whose diameter, *c d*, is 54, *a b*, 28, and height, *f e*, 18 inches?



$\overline{54}^2 = 2916$ Then, $3700 \times 18 \times \cdot 3927 = 26153 \cdot 82$. Ans.

$\overline{28}^2 = 784$

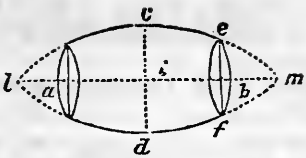
3700

PROBLEM XIV.

To find the Solidity of a Parabolic Spindle.

Rule.—Multiply the square of the middle diameter, *c d*, by the length of the spindle, *l m*, and the product by ·41888 (which is $\frac{8}{15}$ of 7854), and it will give the solidity.

Example.—Required the solidity of the parabolic spindle, $l\ m$, $c\ d$, whose length, $l\ m$, is 100, and diameter, $c\ d$, 40.



$\overline{40}^2=1600$.

And $1600\times 100\times \cdot 41888 = 67020\cdot 8$. Ans.

PROBLEM XV.

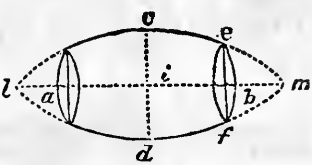
To find the Solidity of the Middle Frustrum of a Parabolic Spindle.

Rule.—Add together 8 times the square of the greatest diameter, $c\ d$, 3 times the square of the least diameter, $f\ e$, and 4 times the product of these two diameters; multiply the sum by the length, $a\ b$, and the product by $\cdot 05236$ (which is $\frac{1}{60}$ of $3\cdot 1416$); this will give the solidity.

Example.—What is the solidity of the frustrum of a parabolic spindle, whose dimensions are as follows: $a\ b$, 60, $c\ d$, 40, $f\ e$, 30 inches?

$$\begin{array}{lr}
\overline{40}^2= & 1600 \\
 & 8 \\
\hline
 & 12800 \\
\overline{30}^2=900\times 3= & 2700 \\
\hline
 & 15500 \\
30\times 40\times 4= & 4800 \\
\hline
\end{array}$$

$20300\times 60\times \cdot 05236=63774\cdot 48$. Ans.



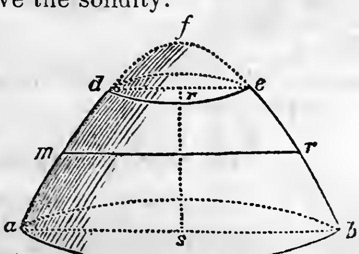
OF HYPERBOLOIDS AND HYPERBOLIC CONOIDS.

PROBLEM XVI.

To find the Solidity of a Hyperboloid.

Rule.—To the square of the radius of the base, $a\ s$, add the square of the middle diameter, $m\ r$; multiply this sum by the height, $s\ f$, and the product by $\cdot 5236$, and it will give the solidity.

Example.—What is the solidity of a hyperboloid, $a\ b\ f$, whose base, $a\ b$, is 40 inches, and height, $s\ f$, 30 inches; and whose middle diameter, $m\ r$, is 30 inches?



$\overline{20}^2=400$

$\overline{30}^2=900$

1300 And $1300\times 30\times \cdot 5236\div 1728=11\cdot 817$ cubic feet.

PROBLEM XVII.

To find the Solidity of the Frustrum of a Hyperbolic Conoid.

(See the foregoing figure.)

Rule.—Add together the squares of the greatest and least semidiameters, $a\ s$ and $d\ r$, and the square of the whole diameter, $m\ r$, in the middle of the two; multiply this sum by the height, $r\ s$, and the product by ·5236, and it will give the solidity.

Example.—Required the solidity of the frustrum of a hyperbola, $a\ b\ d\ c$, whose semidiameter, $a\ s$, is 20 inches, and $d\ r$, 10 inches; the middle diameter, $m\ r$, 30 inches, and whose height is 20 inches?

$\overline{20}^2=400$
$\overline{10}^2=100$
$\overline{30}^2=900$
$\overline{1400}$ Then, $1400\times 20\times \cdot 6236\div 1728=8\cdot 426$ cubic feet.

PROBLEM XVIII.

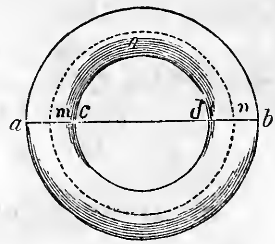
To find the Convex Surface of a Cylindrical Ring.

Rule.—To the thickness of the ring, $a\ c$, add the inner diameter; then multiply this sum by the thickness, and the product by 9·8696 (which is the square of 3·1416), and it will give the convex surface required.

Example.—The thickness, $a\ c$, of a cylindrical ring is 4 inches, and the inner diameter, $c\ d$, is 14 inches; required the convex surface.

$ac+cd=4+14=18.$

Then, $18\times 4\times 9\cdot 8696=710\cdot 612$ square inches = convex surface.



PROBLEM XIX.

To find the Solidity of a Cylindrical Ring.

Rule.—To the thickness of the ring, $a\ c$, add the inner diameter, $c\ d$; then multiply the sum by the square of the thickness, and the product by 2·4674 (which is $\frac{1}{4}$ of the square of 3·1416), and it will give the solidity.

Example.—Required the solidity of an anchor ring, whose inner diameter is 8 inches, and thickness in metal 3 inches.

First, $3+8=11$
$3\times 3=9=$ square of thickness.
$99\times 2\cdot 4674=244\cdot 2726=$ solidity in inches.

GAUGING OF CASKS.

Gauging is a practical art, which does not admit of being treated in a very scientific manner.

Casks are not commonly constructed in exact conformity with any regular mathematical figure. By most writers on this subject, however, they are considered as nearly coinciding with one of the following forms:

1. } The middle frustrum { of a spheroid,
2. } { of a parabolic spindle.

3. } The equal frustrums { of a paraboloid,
4. } { of a cone,

and their contents in cubic inches may be found by the rules in mensuration, for determining the solidity of these figures.

To find the Contents of a Cask by four Dimensions.

Rule.—Add together the squares of the bung and head diameter, and the square of double the diameter, taken in the middle between the bung and head; multiply the sum by the length of the cask, and the product by ·1309.

To find the Contents of a Cask in the form of the Middle Frustrum of a Spheroid.

Rule.—Add together the square of the head diameter and twice the square of the bung diameter; multiply the sum by $\frac{1}{3}$ of the length, and the product by ·00355, for a wine gallon of New York standard measure, or ·0034 for old English gallons. If D and d = the two diameters, and l = the length, the capacity in inches = $(2D^2 \times d^2) \times \frac{1}{3}l \times \cdot 7854$. And by substituting ·00355 for ·7854, we have the capacity in wine gallons.

Example.—What is the capacity of a cask of the *second* form, whose length is 30 inches its head diameter 18 inches, and its bung diameter 24?

$$\begin{aligned} \overline{18}^2 &= 324 \\ 2\times\overline{24}^2 &= 1152 \\ &\overline{1476} \\ \tfrac{1}{3} \text{ of } 30 &= 10 \\ &\overline{14760}\times\cdot 00355 = 52\cdot 39 \text{ wine gallons. Ans.} \end{aligned}$$

To find the Contents of a Cask in the form of two equal Frustrums of a Cone.

Rule.—Add together the square of the head diameter, the square of the bung diameter, and the *product* of the two diameters; multiply the sum by $\frac{1}{3}$ of the length and the product by ·00355 for New York wine gallons, or 0034 for old English gallons of 231 cubic inches.

Example.—What is the capacity of a cask whose dimensions are as follows: 30 inches long, head diameter 18 inches, and bung diameter 24 inches?

$$\begin{aligned} \overline{18}^2 &= 324 \\ \overline{24}^2 &= 576 \\ \text{Product of 2 diam.} &= 432 \\ &\overline{1332}\times 10 = 13320\times\cdot 00355 = 46\cdot 286. \end{aligned}$$

Or $(D^2+d^2+Dd)\times\frac{1}{3}l\times 00355$.

OF ARTIFICERS' WORK.

Artificers compute the contents of their works by several different measures, viz.:

1. *Glazing and mason work*, by the foot.
2. *Painting, plastering, paving, &c.*, by the yard.
3. *Flooring, partitioning, roofing, &c.*, by the foot.
4. *Brickwork*, by the 1000, or cubic foot.

BRICKLAYERS' WORK.

Brickwork is estimated at the rate of a brick and a half thick. So that, if a wall be more or less than this standard thickness, it must be reduced to it, as follows:

Rule.—Multiply the superficial contents of the wall by the number of half bricks in the thickness, and $\frac{1}{3}$ of that product will be the contents required.

BRICKS AND LATHS.

DIMENSIONS.

15	common bricks	to a cubic foot	of 8-inch	wall	when laid.
$22\frac{1}{2}$	"	"	of 12	"	"
30	"	"	of 16	"	"
$37\frac{1}{2}$	"	"	of 20	"	"

Laths are $\frac{1}{4}$ to $\frac{1}{2}$ inches by 4 feet in length, are usually set $\frac{1}{4}$ of an inch apart, and a bundle contains 100.

Stourbridge fire-brick $9\frac{1}{8}$ by $4\frac{5}{8}$ by $2\frac{3}{8}$ inches.

Example.—How many bricks will it require to build a house 30 feet square, 20 feet high, and 12 inches thick, above which is a triangular gable, rising 12 feet, and 8 inches thick?

$30 \times 6 = 180 = 1$ gable end
$30 \times 6 = 180 = 1$ " "
$\overline{360} \times 15 =$ - - - - 5400
$30 + 30 = 60 =$ two side walls.
$28 + 28 = 56 =$ two end "
$\overline{116}$
$20 =$ height.
$\overline{2320} \times 22\frac{1}{2} =$ - - - 52200
57600 Ans.

OF MASONRY.

Masonry is the science of preparing and combining stones, so as to properly touch, indent, or lie on each other, and become masses of walling and arching for the purposes of building.

In stone walling, the bedding joints ought each to be laid horizontally when the top of the wall is to terminate so.

In building bridges and fence walls upon inclined surfaces, the bedding joints ought to follow the general direction of the work.

A wall which consists of unhewn stone, is called a rubble wall, whether or not mortar is used. This species of work is of two kinds—coursed and uncoursed. In the former, the stones are gauged and dressed by the hammer, and the masonry laid in horizontal courses, but not always confined to the same thickness. The uncourse rubble wall is formed by laying the stones in the wall as they come to hand, without any previous gauging or sorting.

Walls, columns, blocks of stone or marble, &c., are measured by the cubic foot; and pavements, slabs, chimney-pieces, &c., by the superficial or square foot. Cubic or solid measure is used for the materials, and square measure for the workmanship. In the solid measure, the true length, breadth and thickness, are taken, and multiplied continually together. In the superficial, there must be taken the length and breadth of every part of the projection which is seen without the general upright face of the building.

DIGGING.

24 cubic feet of loose sand, 17 cubic feet of clay, 18 cubic feet of earth, 13 cubic feet of chalk, equal 1 ton gross. 1 cubic yard of earth before digging will occupy about $1\frac{1}{2}$ cubic yards when dug, and contains 21 striked bushels, which is considered a *single load*, and double these quantities a *double load*.

Weight of a Cubic Foot of various Substances in common use.

Sand - - - - - - - - - -	solid	112·5 lbs.
" - - - - - - - - - -	loose	95·
Earth - - - - - - - - - -	"	93·75
Common soil - - - - - - - - -		124·
Strong soil - - - - - - - - -		127·
Clay - - - - - - - - - - - -		120· to 135·
Clay and stone - - - - - - - -		160·
Common stone - - - - - - - -		158·
Brick - - - - - - - - - - - -		119·
Granite - - - - - - - - - - -		169·
Marble - - - - - - - - - - - -		166· to 169·
1 cubic yard of sand weighs - - -		3037 lbs.
1 " of common soil weighs		3429 "

(See Table of Specific Gravities of Bodies.)

HYDRAULIC CEMENT.

To build a rod of brick work requires $1\frac{1}{2}$ cubic yards of chalk lime and 3 single loads or yards of drift; or 1 cubic yard of stone lime and $3\frac{1}{2}$ single loads of sand; or 36 bushels of cement and 36 bushels of sharp sand.

A load of mortar is 27 cubic feet, and for its preparation requires 9 bushels of lime and 1 cubic yard of sand. Lime and sand lessens about $\frac{1}{3}$ when made into mortar; likewise cement and sand.

Lime, or cement and sand, to make mortar, requires as much water as is equal to $\frac{1}{3}$ of their bulk, or about $5\frac{1}{2}$ barrels for each rod of brick-

work built with mortar. A barrel of cement is 5 struck bushels, and weighs 3 cwt. 1 yard, or 9 superficial feet of the standard thickness ($1\frac{1}{2}$ brick thick), requires of cement about $2\frac{1}{4}$ bushels.

1 yard superficial of pointing to brick work in cement, requires about $\frac{1}{8}$ of a bushel.

1 yard of plastering in cement, requires $\frac{3}{4}$ of a bushel. 1 bushel of cement will cover $1\frac{1}{7}$ square yards at 1 inch thick.

or $1\frac{1}{2}$ " " $\frac{3}{4}$ " "

or $2\frac{1}{4}$ " " $\frac{1}{2}$ " "

BROWN MORTAR.

One-third Thomaston or Rhode Island lime, two-thirds sand, and a small quantity of hair.

TABLE,

Showing the capacity of Cisterns, Wells, &c., in Ale Gallons and Hogsheads, in proportion to their diameters and depths.

Capacity in Gallons.		Capacity in Hogsheads, Ale measure, computed at 63 Gallons per Hogshead.								
Diam. in feet.	Depth. 1 FT.	Feet. 6	Feet. 7	Feet. 8	Feet. 9	Feet. 10	Feet. 11	Feet. 12	Feet. 13	Feet. 14
3	43·3	4·1	4·8	5·5	6·2	6·8	7·5	8·2	9·	9·6
$3\frac{1}{2}$	59·	5·6	6·5	7·4	8·4	9·3	10·3	11·1	12·1	13·1
4	77·6	7·4	8·6	9·8	11·1	12·2	13·5	14·8	16·	17·2
$4\frac{1}{2}$	97·5	9·3	10·8	12·4	13·9	15·5	17·	18·6	20·1	21·6
5	120·3	11·1	13·3	14·8	17·2	19·1	21·	22·2	24·4	26·7
$5\frac{1}{2}$	145·5	13·8	16·1	18·4	20·8	23·1	25·4	27·6	29·9	32·3
6	173·2	16·5	19·2	22·	24·7	27·5	30·2	33·	35·7	38·4
$6\frac{1}{2}$	203·3	19·3	22·6	25·7	29·	32·2	35·5	38·6	41·9	45·1
7	235·8	22·4	26·2	29·8	33·7	37·4	41·1	44·8	48·6	52·4
$7\frac{1}{2}$	270·7	25·8	30·	34·4	38·6	42·9	47·2	51·6	55·8	60·1
8	308·	29·3	34·2	39·	44·	48·9	53·8	58·6	63·5	68·4
$8\frac{1}{2}$	347·7	33·	38·6	44·	49·6	55·2	60·7	66·	71·6	77·2
9	390·	37·1	43·3	49·4	55·7	61·9	68·1	74·2	80·4	86·6
$9\frac{1}{2}$	434·3	41·3	48·2	55·	62·	68·9	75·8	82·6	89·5	96·5
10	481·	45·8	53·4	61·	68·7	76·4	84·	91·6	99·2	106·9
11	583·3	55·5	64·8	74·	83·3	92·6	101·8	111·	120·3	129·6
12	693·	66·	77·	88·	99·	110·	121·	132·	143·	154·

EXPLANATION.—Find the diameter in feet in the left-hand column of the table; then move to the right on the *same line* till you come under the depth in feet, and you will have the answer sought for, in hogsheads. Thus, if the capacity of a cistern be required, whose diameter is $8\frac{1}{2}$ feet, and depth 10, we find opposite $8\frac{1}{2}$, and directly under 10, on the same line, 55·2 which is the true answer sought.

NOTE. The above table will be found useful and convenient in the construction of public *reservoirs*, as well as private cisterns, as it will enable any one to determine at a glance the dimensions in diameter and depth to hold a given number of hogsheads. For a private dwelling, the capacity should not be less than 7 feet diameter by 8 or 9 feet deep. In the construction of reservoirs for the supply of *tenders* on railroads, the *height* should be double that of the diameter, in order to obtain head of water, and thus save time in replenishing the tender.

The preceding table was computed in English ale gallons; although in the State of New York but one standard measure for all liquids exists. The following additional rules for computing liquids will be found useful, and often very convenient:

APPROXIMATING RULES FOR CALCULATING LIQUIDS.

To find the number of Gallons contained in any Square or Rectangular Cistern

Rule.—Multiply the contents of the cistern in cubic feet by 7·812 (*for ale* 6·127), or the contents in cubic inches by ·004521 (for ale ·00353), and the product will be the number of gallons, nearly.

Example.—A cistern that is 6 feet long, 4½ feet wide, and 4 feet deep, required its contents in New York gallons.

$6 \times 4{\cdot}5 \times 4 = 108$ cubic feet.

And $108 \times 7{\cdot}812 = 843{\cdot}69$ gallons $= 13{\cdot}39$ hhds.

Or, 6 feet = 72 inches; 4½ feet = 54 inches; 4 feet = 48 inches.

Then, $72 \times 54 \times 48 = 186624$ cubic inches.

And $186624 \times {\cdot}004521 = 843{\cdot}72$ gallons $= 13{\cdot}39$ hhds.

Any two dimensions of a Square or Rectangular Cistern being given, to find the third that shall contain any number of gallons (ale or wine) required.

Rule.—Divide the number of gallons that the cistern is required to hold by the product of the two dimensions, multiplied by either of the multipliers as above, according as the dimensions are given in feet or inches, or whether ale or wine measure be required, and the quotient will be the third dimension of the cistern, nearly.

Example.—Required the depth of a cistern to contain 1260 gallons, the length being 6 feet, and width 4½ feet?

$6 \times 4{\cdot}5 \times 7{\cdot}812 = (210{\cdot}92$; and $1260 \div 210{\cdot}92 = 5{\cdot}97$ feet deep.

NOTE. For calculating the capacity of right cylinders, or cylinders of the figure of the frustrum of a cone, see Mensuration of the Cylinder, &c.

CARPENTERS' AND JOINERS' WORK.

To this branch belongs all the wood-work of a house, such as flooring, partitioning, roofing, &c. Large and plain articles are usually measured by the square foot or yard, &c.; but enriched moldings, and some other articles, are often estimated by running or lineal measures, and some things are rated by the piece.

PLASTERERS' WORK.

Plasterers' work is of two kinds, namely: *ceiling*, which is plastering upon laths, and *rendering*, which is plastering upon walls, which are measured separately.

NOTE. The contents are estimated either by the foot or yard, or square of 100 feet. Enriched moldings, &c., are rated by running or lineal measure.

Proper deductions are to be made for chimneys, doors, &c. But the windows are seldom deducted, as the plastered returns at the top and sides are allowed to compensate for the window opening.

3 hundred of lime, 4 loads of sand, and 10 bushels of hair to 200 yards of render set; 4½ hundreds of lime, 6 loads of sand, 15 bushels of hair, and 2 loads of laths and nails, to 270 feet of lath plaster work.

1 bundle of laths, and 5 hundred of nails, will cover 4½ yards superficial.

PAINTERS' WORK.

Painters' work is computed in square yards. Every part is measured where the color lies; and the measuring line is forced into all the moldings and corners.

Windows are done at so much a piece; and it is usual to allow double measure for carved moldings, &c.

GLAZIERS' WORK.

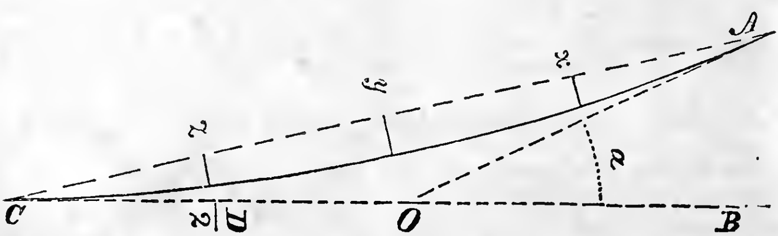
Glaziers' work is done at so much per light, that is, computed at a given price for putting in each pane of glass, according to the size.

(See article on Paints, Lockers &c., under Miscellaneous Notes.)

ENGINEERING.

HINTS ABOUT LAYING OUT CURVES.

In laying out curves, the following method has the advantages of great accuracy and expedition over that of angles taken by an instrument. *The stations are supposed to be* 100 *feet apart.*



In the cut, the chord $A\ C$, is divided into four equal parts: the middle ordinate, y, is expressed by $2 \times 2 = 4$, and x and z are expressed by $1 \times 3 = 3$;

Measure at the intersection the angle, $A\ O\ B$, in the above figure, of the two straight lines to be joined, by observing the quantity of deflection in 100 feet; that is, *the length of the arc, as of a circle comprehended between the angular lines*, the radius of the arc being 100 feet. When the angle is small, the length of the chord is sufficiently

accurate. Divide this quantity by the number of stations you propose to give the curve. The quotient is D hereafter mentioned. $\frac{10000}{D}$ is R, radius of the curve, or reciprocally, $\frac{10000}{R}$ is D deflection. Produce the tangent, 100 feet beyond the commencement of the curve, and set off so as to form an isosceles triangle $\frac{D}{2}$. You have then obtained the first station on the curve. Produce the first chord 100 feet beyond this station, and set off D. Proceed in like manner, setting off D at each station. Having completed the curve, produce the last chord 100 feet, and set off $\frac{D}{2}$. The point thus obtained, and the end of the curve, form the line of the tangent.

Having completed a curve as above, if you do not arrive at the destined point, measure the variation, v, and divide it among the stations in proportion to the square of the distance from the point of commencement.

To find what difference of direction will be obtained for the tangent by making the offset of the above-mentioned variation, v, divide twice the variation by the number of stations. The quotient $\frac{2v}{n}$, is the difference of direction for one station.

In running a curve for laying of track upon a finished grading, if, both ends being adjusted, the curve varies at intermediate points from the grading, measure the variation v, at the point g, where it is greatest, or assume some quantity v, that shall be a judicious average of variations. In making the change, it will not be proper to occupy less than 4 stations on each side of the point g, making 8 stations in all. *If* 8 *stations are occupied*, set off on each side of g, at the 3d station therefrom, $\frac{1}{8}v$; then, proceeding towards g, set off successively at the stations, $\frac{4}{8}$, $\frac{7}{8}$, $\frac{8}{8}$, v. *If* 12 *stations are to be occupied*, set off at 5 stations from g, $\frac{1}{18}v$; then successively toward g, $\frac{4}{18}$, $\frac{9}{18}$, $\frac{14}{18}$, $\frac{17}{18}$, $\frac{18}{18}v$. These fractions can always be obtained as follows: Decide upon some even number of stations, to be used on each side of g. The denominator is $\frac{1}{2}$ the square of the number, n, of stations on one side of g, and the numerators are found by deducting from the denominators successively the square numbers 1, 4, 9, 16, 25, &c., till the remainder is a perfect square, which will be one-half the denominators, and consequently the square of $\frac{1}{2}n$. It will also be arrived at in $\frac{1}{2}n$ from g. *Thus*, if 14 be decided upon as the number of stations on each side g, 98 will be the denominator, and the numerators will be, successively, 98, 97, 94, 89, 82, 73, 62, 49. This last No. is $\frac{1}{2}$ the denominator, and 7 stations from g. It is likewise $\overline{7}^2$. The remaining numerators are, inversely, the succession of square numbers previously deducted from the numerators. *Thus*, continuing the above example from 49, we have 36, 25, 16, 9, 4, 1. If necessary, the numerator may be preserved equal to the denominator on either side of g, for an indefinite period before diminishing.

For distances not exceeding 20° in length on curves, *the offsets from a tangent are* (sufficiently near for practice) *in proportion to the squares of the distances from the commencement.*

It is proper in laying out a curve by this rule to measure the distances, both on the curve and on the tangent. For approximation in the field, this rule is valuable for a distance of 40°; after which it is

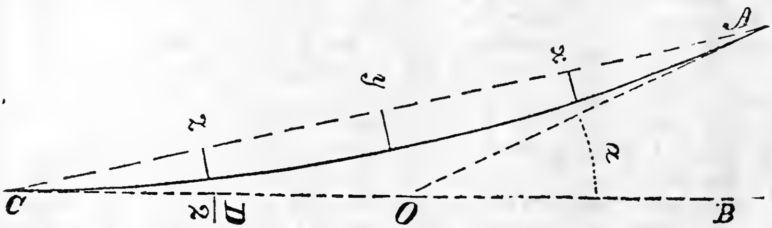
rather unsafe, and for 90° absolutely useless. It will be found serviceable for calculating vertical curves in a profile of grades, measuring the angle of the two planes, as of the lines in horizontal curves. *The calculation by this rule always exceeds the true offset.* The excess for 20° on a 1° curve (radius 5730), is 1·14 feet ; for 30°, 5·86 feet ; for 60°, 94 feet ; and for 90°, 490 feet. *For* 10°, *the variation is scarcely perceptible.*

To obtain ordinates from a chord, observe the three following rules :

1. The deflection D for chords is in proportion to the square o. their lengths.
2. The ordinate at the middle of a chord is always $\frac{D}{8}$.
3. All the ordinates of a chord are in proportion to the rectangle of the two parts into which they divide the chord. *Thus,* divide the chord into 10 parts, equal ; the middle ordinate will be expressed by $5\times5=25$; and the others will be successively expressed by

$$4\times6=24\ ;\ 3\times7=21\ ;\ 2\times8=16\ ;\ \text{and}\ 1\times9=9.$$

Where R is 5000, and the chord 100, D is 2, or $\frac{10000}{R}$, and the middle ordinate is 25 ; the others are 24, 21, 16, and ·09, on each side the center.



In the cut, the chord $A\ C$, is divided into four equal parts : the middle ordinate, y, is expressed by $2\times2=4$, and x and z are expressed by $1\times3=3$; and x z are therefore each $\frac{3}{4}$ of y.

The 1st and 2d of these rules will be found useful *for springing rails of any length, or any radius of curvature.* Thus, for 100 feet chord, the middle ordinate is ·25 ; for an 18 feet rail, the middle ordinate : ·25 as $18\times18=$·324 ; $100\times100=10000$; or ·0324 —— ·0324 × 25 = ·0081 feet ; or ·0972 inch, or for practice $\frac{1}{10}$ inch.

The difference in length, between the inner and the outer rail, may be determined as follows : Where R is radius, C the length of the inner rail, and W the width of the track, the difference is $\frac{CW}{R}$.

Where 5 feet is the width, $\frac{500}{R}$ is the difference for 100 feet, *and* $\frac{R}{20}$ *is the length required to gain 3 inches.* This formula will be found convenient for *distributing overlength iron for curves.*

TABLE

Of Squares, Cubes, Square and Cube Roots of Numbers.

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1·0	1·0
2	4	8	1·414213	1·25992
3	9	27	1·732050	1·44225
4	16	64	2·0	1·58740
5	25	125	2·236068	1·70997
6	36	216	2·449489	1·81712
7	49	343	2·645751	1·91293
8	64	512	2·828427	2·0
9	81	729	3·0	2·08008
10	100	1000	3·162277	2·15443
11	121	1331	3·316624	3·22398
12	144	1728	3·464101	2·28942
13	169	2197	3·605551	2·35133
14	196	2744	3·741657	2·41014
15	225	3375	3·872983	2·46621
16	256	4096	4·0	2·51984
17	289	4913	4·123105	2·57128
18	324	5832	4·242640	2·62074
19	361	6859	4·358898	2·66840
20	400	8000	4·472136	2·71441
21	441	9261	4·582575	2·75892
22	484	10648	4·690415	2·80203
23	529	12167	4·795831	2·84386
24	576	13824	4·898979	2·88449
25	625	15625	5·0	2·92401
26	676	17576	5·099019	2·96249
27	729	19683	5·196152	3·0
28	784	21952	5·291502	3·03658
29	841	24389	5·385164	3·07231
30	900	27000	5·477225	3·10723
31	961	29791	5·567764	3·14138
32	1024	32768	5·656854	3·17480
33	1089	35937	5·744562	3·20753
34	1156	39304	5·830951	3·23961
35	1225	42875	5·916079	3·27106
36	1296	46656	6·0	3·30192
37	1369	50653	6·082762	3·33222
38	1444	54872	6·164414	3·36197
39	1521	59319	6·244998	3·39121
40	1600	64000	6·324555	3·41995
41	1681	68921	6·403124	3·44821
42	1764	74088	6·480740	3·47602
43	1849	79507	6·557438	3·50339
44	1936	85184	6·633249	3·53034
45	2025	91125	6·708203	3·55689
46	2116	97336	6·782330	3·58304
47	2209	103823	6·855654	3·60882
48	2304	110592	6·928203	3·63424
49	2401	117649	7·0	3·65930

Number.	Square.	Cube.	Square Root.	Cube Root.
50	2500	125000	7·071067	3·68403
51	2601	132651	7·141428	3·70843
52	2704	140608	7·211102	3·73251
53	2809	148877	7·280109	3·75628
54	2916	157464	7·348469	3·77976
55	3025	166375	7·416198	3·80295
56	3136	175616	7·483314	3·82586
57	3249	185193	7·549834	3·84850
58	3364	195112	7·615773	3·87087
59	3481	205379	7·681145	3·89299
60	3600	216000	7·745966	3·91486
61	3721	226981	7·810249	3·93649
62	3844	238328	7·874007	3·95789
63	3969	250047	7·937253	3·97905
64	4096	262144	8·0	4·0
65	4225	274625	8·062257	4·02072
66	4356	287496	8·124038	4·04124
67	4489	300763	8·185352	4·06154
68	4624	314432	8·246211	4·08165
69	4761	328509	8·306623	4·10156
70	4900	343000	8·366600	4·12128
71	5041	357911	8·426149	4·14081
72	5184	373248	8·485281	4·16016
73	5329	389017	8·544003	4·17933
74	5476	405224	8·602325	4·19833
75	5625	421875	8·660254	4·21716
76	5776	438976	8·717797	4·23582
77	5929	456533	8·774964	4·25432
78	6084	474552	8·831760	4·27265
79	6241	493039	8·888194	4·29084
80	6400	512000	8·944271	4·30887
81	6561	531441	9·0	4·32674
82	6724	551368	9·055385	4·34448
83	6889	571787	9·110433	4·36207
84	7056	592704	9·165151	4·37951
85	7225	614125	9·219544	4·39683
86	7396	636056	9·273618	4·41400
87	7569	658503	9·327379	4·43104
88	7744	681472	9·380831	4·44796
89	7921	704969	9·433981	4·46474
90	8100	729000	9·486833	4·48140
91	8281	753571	9·539392	4·49794
92	8464	778688	9·591663	4·51435
93	8649	804357	9·643650	4·53065
94	8836	830584	9·695359	4·54683
95	9025	857375	9·746794	4·56290
96	9216	884736	9·797959	4·57785
97	9409	912673	9·848857	4·59470
98	9604	941192	9·899494	4·61043

Number.	Square.	Cube.	Square Root.	Cube Root.
99	9801	970299	9·949874	4·62606
100	10000	1000000	10·0	4·64158
101	10201	1030301	10·049875	4·65701
102	10404	1061208	10·099504	4·67233
103	10609	1092727	10·148891	4·68754
104	10816	1124864	10·198039	4·70266
105	11025	1157625	10·246950	4·71769
106	11236	1191016	10·295630	4·73262
107	11449	1225043	10·344080	4·74745
108	11664	1259712	10·392304	4·76220
109	11881	1295029	10·440306	4·77685
110	12100	1331000	10·488088	4·79142
111	12321	1367631	10·535653	4·80589
112	12544	1404928	10·583005	4·82028
113	12769	1442897	10·630145	4·83458
114	12996	1481544	10·677078	4·84880
115	13225	1520875	10·723805	4·86294
116	13456	1560896	10·770329	4·87699
117	13689	1601613	10·816653	4·89097
118	13924	1643032	10·862780	4·94086
119	14161	1685159	10·908712	4·91868
120	14400	1728000	10·954451	4·93242
121	14641	1771561	11·0	4·94608
122	14884	1815848	11·045361	4·95967
123	15129	1860867	11·090536	4·97319
124	15376	1906624	11·135528	4·98663
125	15625	1953125	11·180339	5·0
126	15876	2000376	11·224972	5·01329
127	16129	2048383	11·269427	5·02652
128	16384	2097152	11·313708	5·03968
129	16641	2146689	11·357816	5·05277
130	16900	2197000	11·401754	5·06579
131	17161	2248091	11·445523	5·07875
132	17424	2299968	11·489125	5·09164
133	17689	2352637	11·532562	5·10446
134	17956	2406104	11·575836	5·11723
135	18225	2460375	11·618950	5·12992
136	18496	2515456	11·661903	5·14256
137	18769	2571353	11·704699	5·15513
138	19044	2628072	11·747344	5·16764
139	19321	2685619	11·789826	5·18010
140	19600	2744000	11·832159	5·19249
141	19881	2803221	11·874342	5·20482
142	20164	2863288	11·916375	5·21710
143	20449	2924207	11·958260	5·22932
144	20736	2985984	12·0	5·24148
145	21025	3048625	12·041594	5·25358
146	21316	3112136	12·083046	5·26563
147	21609	3176523	12·124355	5·27763
148	21904	3241792	12·165525	5·28957

Number.	Square.	Cube.	Square Root.	Cube Root.
149	22201	3307949	12·206555	5·30145
150	22500	3375000	12·247448	5·31329
151	22801	3442951	12·288205	5·32507
152	23104	3511808	12·328828	5·33680
153	23409	3581577	12·369316	5·34848
154	23716	3652264	12·409673	5·36010
155	24025	3723875	12·449899	5·37168
156	24336	3796416	12·489996	5·38323
157	24649	3869893	12·529964	5·39469
158	24964	3944312	12·569805	5·40612
159	25281	4019679	12·609520	5·41750
160	25600	4096000	12·649110	5·42883
161	25921	4173281	12·688577	5·44012
162	26244	4251528	12·727922	5·45136
163	26569	4330747	12·767145	5 46255
164	26896	4410944	12·806248	5·47370
165	27225	4492125	12·845232	5·48480
166	27556	4574296	12·884098	5·49586
167	27889	4657463	12·922848	5·50687
168	28224	4741632	12·961481	5·51784
169	28561	4826809	13·0	5·52877
170	28900	4913000	13·038404	5·53965
171	29241	5000211	13·076696	5·55049
172	29584	5088448	13·114877	5·56129
173	29929	5177717	13·152946	5·57205
174	30276	5268024	13·190906	5·58277
175	30625	5359375	13·228756	5·59344
176	30976	5451776	13·266499	5·60407
177	31329	5545233	13·304134	5·61467
178	31684	5639752	13 341664	5·62522
179	32041	5735339	13·379088	5·63574
180	32400	5832000	13·416407	5·64621
181	32761	5929741	13·453624	5·65665
182	33124	6028568	13·490737	5·66705
183	33489	6128487	13·527749	5·67741
184	33856	6229504	13·564660	5·68773
185	34225	6331625	13·601470	5·69801
186	34596	6434856	13·638181	5·70826
187	34969	6539203	13·674794	5·71847
188	35344	6644672	13·711309	5·72865
189	35721	6751269	13·747727	5·73879
190	36100	6859000	13·784048	5·74889
191	36481	6967871	13·820275	5·75896
192	36864	7077888	13·856406	5·76899
193	37249	7189057	13·892444	5·77899
194	37636	7301384	13·928388	5·78896
195	38025	7414875	13·964240	5·79889
196	38416	7529536	14·0	5·80878
197	38809	7645373	14·035668	5·81864
198	39204	7762392	14·071247	5·82847

Number.	Square.	Cube.	Square Root.	Cube Root.
199	39601	7880599	14·106736	5·83827
200	40000	8000000	14·142135	5·84803
201	40401	8120601	14·177446	5·85776
202	40804	8242408	14·212670	5·86746
203	41209	8365427	14·247806	5·87713
204	41616	8489664	14·282856	5·88676
205	42025	8615125	14·317821	5·89636
206	42436	8741816	14·352700	5·90594
207	42849	8869743	14·387494	5·91548
208	43264	8998912	14·422205	5·92499
209	43681	9123329	14·456832	5·93447
210	44100	9261000	14·491376	5·94391
211	44521	9393931	14·525839	5·95334
212	44944	9528128	14·560219	5·96273
213	45369	9663597	14·594519	5·97209
214	45796	9800344	14·628738	5·98142
215	46225	9938375	14·662878	5·99072
216	46656	10077696	14·696938	6·0
217	47089	10218313	14·730919	6·00924
218	47524	10360232	14·764823	6·01836
219	47961	10503459	14·798648	6·02765
220	48400	10648000	14·832397	6·03681
221	48841	10793861	14·866068	6·04594
222	49284	10941048	14·899664	6·05504
223	49729	11089567	14·933184	6·06412
224	50176	11239424	15·966629	6·07317
225	50625	11390625	15·0	6·08220
226	51076	11543176	15·033296	6·09119
227	51529	11697083	15·066519	6·10017
228	51984	11852352	15·099668	6·10911
229	52441	12008989	15·132746	6 11803
230	52900	12167000	15·165750	6·12692
231	53361	12326391	15·198684	6·13579
232	53824	12487168	15·231546	6·14463
233	54289	12649337	15·264337	6·15344
234	54756	12812904	15·297058	6·16223
235	55225	12977875	15·329709	6·17100
236	55696	13144256	15·362291	6·17974
237	56169	13312053	15·394804	6·18846
238	56644	13481272	15·427248	6·19715
239	57121	13651919	15·459624	6·20582
240	57600	13824000	15·491933	6·21446
241	58081	13997521	15·524174	6·22308
242	58564	14172488	15·556349	6·23167
243	59049	14348907	15·588457	6·24025
244	59536	14526784	15·620499	6·24880
245	60025	14706125	15·652475	6·25732
246	60516	14886936	15·684387	6·26582
247	61009	15069223	15·716233	6·27430
248	61504	15252992	15·748015	6·28276

Number.	Square.	Cube.	Square Root.	Cube Root.
249	62001	15438249	15·779733	6·29119
250	62500	15625000	15·811388	6·29960
251	63001	15813251	15·842979	6·30799
252	63504	16003008	15·874507	6·31635
253	64009	16194277	15·905973	6·32470
254	64516	16387064	15·937377	6·33302
255	65025	16581375	15·968719	6·34132
256	65536	16777216	16·0	6·34960
257	66049	16974593	16·031219	6·35785
258	66564	17173512	16·062378	6·36609
259	67081	17373979	16·093476	6·37431
260	67600	17576000	16·124515	6·38250
261	68121	17779581	16·155494	6·39067
262	68644	17984728	16·186414	6·39882
263	69169	18191447	16·217274	6·40695
264	69696	18399744	16·248076	6·41506
265	70225	18609625	16·278820	6·42315
266	70756	18821096	16·309506	6·43122
267	71289	19034163	16·340134	6·43927
268	71824	19248832	16·370705	6·44730
269	72361	19465109	16·401219	6·45531
270	72900	19683000	16·431676	6·46330
271	73441	19902511	16·462077	6·47127
272	73984	20123648	16·492422	6·47922
273	74529	20346417	16·522711	6·48715
274	75076	20570824	16·552945	6·49506
275	75625	20796875	16·583124	6·50295
276	76176	21024576	16·613247	6·51082
277	76729	21253933	16·643317	6·51868
278	77284	21484952	16·673332	6·52651
279	77841	21717639	16·703293	6·53433
280	78400	21952000	16·733200	6·54213
281	78961	22188041	16·763054	6·54991
282	79524	22425768	16·792855	6·55767
283	80089	22665187	16·822603	6·56541
284	80656	22906304	16·852299	6·57313
285	81225	23149125	16·881943	6·58084
286	81796	23393656	16·911534	6·58853
287	82369	23639903	16·941074	6·59620
288	82944	23887872	16·970562	6·60385
289	83521	24137569	17·0	6·61148
290	84100	24389000	17·029386	6·61910
291	84681	24642171	17·058722	6·62670
292	85264	24897088	17·088007	6·63428
293	85849	25153757	17·117242	6·64185
294	86436	25412184	17·146428	6·64939
295	87025	25672375	17·175564	6·65693
296	87616	25934336	17·204650	6·66444
297	88209	26198073	17·233687	6·67194
298	88804	26463592	17·262676	6·67941

Number.	Square.	Cube.	Square Root.	Cube Root.
299	89401	26730899	17·291616	6·68688
300	90000	27000000	17·320508	6·69432
301	90601	27270901	17·349351	6·70175
302	91204	27543608	17·378147	6·70917
303	91809	27818127	17·406895	6·71656
304	92416	28094464	17·435595	6·72395
305	93025	28372625	17·464249	6·73131
306	93636	28652616	17·492855	6·73866
307	94249	28934443	17·521415	6·74599
308	94864	29218112	17·549928	6·75331
309	95481	29503629	17·578395	6·76061
310	96100	29791000	17·606816	6·76789
311	96721	30080231	17·635192	6·77516
312	97344	30371328	17·663521	6·78242
313	97969	30664297	17·691806	6·78966
314	98596	30959144	17·720045	6·79688
315	99225	31255875	17·748239	6·80409
316	99856	31554496	17·776388	6·81128
317	100489	31855013	17·804493	6·81846
318	101124	32157432	17·832554	6·82562
319	101761	32461759	17·860571	6·83277
320	102400	32768000	17·888543	6·83990
321	103041	33076161	17·916472	6·84702
322	103684	33386248	17·944358	6·85412
323	104329	33698267	17·972200	6·86121
324	104976	34012224	18·0	6·86828
325	105625	34328125	18·027756	6·87534
326	106276	34645976	18·055470	6·88238
327	106929	34965783	18·083141	6·88941
328	107584	35287552	18·110770	6·89643
329	108241	35611289	18·138357	6·90343
330	108900	35937000	18·165902	6·91042
331	109561	36264691	18·193405	6·91739
332	110224	36594368	18·220867	6·92435
333	110889	36926037	18·248287	6·93130
334	111556	37259704	18·275666	6·93823
335	112225	37595375	18·303005	6·94514
336	112896	37933056	18·330302	6·95205
337	113569	38272753	18·357559	6·95894
338	114244	38614472	18·384776	6·96581
339	114921	38958219	18·411952	6·97268
340	115600	39304000	18·439088	6·97953
341	116281	39651821	18·466185	6·98636
342	116964	40001688	18·493242	6·99319
343	117649	40353607	18·520259	7·0
344	118336	40707584	18·547237	7·00679
345	119025	41063625	18·574175	7·01357
346	119716	41421736	18·601075	7·02034
347	120409	41781923	18·627936	7·02710
348	121104	42144192	18·654758	7·03385

Number.	Square.	Cube.	Square Root.	Cube Root.
349	121801	42508549	18·681541	7·04058
350	122500	42875000	18·708286	7·04720
351	123201	43243551	18·734994	7·05400
352	123904	43614208	18·761663	7·06069
353	124609	43986977	18·788294	7·06737
354	125316	44361864	18·814887	7·07404
355	126025	44738875	18·841443	7·08069
356	126736	45118016	18·867962	7·08734
357	127449	45499293	18·894443	7·09397
358	128164	45882712	18·920887	7·10058
359	128881	46268279	18·947295	7·10719
360	129600	46656000	18·973666	7·11378
361	130321	47045881	19·0	7·12036
362	131044	47437928	19·026297	7·12693
363	131769	47832147	19·052558	7·13349
364	132496	48228544	19·078784	7·14003
365	133225	48627125	19·104973	7·14656
366	133956	49027896	19·131126	7·15309
367	134689	49430863	19·157244	7·15959
368	135424	49836032	19·183326	7·16609
369	136161	50243409	19·209372	7·17258
370	136900	50653000	19·235384	7·17905
371	137641	51064811	19·261360	7·18551
372	138384	51478848	19·287301	7·19196
373	139129	51895117	19·313207	7·19840
374	139876	52313624	19·339079	7·20483
375	140625	52734375	19·364916	7·21124
376	141376	53157376	19·390719	7·21765
377	142129	53582633	19·416487	7·22404
378	142884	54010152	19·442222	7·23042
379	143641	54439939	19·467922	7·23679
380	144400	54872000	19·493588	7·24315
381	145161	55306341	19·519221	7·24950
382	145924	55742968	19·544820	7·25584
383	146689	56181887	19·570385	7·26216
384	147456	56623104	19·595917	7·26848
385	148225	57066625	19·621416	7·27478
386	148996	57512456	19·646882	7·28107
387	149769	57960603	19·672315	7·28736
388	150544	58411072	19·697715	7·29363
389	151321	58863869	19·723082	7·29989
390	152100	59319000	19·748417	7·30614
391	152881	59776471	19·773719	7·31238
392	153664	60236288	19·798989	7·31861
393	154449	60698457	19·824227	7·32482
394	155236	61162984	19·849433	7·33103
395	156025	61629875	19·874606	7·33723
396	156816	62099136	19·899748	7·34342
397	157609	62570773	19·924858	7·34959
398	158404	63044792	19·949937	7·35576

Number.	Square.	Cube.	Square Root.	Cube Root.
399	159201	63521199	19·974984	7·36191
400	160000	64000000	20·0	7·36806
401	160801	64481201	20·024984	7·37419
402	161604	64964808	20.049937	7·38032
403	162409	65450827	20·074859	7·38643
404	163216	65939264	20·099751	7·39254
405	164025	66430125	20·124611	7·39863
406	164836	66923416	20·149441	7·40472
407	165649	67419143	20·174241	7·41079
408	166464	67917312	20·199009	7·41685
409	167281	68417929	20·223748	7·42291
410	168100	68921000	20·248456	7·42895
411	168921	69426531	20·273134	7·43499
412	169744	69934528	20·297783	7·44101
413	170569	70444997	20·322401	7·44703
414	171396	70951944	20·346980	7·45303
415	172225	71473375	20·371548	7·45903
416	173056	71991296	20·396078	7·46502
417	173889	72511713	20·420577	7·47099
418	174724	73034632	20·445048	7·47696
419	175561	73560059	20·469489	7·48292
420	176400	74088000	20·493901	7·48887
421	177241	74618461	20·518284	7·49481
422	178084	75151448	20·542638	7·50074
423	178929	75686967	20·566963	7·50666
424	179776	76225024	20·591260	7·51257
425	180625	76765625	20·615528	7·51847
426	181476	77308776	20·639767	7·52436
427	182329	77854483	20·663978	7·53024
428	183184	78402752	20·688160	7·53612
429	184041	78953589	20·712315	7·54198
430	184900	79507000	20·736441	7·54784
431	185761	80062991	20·760539	7·55368
432	186624	80621568	20·784609	7·55952
433	187489	81182737	20·808652	7·56535
434	188356	81746504	20·832666	7·57117
435	189225	82312875	20·856653	7·57698
436	190096	82881856	20·880613	7·58278
437	190969	83453453	20·904545	7·58857
438	191844	84027672	20·928449	7·59436
439	192721	84604519	20·952326	7·60013
440	193600	85184000	20·976177	7·60590
441	194481	85766121	21·0	7·61166
442	195364	86350888	21·023796	7·61741
443	196249	86938307	21·047565	7·62315
444	197136	87528384	21·071307	7·62888
445	198025	88121125	21·095023	7·63460
446	198916	88716536	21·118712	7·64032
447	199809	89314623	21·142374	7·64602
448	200704	89915392	21·166010	7·65172

Number.	Square.	Cube.	Square Root.	Cube Root.
449	201601	90518849	21·189620	7·65741
450	202500	91125000	21·213204	7·66309
451	203401	91733851	21·236760	7·66876
452	204304	92345408	21·260291	7·67443
453	205209	92959677	21·283796	7·68008
454	206116	93576664	21·307275	7·68573
455	207025	94196375	21·330729	7·69137
456	207936	94818816	21·354156	7·69700
457	208849	95443993	21·377558	7·70262
458	209764	96071912	21·400934	7·70823
459	210681	96702579	21·424285	7·71384
460	211600	97336000	21·447610	7·71944
461	212521	97972181	21·470910	7·72503
462	213444	98611128	21·494185	7·73061
463	214369	99252847	21·517434	7·73618
464	215296	99897344	21·540659	7·74175
465	216225	100544625	21·563858	7·74731
466	217156	101194696	21·587033	7·75286
467	218089	101847563	21·610182	7·75840
468	219024	102503232	21·633307	7·76393
469	219961	103161709	21·656407	7·76946
470	220900	103823000	21·679483	7·77498
471	221841	104487111	21·702534	7·78049
472	222784	105154048	21·725561	7·78599
473	223729	105823817	21·748563	7·79148
474	224676	106496424	21·771541	7·79697
475	225625	107171875	21·794494	7·80245
476	226576	107850176	21·817424	7·80792
477	227529	108531333	21·840329	7·81338
478	228484	109215352	21·863211	7·81884
479	229441	109902239	21·886068	7·82429
480	230400	110592000	21·908902	7·82973
481	231361	111284641	21·931712	7·83516
482	232324	111980168	21·954498	7·84059
483	233289	112678587	21·977261	7·84601
484	234256	113379904	22·0	7·85142
485	235225	114084125	22·022715	7·85682
486	236196	114791256	22·045407	7·86222
487	237169	115501303	22·068076	7·86761
488	238144	116214272	22·090722	7·87299
489	239121	116930169	22·113344	7·87836
490	240100	117649000	22·135943	7·88373
491	241081	118370771	22·158519	7·88909
492	242064	119095488	22·181073	7·89444
493	243049	119823157	22·203603	7·89979
494	244036	120553784	22·226110	7·90512
495	245025	121287375	22·248595	7·91046
496	246016	122023936	22·271057	7·91578
497	247009	122763473	22·293496	7·92110
498	248004	123505992	22·315913	7·92640

Number.	Square.	Cube.	Square Root.	Cube Root.
499	249001	124251499	22·338307	7·93171
500	250000	125000000	22·360679	7·93700
501	251001	125751501	22·383029	7·94229
502	252004	126506008	22·405356	7·94757
503	253009	127263527	22·427661	7·95284
504	254016	128024064	22·449944	7·95811
505	255025	128787625	22·472205	7·96337
506	256036	129554216	22·494443	7·96862
507	257049	130323843	22·516660	7·97387
508	258064	131096512	22·538855	7·97911
509	259081	131872229	22·561028	7·98434
510	260100	132651000	22·583179	7·98956
511	261121	133432831	22·605309	7·99478
512	262144	134217728	22·627417	8·0
513	263169	135005697	22·649503	8·00520
514	264196	135796744	22·671568	8·01040
515	265225	136590875	22·693611	8·01559
516	266256	137388096	22·715633	8·02077
517	267289	138188413	22·737634	8·02595
518	268324	138991832	22·759613	8·03112
519	269361	139798359	22·781571	8·03629
520	270400	140608000	22·803508	8·04145
521	271441	141420761	22·825424	8·04660
522	272484	142236648	22·847319	8·05174
523	273529	143055667	22·869193	8·05688
524	274576	143877824	22·891046	8·06201
525	275625	144703125	22·912878	8·06714
526	276676	145531576	22·934689	8·07226
527	277729	146363183	22·956480	8·07737
528	278784	147197952	22·978250	8·08248
529	279841	148035889	23·0	8·08757
530	280900	148877000	23·021728	8·09267
531	281961	149721291	23·043437	8·09775
532	283024	150568768	23·065125	8·10283
533	284089	151419437	23·086792	8·10791
534	285156	152273304	23·108440	8·11298
535	286225	153130375	23·130067	8·11804
536	287296	153990656	23·151673	8·12309
537	288369	154854153	23·173260	8·12814
538	289444	155720872	23·194827	8·13318
539	290521	156590819	23·216373	8·13822
540	291600	157464000	23·237900	8·14325
541	292681	158340421	23·259406	8·14827
542	263764	159220088	23·280893	8·15329
543	294849	160103007	23·302360	8·15830
544	295936	160989184	23·323807	8·16330
545	297025	161878625	23·345235	8·16830
546	298116	162771336	23·366642	8·17330
547	299209	163667323	23·388031	8·17828
548	300304	164566592	23·409399	8·18326

Number.	Square.	Cube.	Square Root.	Cube Root.
549	301401	165469149	23·430749	8·18824
550	302500	166375000	23·452078	8·19321
551	303601	167284151	23·473389	8·19817
552	304704	168196608	23·494680	8·20313
553	305809	169112377	23·515952	8·20808
554	306916	170031464	23·537204	8·21302
555	308025	170953875	23·558438	8·21796
556	309136	171879616	23·579652	8·22289
557	310249	172808693	23·600847	8·22782
558	311364	173741112	23·622023	8·23274
559	312481	174676879	23·643180	8·23766
560	313600	175616000	23·664319	8·24257
561	314721	176558481	23·685438	8·24747
562	315844	177504328	23·706539	8·22237
563	316969	178453547	23·727621	8·25726
564	318096	179406144	23·748684	8·26214
565	319225	180362125	23·769728	8·26702
566	320356	181321496	23·790754	8·27190
567	321489	182284263	23·811761	8·27677
568	322624	183250432	23·832750	8·28163
569	323761	184220009	23·853720	8·28649
570	324900	185193000	23·874672	8·29134
571	326041	186169411	23·895606	8·29619
572	327184	187149248	23·916521	8·30103
573	328329	188132517	23·937418	8·30586
574	329476	189119224	23·958297	8·31069
575	330625	190109375	23·979157	8·31551
576	331776	191102976	24·0	8·32033
577	332929	192100033	24·020824	8·32514
578	334084	193100552	24·041630	8·32995
579	335241	194104539	24·062418	8·33475
580	336400	195112000	24·083189	8·33955
581	337561	196122941	24·103941	8·34434
582	338724	197137368	24·124676	8·34912
583	339889	198155287	24·145392	8·35390
584	341056	199176704	24·166091	8·35867
585	342225	200201625	24·186773	8·36344
586	343396	201230056	24·207436	8·36820
587	344569	202262003	24·228082	8·37296
588	345744	203297472	24·248711	8·37771
589	346921	204336469	24·269322	8·38246
590	348100	205379000	24·289915	8·38720
591	349281	206425071	24·310491	8·39194
592	350464	207474688	24·331050	8·39667
593	351649	208527857	24·351591	8·40139
594	352836	209584584	24·372115	8·40611
595	354025	210644875	24·392621	8·41083
596	355216	211708736	24·413111	8·41554
597	356409	212776173	24·433583	8·42024
598	357604	213847192	24·454038	8·42494

Number.	Square.	Cube.	Square Root.	Cube Root.
599	358801	214921799	24·474476	8·42963
600	360000	216000000	24·494897	8·43432
601	361201	217081801	24·515301	8·43900
602	362404	218167208	24·535688	8·44368
603	363609	219256227	24·556058	8·44836
604	364816	220348864	24·576411	8·45302
605	366025	221445125	24·596747	8·45768
606	367236	222545016	24·617067	8·46234
607	368449	223648543	24·637370	8·46699
608	369664	224755712	24·657656	8·47164
609	370881	225866529	24·677925	8·47628
610	372100	226981000	24·698178	8·48092
611	373321	228099131	24·718414	8·48555
612	374544	229220928	24·738633	8·49018
613	375769	230346397	24·758836	8·49480
614	376996	231475544	24·779023	8·49942
615	378225	232608375	24·799193	8·50403
616	379456	233744896	24·819347	8·50864
617	380689	234885113	24·839484	8·51324
618	381924	236029032	24·859605	8·51784
619	383161	237176659	24·879710	8·52243
620	384400	238328000	24·899799	8·52701
621	385641	239483061	24·919871	8·53160
622	386884	240641848	24·939927	8·53617
623	388129	241804367	24·959967	8·54074
624	389376	242970624	24 979992	8·54531
625	390625	244140625	25·0	8·54987
626	391876	245314376	25·019992	8·55443
627	393129	246491883	25·039968	8·55899
628	394384	247673152	25·059928	8·56353
629	395641	248858189	25·079872	8·56808
630	396900	250047000	25·099800	8·57261
631	398161	251239591	25·119713	8·57715
632	399424	252435968	25·139610	8·58168
633	400689	253636137	25·159491	8·58620
634	401956	254840104	25·179356	8·59072
635	403225	256047875	25·199206	8·59523
636	404496	257259456	25·219040	8·59974
637	405769	258474853	25·238858	8·60425
638	407044	259694072	25·258661	8·60875
639	408321	260917119	25·278449	8·61324
640	409600	262144000	25·298221	8·61773
641	410881	263374721	25·317977	8·62222
642	412164	264609288	25·337718	8·62670
643	413449	265847707	25·357444	8·63118
644	414736	267089984	25·377155	8·63565
645	416025	268336125	25·396850	8·64012
646	417316	269586136	25·416530	8·64458
647	418609	270840023	25·436194	8·64904
648	419904	272097792	25·455844	8·65349

Additional use can be made of these tables by the aid of the following rules :

To find the Cube or Square Root of a higher number than is contained in the Table.

Rule.—Refer to the table, and seek in the column of squares or cubes the number nearest to that number whose root is sought, and the number from which that square or cube is derived will be the answer required, when decimals are not of importance.

Examples.—1. Required the square root of 352836, and the number from which that square has been obtained is 594.

Therefore, $\sqrt{352836}=594$. Ans.

2. Required the cube root of 201236656.

In the tables of cubes, the nearest number is 201230056, and the number opposite is 586.

Therefore, $\sqrt[3]{201236656}=586$, nearly. Ans.

To find the Sixth Root of a Number.

Rule.—Take the cube root of its square root.

Example.—What is the $\sqrt[6]{}$ of 441 ?

$\sqrt{441}=21$; and $\sqrt[3]{21}=2{\cdot}7589$. Ans

To find the Cube or Square Root of a Number, consisting of Integers and Decimals.

Rule.—Multiply the difference between the root of the integer part, and the root of the next higher integer by the *decimal*, and add the product to the root of the integer given ; the sum will be the root of the number required.

This is correct in all cases of the square root to three places of decimals, and in the cube root to seven.

Example.—Required the square root of 60·2, and the cube root of 843·75.

$$\begin{array}{l}\sqrt{61}=7{\cdot}8102\\ \sqrt{60}=\underline{7{\cdot}7459}\\ \quad{\cdot}0643\text{, diff.}\end{array}\qquad \begin{array}{r}{\cdot}0643\\ \underline{2}\\ 0{\cdot}1286\\ \underline{7{\cdot}7459}\end{array}$$

$\sqrt{60{\cdot}2}=7{\cdot}75876$, as required; correct to 3 places of decimals.

$$\begin{array}{lcr}\sqrt[3]{844} & = & 9{\cdot}4503\\ \sqrt[3]{843} & = & \underline{9{\cdot}4466}\\ & & {\cdot}0037=\text{diff.}\\ & & \underline{{\cdot}75}\\ & & 002775\\ \sqrt[3]{843} & = & \underline{9{\cdot}4466}\end{array}$$

$\sqrt[3]{843{\cdot}75}=9{\cdot}449375$, as required.

TABLE

Of the Circumferences, Areas, &c. of Circles.

Diam.	Circum.	Area.	Side of equal Square.	Diam.	Circum.	Area.	Side of equal Square.
				5 in.	15·708	19·635	4·4310
⅛	·3927	·0122	·1107	⅛	16 100	20·629	4·5417
¼	·7854	·0490	·2115	¼	16·493	21·647	4·6525
⅜	1.1781	·1104	·3223	⅜	16·886	22·690	4·7633
½	1·5708	·1963	·4331	½	17·278	23·758	4·8741
⅝	1·9635	·3068	·5438	⅝	17·671	24·850	4·9848
¾	2·3562	·4417	·6646	¾	18·064	25·967	5·0956
⅞	2.7489	·6013	·7754	⅞	18·457	27·108	5·2064
1 in.	3·1416	·7854	·8862	6 in.	18·849	28·274	5·3172
⅛	3.5343	·9940	·9969	⅛	19·242	29·464	5·4280
¼	3 9270	1·2271	1·0775	¼	19·635	30·679	5·5388
⅜	4·3197	1·4848	1·2185	⅜	20·027	31·919	5·6495
½	4·7124	1·7671	1·3293	½	20·420	33·183	5·7603
⅝	5·1051	2·0739	1·4401	⅝	20·813	34·471	5·8711
¾	5.4978	2·4052	1·5508	¾	21·205	35·784	5·9819
⅞	5·8905	2·7611	1·6616	⅞	21·598	37·122	6·0927
2 in.	6·2832	3·1416	1·7724	7 in.	21·991	38·484	6·2034
⅛	6·6759	3·5465	1·8831	⅛	22·383	39·871	6·3142
¼	7·0686	3·9760	1·9939	¼	22·776	41·282	6·4350
⅜	7·4613	4·4302	2·1047	⅜	23·169	42·718	6·5358
½	7·8540	4·9087	2·2155	½	23·562	44·178	6·6465
⅝	8·2467	5·4119	2·3262	⅝	23·954	45·663	6·7573
¾	8·6394	5·9395	2·4370	¾	24·347	47·173	6·8681
⅞	9.0321	6·4918	2·5478	⅞	24·740	48·707	6·9789
3 in.	9·4248	7·0686	2·6586	8 in.	25·132	50·265	7·0897
⅛	9·8175	7·6699	2·7694	⅛	25·515	51·848	7·2005
¼	10·210	8·2957	2·8801	¼	25·918	53·456	7·3112
⅜	10·602	8·9462	2·9909	⅜	26·310	55·088	7·4220
½	10·995	9·6211	3·1017	½	26·703	56·745	7·5328
⅝	11·388	10·320	3·2124	⅝	27·096	58·426	7·6436
¾	11·781	11·044	3·3232	¾	27·489	60·132	7·7544
⅞	12·173	11·793	3·4340	⅞	27·881	61·862	7·8651
4 in.	12·566	12·566	3·5448	9 in.	28·274	63·617	7·9760
⅛	12·959	13·364	3·6555	⅛	28·667	65·396	8·0866
¼	13·351	14·186	3·7663	¼	29·059	67·200	8·1974
⅜	13·744	15·033	3·8771	⅜	29·452	69·029	8·3081
½	14·137	15·904	3·9880	½	29·845	70 882	8·4190
⅝	14·529	16·800	4·0987	⅝	30·237	72·759	8·5297
¾	14·922	17·720	4·2095	¾	30·630	74·662	8·6405
⅞	15·315	18·665	4·3202	⅞	31·023	76·588	8·7513

Diam.	Circum.	Area.	Side of equal Square.
10 in.	31·416	78·540	8·8620
⅛	31·808	80·515	8·9728
¼	32·201	82·516	9·0836
⅜	32·594	84·540	9·1943
½	32·986	86·590	9·3051
⅝	33·379	88·664	9·4159
¾	33·772	90·762	9·5267
⅞	34·164	92·885	9·6375
11 in.	34·557	95·033	9·7482
⅛	34·950	97·205	9·8590
¼	35·343	99·402	9·9698
⅜	35·735	101·623	10·080
½	36·128	103·869	10·191
⅝	36·521	106·139	10·302
¾	36·913	108·434	10·413
⅞	37·306	110·753	10·523
12 in.	37·699	113·097	10·634
⅛	38·091	115·466	10·745
¼	38·484	117·859	10·856
⅜	38·877	120·276	10·966
½	39·270	122·718	11·077
⅝	39·662	125·184	11·188
¾	40·055	127·676	11·299
⅞	40·448	130·192	11·409
13 in.	40·840	132·732	11·520
⅛	41·233	135·297	11·631
¼	41·626	137·886	11·742
⅜	42·018	140·500	11·853
½	42·411	143·139	11·963
⅝	42·804	145·802	12·074
¾	43·197	148·489	12·185
⅞	43·589	151·201	12·296
14 in.	43·982	153·938	12·406
⅛	44·375	156·699	12·517
¼	44·767	159·485	12·628
⅜	45·160	162·295	12·739
½	45·553	165·130	12·850
⅝	45·945	167·989	12·960
¾	46·338	170·873	13·071
⅞	46·731	173·782	13·182
15 in.	47·124	176·715	13·293
⅛	47·516	179·672	13·403
¼	47·909	182·654	13·514
⅜	48·302	185·661	13·625

Diam.	Circum.	Area.	Side of equal Square.
15½ in.	48·694	188·692	13·736
⅝	49 087	191·748	13·847
¾	49·480	194·828	13·957
⅞	49·872	197·933	14·068
16 in.	50·265	201·062	14.179
⅛	50·658	204·216	14·290
¼	51·051	207·394	14·400
⅜	51·443	210·597	14·511
½	51·836	213·825	14·622
⅝	52·229	217·077	14·732
¾	52·621	220·353	14·843
⅞	53·014	223·654	14·954
17 in.	53·407	226·980	15·065
⅛	53·799	230·330	15·176
¼	54·192	233·705	15·286
⅜	54·585	237·104	15·397
½	54·978	240·528	15·508
⅝	55·370	243·977	15·619
¾	55·763	247·450	15·730
⅞	56·156	250·947	15·840
18 in.	56·548	254·469	15·951
⅛	56·941	258·016	16·062
¼	57·334	261·587	16·173
⅜	57·726	265·182	16·283
½	58·119	268·803	16·394
⅝	58·512	272·447	16·505
¾	58·905	276·117	16·616
⅞	59·297	279·811	16·727
19 in.	59·690	283·529	16·837
⅛	60·083	287·272	16·948
¼	60·475	291·039	17·060
⅜	60·868	294·831	17·170
½	61·261	298·648	17·280
⅝	61·653	302·489	17·391
¾	62·046	306·355	17·502
⅞	62·439	310·245	17·613
20 in.	62·832	314·160	17·724
⅛	63·224	318·099	17·834
¼	63·617	322·063	17·945
⅜	64·010	326·051	18·056
½	64·402	330·064	18·167
⅝	64·795	334·101	18·277
¾	65·188	338·163	18·388
⅞	65·580	342·250	18·499

Diam.	Circum.	Area.	Side of equal Square.	Diam.	Circum.	Area.	Side of equal Square.
21 in.	65·793	346·361	18·610	26½ in.	83·252	551·547	23·484
⅛	66·366	350·497	18·721	⅝	83·645	556·762	23·595
¼	66·759	354·657	18·831	¾	84·037	562·002	23·708
⅜	67·151	358·841	18·942	⅞	84·430	567·267	23·816
½	67·544	363·051	19·053	27 in.	84·823	572·556	23·927
⅝	67·937	367·284	19·164	⅛	85·215	577·870	24·038
¾	68·329	371·543	19·274	¼	85·608	583·208	24·149
⅞	68·722	375·826	19·385	⅜	86·001	588·571	24·259
22 in.	69·115	380·133	19·496	½	86·394	593·958	24·370
⅛	69·507	384·465	19·607	⅝	86·786	599·370	24·481
¼	69·900	388·822	19·718	¾	87·179	604·807	24·592
⅜	70·293	393·203	19·828	⅞	87·572	610·268	24·703
½	70·686	397·608	19·939	28 in.	87·964	615·753	24·813
⅝	71·078	402·038	20·050	⅛	88·357	621·263	24·924
¾	71·471	406·493	20·161	¼	88·750	626·798	25·035
⅞	71·864	410·972	20·271	⅜	89·142	632·357	25·146
23 in.	72·256	415·476	20·382	½	89·535	637·941	25·256
⅛	72·649	420·004	20·493	⅝	89·928	643·594	25·367
¼	73·042	424·557	20·604	¾	90·321	649·182	25·478
⅜	73·434	429·135	20·715	⅞	90·713	654·839	25·589
½	73·827	433·731	20·825	29 in.	91·106	660·521	25·699
⅝	74·220	438·363	20·936	⅛	91·499	666·227	25·810
¾	74·613	443·014	21·047	¼	91·891	671·958	25·921
⅞	75·005	447·699	21·158	⅜	92·284	677·714	26·032
24 in.	75·398	452·390	21·268	½	92·677	683·494	26·143
⅛	75·791	457·115	21·379	⅝	93·069	689·298	26·253
¼	76·183	461·864	21·490	¾	93·462	695·128	26·364
⅜	76·576	466·638	21·601	⅞	93·855	700·981	26·478
½	76·969	471·436	21·712	30 in.	94·248	706·860	26·586
⅝	77·361	476·259	21·822	⅛	94·640	712·762	26·696
¾	77·754	481·106	21·933	¼	95·033	718·690	26·807
⅞	78·147	485·978	22·044	⅜	95·426	724·641	26·918
25 in.	78·540	490·875	22·155	½	95·818	730·618	27·029
⅛	78·932	495·796	22·265	⅝	96·211	736·619	27·139
¼	79·325	500·741	22·376	¾	96·604	742·644	27·250
⅜	79·718	505·711	22·487	⅞	96·996	748·694	27·361
½	80·110	510·706	22·598	31 in.	97·389	754·769	27·472
⅝	80·503	515·725	22·709	⅛	97·782	760·868	27·583
¾	80·896	520·769	22·819	¼	98·175	766·992	27·693
⅞	81·288	525·837	22·930	⅜	98·567	773·140	27·804
26 in.	81·681	530·930	23·041	½	98·968	779·313	27·915
⅛	82·074	536·047	23.152	⅝	99·353	785·510	28·026
¼	82·467	541·189	23 062	¾	99·745	791·732	28·136
⅜	82 859	546·356	23·373	⅞	100·138	797·978	28·247

Diam.	Circum.	Area.	Side of equal Square.
32 in.	100·531	804·249	28·358
1/8	100·924	810·545	28·469
1/4	101·316	816·865	28·580
3/8	101·709	823·209	28·691
1/2	102·102	829·578	28·801
5/8	102·494	835·972	28·912
3/4	102·887	842·390	29·023
7/8	103·280	848·833	29·133
33 in.	103·672	855·30	29·244
1/8	104·055	861·79	29·355
1/4	104·458	868·30	29·466
3/8	104·850	874·84	29·577
1/2	105 243	881·41	29·687
5/8	105·636	888·00	29·798
3/4	106·029	894·61	29·909
7/8	106·421	901·25	30·020
34 in.	106·814	907·92	30·131
1/8	107·207	914·61	30·241
1/4	107·599	921·32	30·352
3/8	107·992	928·06	30·463
1/2	108·385	934·82	30·574
5/8	108·777	941·60	30·684
3/4	109·170	948·41	30·795
7/8	109·563	955·25	30·906
35 in.	109·956	962·11	31·017
1/8	110·348	968·99	31·128
1/4	110·741	975·90	31·238
3/8	111·134	982·84	31·349
1/2	111·526	989·80	31·460
5/8	111·919	996·78	31·571
3/4	112·312	1003·7	31·681
7/8	112·704	1010·8	31·792
36 in.	113·097	1017·87	31·903
1/8	113·490	1024·95	32·014
1/4	113·883	1032·06	32·124
3/8	114·275	1039·19	32·235
1/2	114·668	1046·39	32·346
5/8	115·061	1053·52	32·457
3/4	115·453	1060·73	32·567
7/8	115·846	1067·95	32·678
37 in.	116·239	1075·21	32·789
1/8	116·631	1082·48	32·900
1/4	117·024	1089·79	33·011
3/8	117·417	1097·11	33·021

Diam.	Circum.	Area.	Side of equal Square.
37½ in.	117·810	1104·46	33·232
5/8	118·202	1111·84	33·343
3/4	118·595	1119·24	33·454
7/8	118·988	1126·66	33·564
38 in.	119·380	1134·11	33·675
1/8	119·773	1141·59	33·786
1/4	120·166	1149·08	33·897
3/8	120·558	1156·61	34·008
1/2	120·951	1164·15	34·118
5/8	121·344	1171·73	34·229
3/4	121·737	1179·32	34·340
7/8	122·129	1186·94	34·451
39 in.	122·522	1194·59	34·561
1/8	122·915	1202·26	34·672
1/4	123·307	1209·95	34·783
3/8	123·700	1217·67	34·894
1/2	124·093	1225·42	35·005
5/8	124·485	1233·18	35·115
3/4	124·878	1240·98	35·226
7/8	125·271	1248·79	35·337
40 in.	125·664	1256·64	35·448
1/8	126·056	1264·50	35·558
1/4	126·449	1272·39	35·669
3/8	126·842	1280·31	35·780
1/2	127·234	1288·25	35·891
5/8	127·627	1296·21	36·002
3/4	128·020	1304·20	36·112
7/8	128·412	1312·21	36·223
41 in.	128·805	1320·25	36·334
1/8	129·198	1328·32	36·445
1/4	129·591	1336·40	36·555
3/8	129·983	1344·51	36·666
1/2	130·376	1352·65	36·777
5/8	130·769	1360·81	36·888
3/4	131·161	1369·00	36·999
7/8	131·554	1377·21	37·109
42 in.	131·947	1385·44	37·220
1/8	132·339	1393·70	37·331
1/4	132·732	1401·98	37·442
3/8	133·125	1410·29	37·552
1/2	133·518	1418·62	37·663
5/8	133·910	1426·98	37·774
3/4	134·303	1435·36	37·885
7/8	134·696	1443·77	37·996

Diam.	Circum.	Area.	Side of equal Square.
43 in.	135·038	1452·20	38·106
⅛	135·481	1460·65	38·217
¼	135·874	1469·13	38·328
⅜	136·266	1477·63	38·439
½	136·659	1486·17	38·549
⅝	137·052	1494·72	38·660
¾	137·445	1503·30	38·771
⅞	137·837	1511·90	38·882
44 in.	138·230	1520·53	38·993
⅛	138·623	1529·18	39·103
¼	139·015	1537·86	39·214
⅜	139·408	1546·55	39·325
½	139 801	1555·28	39·436
⅝	140·193	1564·03	39·546
¾	140·586	1572·81	39·657
⅞	140·979	1581·61	39·768
45 in.	141·372	1590·43	39·879
⅛	141·764	1599·28	39·989
¼	142·157	1608·15	40·110
⅜	142·550	1617·04	40·211
½	142·942	1625·97	40·322
⅝	143·335	1634·92	40·432
¾	143·728	1643·89	40·543
⅞	144·120	1652·88	40·654
46 in.	144·513	1661·90	40·765
⅛	144·906	1670·95	40·876
¼	145·299	1680·01	40·986
⅜	145·691	1689·10	41·097
½	146·084	1698·23	41·208
⅝	146·477	1707·37	41·319
¾	146·869	1716·54	41·429
⅞	147·262	1725·73	41·540
47 in.	147·655	1734·94	41·651
⅛	148·047	1744·18	41·762
¼	148·440	1753·45	41·873
⅜	148·833	1762·73	41·983
½	149·226	1772·05	42·094
⅝	149·618	1781·39	42·205
¾	150·011	1790·76	42·316
⅞	150·404	1800·14	42·427
48 in.	150·796	1809·56	42·537
⅛	151·189	1818·99	42·648
¼	151·582	1828·46	42·759
⅜	151·974	1837·93	42·870
48½ in.	152·367	1847·45	42·980
⅝	152·760	1856·99	43·091
¾	153·153	1866·55	43·202
⅞	153·545	1876·13	43·313
49 in.	153·938	1885·74	43·423
⅛	154·331	1895·37	43·534
¼	154·723	1905·03	43·645
⅜	155·116	1914·70	43·756
½	155·509	1924·42	43·867
⅝	155·901	1934·15	43·977
¾	156·294	1943·91	44·088
⅞	156·687	1953·69	44·199
50 in.	157·080	1963·50	44·310
¼	157·865	1983·18	44·531
½	158·650	2002·96	44·753
¾	159·436	2022·84	44·974
51 in.	160·221	2042·82	45·196
¼	161·007	2062·90	45·417
½	161·792	2083·07	45·639
¾	162·577	2103·35	45·861
52 in.	163·363	2123·72	46·082
¼	164·148	2144·19	46·304
½	164·934	2164·75	46·525
¾	165·719	2185·42	46·747
53 in.	166·504	2206·18	46·968
¼	167·290	2227·05	47·190
½	168·075	2248·01	47·411
¾	168·861	2269 06	47·633
54 in.	169·646	2290·22	47·854
¼	170·431	2311·48	48·076
½	171·217	2332·83	48·298
¾	172·002	2354·28	48·519
55 in.	172·788	2375·83	48·741
¼	173·573	2397·48	48·962
½	174·358	2419·22	49·184
¾	175·144	2441·07	49·405
56 in.	175·929	2463·01	49·627
¼	176·715	2485·05	49·848
½	177·500	2507·19	50·070
¾	178·285	2529·42	50·291

Diam.	Circum.	Area.	Side of equal Square.	Diam.	Circum.	Area.	Side of equal Square.
57 in.	179·071	2551·76	50·513	67 in.	210·487	3525·66	59·375
¼	179·856	2574·19	50·735	¼	211·272	3552·01	59·597
½	180·642	2596·72	50·956	½	212·058	3578·47	59·818
¾	181·427	2619·35	51·178	¾	212·843	3605·03	60·040
58 in.	182·212	2642·08	51·399	68 in.	213·628	3631·68	60·261
¼	182·998	2664·91	51·621	¼	214·414	3658·44	60·483
½	183·783	2687·83	51·842	½	215·199	3685·29	60·704
¾	184·569	2710·85	52·064	¾	215·985	3712·24	60·926
59 in.	185·354	2733·97	52·285	69 in.	216·770	3739·28	61·147
¼	186·139	2757·19	52·507	¼	217·555	3766·43	61·369
½	186·925	2780·51	52·729	½	218·341	3793·67	61·591
¾	187·710	2803·92	52·950	¾	219·126	3821·02	61·812
60 in.	188·496	2827·44	53·172	70 in.	219·912	3848·46	62·034
¼	189·281	2851·05	53·393	¼	220·697	3875·99	62·255
½	189·066	2874·76	53·615	½	221 482	3903·63	62·477
¾	190·852	2898·56	53·836	¾	222·268	3931·36	62·968
61 in.	191·637	2922·47	54·048	71 in.	223·053	3959·20	69·920
¼	192·423	2946·47	54·279	¼	223·839	3987·13	63·141
½	193·208	2970·57	54·501	½	224·624	4015·16	63·363
¾	193·993	2994·77	54·723	¾	225·409	4043·28	63·545
62 in.	194·779	3019·07	54·944	72 in.	226·195	4071·51	63·806
¼	195·564	3043·47	55·166	¼	226·980	4099·83	64·028
½	196·350	3067·96	55·387	½	227·766	4128·25	64·249
¾	197·135	3092·56	55·609	¾	228·551	4156·77	64·471
63 in.	197·920	3117·25	55·830	73 in.	229·336	4185·39	64·692
¼	198·706	3142·04	56·052	¼	230·122	4214·11	64·914
½	199·491	3166·92	56·273	½	230·907	4242·92	65·135
¾	200·277	3191·91	56·495	¾	231·693	4271·83	65·357
64 in.	201·062	3216·99	56·716	74 in.	232·478	4300·85	65·578
¼	201·847	3242·17	56·938	¼	233·263	4329·95	65·800
½	202·633	3267·46	57·159	½	234·049	4359·16	66·022
¾	203·418	3292·83	57·381	¾	234·834	4388·47	66·243
65 in.	204·204	3318·31	57·603	75 in.	235·620	4417·87	66·465
¼	204·989	3343·88	57·824	¼	236·405	4447·37	66·686
½	205·774	3369·56	58·046	½	237·190	4476·97	66·908
¾	206·560	3395·33	58·267	¾	237·976	4506·67	67·129
66 in.	207·345	3421·20	58·489	76 in.	238·761	4536·47	67·351
¼	208·131	3447·16	58·710	¼	239·547	4566·36	67·572
½	208·916	3473·23	58·932	½	240·332	4596·35	67·794
¾	209·701	3499·39	59·154	¾	241·117	4626·44	68·016

Diam.	Circum.	Area.	Side of equal Square.
77 in.	241·903	4656·63	68·237
¼	242·688	4686·92	68·459
½	243·474	4717·30	68·680
¾	244·259	4747·79	68·902
78 in.	245·044	4778·37	69·123
¼	245·830	4809·05	69·345
½	246·615	4839 83	69·566
¾	247·401	4870·70	69·788
79 in.	248·186	4901·68	70·009
¼	248·971	4932·75	70·231
½	249·757	4963·92	70·453
¾	250·542	4995·19	70·674
80 in.	251·328	5026·56	70·896
½	252·898	5089·58	71·339
81 in.	254·469	5153·00	71·782
½	256·040	5216·82	72·225
82 in.	257·611	5281·02	72·668
½	259·182	5345·62	73·111
83 in.	260·752	5410·62	73·554
½	262·323	5476·00	73·997
84 in.	263·894	5541·78	74·440
½	265·465	5607·95	74·884
85 in.	267·036	5674·51	75·327
½	268·606	5741·47	75·770
86 in.	270·177	5808·81	76·213
½	271·748	5876·55	76·656
87 in.	273·319	5944·69	77·099
½	274·890	6013·21	77·542
88 in.	276·460	6082·13	77·985
½	278·031	6151·44	78·428
89 in.	279·602	6221·15	78·871
½	281·173	6291·25	79·315
90 in.	282·744	6361·74	79·758
½	284·314	6432·62	80·201
91 in.	285·885	6503·89	80·644
½	287·456	6573·56	81·087
92 in.	289·027	6647·62	81·530
½	290·598	6720·07	81·973

Diam.	Circum.	Area.	Side of equal Square.
93 in.	292·168	6792·92	82·416
½	293·739	6866·16	82·859
94 in.	295·310	6939·79	83·302
½	296·881	7013·81	83·746
95 in.	298·452	7088·23	84·189
½	300·022	7163·04	84·632
feet. in.	feet. in.	feet.	feet. in.
8	25 1½	50·265	7 0⅛
1	25 4⅝	51·317	7 1¾
2	25 7⅞	52·381	7 2⅞
3	25 11	53·456	7 3¾
4	26 2⅛	54·541	7 4⅝
5	26 5¼	55·637	7 5½
6	26 8⅜	56·745	7 6⅜
7	26 11½	57·862	7 7¼
8	27 2¾	58·992	7 8⅛
9	27 5¾	60·132	7 9⅛
10	27 9	61·282	7 9⅞
11	28 0⅛	62·444	7 10¾
9	28 3¼	63·617	7 11⅝
1	28 6⅜	64·800	8 0½
2	28 9½	65·995	8 1½
3	29 0⅝	67·200	8 2⅜
4	29 3¾	68·416	8 3¼
5	29 7	69·644	8 4⅛
6	29 10⅛	70·882	8 5
7	30 1¼	72·130	8 5⅞
8	30 4⅜	73·391	8 6¾
9	30 7½	74·662	8 7⅝
10	30 11⅝	75·943	8 8½
11	31 1¾	77·236	8 9½
10	31 5	78·540	8 10¼
1	31 8⅛	79·854	8 11¼
2	31 11¼	81·179	9 0⅛
3	32 2⅜	82·516	9 1
4	32 5½	83·862	9 1⅞
5	32 8⅝	85·221	9 2¾
6	32 11¾	86·590	9 3⅝
7	33 2⅞	87·969	9 4½
8	33 6⅛	89·360	9 5⅜
9	33 9¼	90·762	9 6¼
10	34 0⅜	92·174	9 7¼
11	34 3½	93·598	9 8⅛

Diam. feet.	Diam. in.	Circum. feet.	Circum. in.	Area. feet.	Side of equal Square. feet.	Side of equal Square. in.
11		34	6⅝	95·033	9	8⅞
	1	34	9¾	96·478	9	9⅞
	2	35	0⅞	97·934	9	10¾
	3	35	4⅛	99·402	9	11⅝
	4	35	7¼	100·879	10	0½
	5	35	10⅝	102·368	10	1⅜
	6	36	1½	103·869	10	2¼
	7	36	4½	105·379	10	3⅛
	8	36	7¾	106·901	10	4
	9	36	10⅞	108·434	10	5
	10	37	2¾	109·977	10	5⅞
	11	37	5¼	111·531	10	6¾
12		37	8⅜	113·097	10	7⅝
	1	37	11½	114·673	10	8½
	2	38	2⅝	116·260	10	9⅜
	3	38	5¾	117·859	10	10¼
	4	38	8⅞	119·467	10	11⅛
	5	39	0	121·087	11	0
	6	39	3¼	122·718	11	0⅞
	7	39	6⅜	124·359	11	1⅞
	8	39	9½	126·012	11	2⅝
	9	40	0⅝	127·676	11	3⅝
	10	40	3¾	129·350	11	4½
	11	40	6⅞	131·036	11	5⅜
13		40	10	132·732	11	6¼
	1	41	1⅓	134·439	11	7⅛
	2	41	4⅜	136·157	11	8⅛
	3	41	7½	137·886	11	8⅞
	4	41	10⅝	139·626	11	9¾
	5	42	1⅝	141·377	11	10⅝
	6	42	4⅞	143·139	11	11⅝
	7	42	8	144·911	12	0½
	8	42	11⅛	146·694	12	1⅜
	9	43	2¼	148·489	12	2¼
	10	43	5½	150·294	12	3⅛
	11	43	8⅝	152·110	12	4
14		43	11¾	153·938	12	4⅞
	1	44	2⅞	155·775	12	5⅝
	2	44	6	157·625	12	6½
	3	44	9⅛	159·485	12	7½
	4	45	0¼	161·355	12	8⅜
	5	45	3½	163·237	12	9⅜
	6	45	6⅝	165·130	12	10¼
	7	45	9¾	167·033	12	11⅛
	8	46	0⅞	168·947	13	0
14	9	46	4	170·873	13	1⅛
	10	46	7⅛	172·809	13	1¾
	11	46	11¼	174·756	13	2⅝
15		47	1½	176·715	13	3½
	1	47	4⅝	178·683	13	4⅜
	2	47	7¾	180·663	13	5¼
	3	47	10⅞	182·654	13	6⅛
	4	48	2½	184·655	13	7⅛
	5	48	5⅛	186·668	13	8
	6	48	8¼	188·692	13	8⅞
	7	48	11⅜	190·726	13	9¾
	8	49	2⅝	192·771	13	10⅝
	9	49	5¾	194·828	13	11½
	10	49	8⅞	196·894	14	0⅜
	11	50	0	198·973	14	1¼
16		50	3⅛	201·062	14	2⅛
	1	50	6¼	203·161	14	3
	2	50	9⅝	205·272	14	3⅞
	3	51	0½	207·394	14	4⅞
	4	51	3¾	209·526	14	5¾
	5	51	6½	211·670	14	6⅜
	6	51	10	213·825	14	7½
	7	52	1⅛	215·989	14	8¾
	8	52	4¼	218·166	14	9¼
	9	52	7⅜	220·353	14	10⅛
	10	52	10½	222·551	14	11
	11	53	1⅝	224·760	14	11⅞
17		53	4⅞	226·980	15	0¾
	1	53	8	229·210	15	1⅝
	2	53	11⅛	231·452	15	2⅝
	3	54	2⅛	233·705	15	3½
	4	54	5⅜	235·968	15	4⅜
	5	54	8½	238·243	15	5¼
	6	54	11⅝	240·528	15	6⅛
	7	55	2⅞	242·824	15	7
	8	55	6	245·131	15	7⅞
	9	55	9⅛	247·450	15	8¾
	10	56	0¼	249·778	15	9⅝
	11	56	3½	252·118	15	10½
18		56	6½	254·469	15	11⅜
	1	56	9⅝	256·830	16	0⅜
	2	57	0⅞	259·203	16	1¼
	3	57	4	261·587	16	2⅛
	4	57	7⅛	263·980	16	3⅛
	5	57	10¼	266·386	16	3⅞
	6	58	1⅜	268·803	16	4¾

Diam. feet.	Diam. in.	Circum. feet.	Circum. in.	Area. feet.	Side of equal Square. feet.	Side of equal Square. in.
18	7	58	4½	271·229	16	5⅝
	8	58	7¾	273·667	16	6½
	9	58	10⅞	276·117	16	7⅜
	10	59	2	278·576	16	8¼
	11	59	5⅛	281·047	16	9¼
19		59	8¼	283·529	16	10
	1	59	11½	286·021	16	11
	2	60	2½	288·524	16	11⅞
	3	60	5⅝	291·039	17	0¾
	4	60	8¾	293·564	17	1⅝
	5	60	11⅞	296·110	17	2½
	6	61	3⅛	298·648	17	3⅜
	7	61	6¼	301·205	17	4¼
	8	61	9½	303·774	17	5⅛
	9	62	0½	306·355	17	6
	10	62	3⅝	308·944	17	7
	11	62	6¾	311·546	17	7⅞
20		62	9⅞	314·160	17	8⅝
	1	63	1⅛	316·782	17	9⅝
	2	63	4¼	319·417	17	10½
	3	63	7⅜	322·063	17	11⅜
	4	63	11½	324·718	18	0¼
	5	64	1⅝	327·385	18	1⅛
	6	64	4¾	330·064	18	2
	7	64	7⅞	332·752	18	2⅞
	8	64	11	335·452	18	3¾
	9	65	2¼	338·163	18	4¾
	10	65	5⅜	340·884	18	5⅝
	11	65	8¼	343·617	18	6½
21		65	11⅝	346·361	18	7¼
	1	66	2¾	349·114	18	8¼
	2	66	5⅞	351·880	18	9⅛
	3	66	9	354·657	18	10
	4	67	0¼	357·443	18	10⅞
	5	67	3⅜	360·241	18	11¾
	6	67	6½	363·051	19	0⅝
	7	67	9⅝	365·869	19	1⅝
	8	68	0¾	368·701	19	2½
	9	68	3⅞	371·543	19	3⅜
	10	68	7	374·394	19	4¼
	11	68	10¼	377·258	19	5⅛
22		69	1⅜	380·133	19	5⅞
	1	69	4½	383·017	19	6⅞
	2	69	7⅝	385·914	19	7¾
22	3	69	10¾	388·822	19	8⅝
	4	70	1⅞	391·738	19	9½
	5	70	5	394·668	19	10⅜
	6	70	8¼	397·608	19	11¼
	7	70	11⅛	400·558	20	0¼
	8	71	2½	403·520	20	1⅛
	9	71	5⅝	406·493	20	2
	10	71	8¾	409·475	20	2⅞
	11	71	11⅞	412·470	20	3¾
23		72	3	415·476	20	4½
	1	72	6⅛	418·491	20	5½
	2	72	9⅜	421·519	20	6⅜
	3	73	0½	424·557	20	7¼
	4	73	3⅝	427·605	20	8⅛
	5	73	6¾	430·665	20	9⅛
	6	73	9⅞	433·737	20	10
	7	74	1	436·817	20	10⅞
	8	74	4⅛	439·910	20	11¾
	9	74	7¼	443.014	21	0⅝
	10	74	10⅜	446·127	21	1½
	11	75	1⅝	449·253	21	2⅜
24		75	4¾	452·390	21	3¼
	1	75	7⅞	455·536	21	4⅛
	2	75	11	458·694	21	5
	3	76	2⅛	461·864	21	6
	4	76	5¼	465·042	21	6⅞
	5	76	8½	468·234	21	7¾
	6	76	11⅝	471·436	21	8⅝
	7	77	2¾	474·647	21	9½
	8	77	5⅞	477·871	21	10⅜
	9	77	9	481·106	21	11¼
	10	78	0⅛	484·350	22	0⅛
	11	78	3¼	487·607	22	1
25		78	6⅜	490·875	22	1⅞
	1	78	9½	494·151	22	2¾
	2	79	0¾	497·441	22	3¾
	3	79	3⅞	500·741	22	4⅝
	4	79	7⅛	504·051	22	5½
	5	79	11⅛	507·373	22	6⅜
	6	80	1¼	510·706	22	7¼
	7	80	4⅜	514·048	22	8⅛
	8	80	7⅝	517·403	22	9
	9	80	10¾	520·769	22	9⅞
	10	81	1⅞	524·144	22	10¾
	11	81	5	527·531	22	11⅝

USE OF THE ABOVE TABLE.—To find, by inspection, the area of any circle, from ⅛ to 100 inches, of which the diameter is given :—Calling the diameters feet, the area will be feet; if rods, or yards, the area will be of a corresponding denomination.

TABLE

OF

CIRCUMFERENCES AND AREAS OF CIRCLES,

In Feet and 10ths of a foot, or in Inches and 10ths of an inch.

Diam	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
·0	·000	·000	5 ·0	15·708	19·635	10 ·0	31·416	78·540
·1	·314	·007	·1	16·022	20·428	·1	31·730	80·118
·2	·628	·031	·2	16·336	21·237	·2	32·044	81·713
·3	·942	·070	·3	16·650	22·061	·3	32·358	83·323
·4	1·256	·125	·4	16·964	22·902	·4	32·672	84·948
·5	1·571	·196	·5	17·278	23·758	·5	32·986	86·590
·6	1·884	·282	·6	17·592	24·630	·6	33·300	88·248
·7	2·199	·384	·7	17·906	25·517	·7	33·614	89·920
·8	2·513	·502	·8	18·221	26·420	·8	33·929	91·609
·9	2·827	·636	·9	18·535	27·339	·9	34·243	93·313
1 ·0	3·141	·785	6 ·0	18·849	28·274	11 ·0	34·557	95·033
·1	3·456	·950	·1	19·163	29·224	·1	34·871	96·769
·2	3·769	1·130	·2	19·477	30·190	·2	35·185	98·520
·3	4·084	1·327	·3	19·792	31·172	·3	35·499	100·287
·4	4·399	1·539	·4	20·106	32·169	·4	35·814	102·070
·5	4·713	1·767	·5	20·420	33·183	·5	36·128	103·860
·6	5·027	2·010	·6	20·734	34·212	·6	36·442	105·683
·7	5·341	2·269	·7	21·048	35·256	·7	36·756	107·513
·8	5·655	2·544	·8	21·362	36·316	·8	37·070	109·359
·9	5·969	2·835	·9	21·677	37·412	·9	37·384	111·220
2 ·0	6·283	3·141	7 ·0	21·991	38·484	12 ·0	37·699	113·097
·1	6·596	3·463	·1	22·305	39·592	·1	38·013	114·990
·2	6·910	3·801	·2	22·619	40·715	·2	38·327	116·898
·3	7·224	4·154	·3	22·933	41·853	·3	38·641	118·823
·4	7·538	4·523	·4	23·247	43·008	·4	38·955	120·763
·5	7·853	4·908	·5	23·562	44·178	·5	39·269	122·718
·6	8·167	5·309	·6	23·876	45·364	·6	39·583	124·690
·7	8·481	5·725	·7	24·190	46·566	·7	39·898	126·677
·8	8·795	6·157	·8	24·504	47·783	·8	40·212	128·679
·9	9·109	6·605	·9	24·818	49·016	·9	40·526	130·698
3 ·0	9·424	7·068	8 ·0	25·132	50·265	13 ·0	40·840	132·732
·1	9·737	7·547	·1	25·446	51·530	·1	41·154	134·782
·2	10·051	8·042	·2	25·760	52·810	·2	41·468	136·848
·3	10·366	8·553	·3	26·074	54·106	·3	41·782	138·929
·4	10·680	9·079	·4	26·388	55·417	·4	42·097	141·026
·5	10·994	9·621	·5	26·702	56·745	·5	42·411	143·139
·6	11·308	10·178	·6	27·016	58·082	·6	42·725	145·267
·7	11·622	10·752	·7	27·331	59·446	·7	43·039	147·411
·8	11·936	11·341	·8	27·645	60·821	·8	43·353	149·571
·9	12·251	11·945	·9	27·959	62·212	·9	43·667	151·747
4 ·0	12·566	12·566	9 ·0	28·274	63·617	14 ·0	43·982	153·938
·1	12·880	13·202	·1	28·587	65·038	·1	44·296	156·145
·2	13·194	13·854	·2	28·901	66·476	·2	44·610	158·368
·3	13·508	14·522	·3	29·215	67·929	·3	44·924	160·606
·4	13·822	15·205	·4	29·530	69·397	·4	45·238	162·860
·5	14·137	15·904	·5	29·844	70·882	·5	45·552	165·130
·6	14·451	16·619	·6	30·158	72·387	·6	45·866	167·415
·7	14·765	17·349	·7	30·472	73·898	·7	46·180	169·717
·8	15·079	18·095	·8	30·786	75·429	·8	46·495	172·034
·9	15·393	18·857	·9	31·100	76·977	·9	46·809	174·366

Diam.	Circum.	Area.	Diam.	Circum.	Area.	Diam.	Circum.	Area.
15 ·0	47·124	176·715	21 ·0	65·793	346·361	27 ·0	84·823	572·556
·1	47·438	179·079	·1	66·287	349·667	·1	85·137	576·805
·2	47·752	181·358	·2	66·601	352·990	·2	85·451	581·070
·3	48·066	183·854	·3	66·915	356·328	·3	85·765	585·350
·4	48·380	186·265	·4	67·229	359·681	·4	86·079	589·646
·5	48·694	188·692	·5	67·543	363·051	·5	86·393	593·958
·6	49·008	191·134	·6	67·857	366·435	·6	86·707	598·286
·7	49·323	193·593	·7	68·171	369·837	·7	87·021	602·729
·8	49·637	196·067	·8	68·486	373·253	·8	87·335	606·988
·9	49·951	198·556	·9	68·800	376·685	·9	87·649	611·363
16 0	50·265	201·062	22 ·0	69·115	380·133	28 ·0	87·964	615·753
·1	50·579	203·583	·1	69·429	383·597	·1	88·277	620·159
·2	50·893	206·120	·2	69·743	387·076	·2	88·591	624·581
·3	51·207	208·672	·3	70·057	390·571	·3	88·905	629·019
·4	51·522	211·241	·4	70·371	394·082	·4	89·220	633·472
·5	51·836	213·825	·5	70·686	397·608	·5	89·534	637·941
·6	52·150	216·424	·6	71·000	401·150	·6	89·848	642·425
·7	52·464	219·040	·7	71·314	404·708	·7	90·162	646·926
·8	52·778	221·671	·8	71·629	408·282	·8	90·476	651·442
·9	53·092	224·318	·9	71·943	411·871	·9	90·790	655·973
17 ·0	53·407	226·980	23 ·0	72·256	415·476	29 ·0	91·106	660·521
·1	53·721	229·658	·1	72·570	419·097	·1	91·420	665·084
·2	54·035	232·352	·2	72·885	422·733	·2	91·734	669·663
·3	54·349	235·062	·3	73·199	426·385	·3	92·048	674·258
·4	54·663	237·787	·4	73·513	430·053	·4	92·362	678·867
·5	54·977	240·528	·5	73·827	433·737	·5	92·676	683·494
·6	55·291	243·285	·6	74·141	437·436	·6	92·990	688·136
·7	55·606	246·057	·7	74·455	441·151	·7	93·305	692·793
·8	55·920	248·846	·8	74·770	444·881	·8	93·619	697·466
·9	56·234	251·650	·9	75·084	448·628	·9	93·933	702·155
18 ·0	56·548	254·469	24 ·0	75·398	452·390	30 ·0	94·248	706·860
·1	56·862	257·324	·1	75·712	456·168	·1	94·561	711·580
·2	57·176	260·155	·2	76·026	459·961	·2	94·875	716·316
·3	57·490	263·022	·3	76·340	463·770	·3	95·189	721·067
·4	57·805	265·905	·4	76·655	467·595	·4	95·504	725·835
·5	58·119	268·803	·5	76·969	471·436	·5	95·818	730·618
·6	58·433	271·716	·6	77·284	475·292	·6	96·132	735·417
·7	58·747	274·646	·7	77·598	479·164	·7	96·446	740·231
·8	59·061	277·591	·8	77·912	483·052	·8	96·760	745·061
·9	59·375	280·552	·9	78·226	486·955	·9	97·074	749·907
19 ·0	59·690	283·529	25 ·0	78·540	490·875	31 ·0	97·389	754·769
·1	60·004	286·521	·1	78·855	494·809	·1	97·703	759·646
·2	60·318	289·529	·2	79·169	498·760	·2	98·017	764·539
·3	60·632	292·553	·3	79·483	502·726	·3	98·331	769·448
·4	60·946	295·593	·4	79·797	506·708	·4	98·645	774·372
·5	61·260	298·698	·5	80·111	510·706	·5	98·959	779·313
·6	61·574	301·719	·6	80·425	514·719	·6	99·273	784·269
·7	61·888	304·805	·7	80·739	518·748	·7	99·588	789·240
·8	62·203	307·908	·8	81·053	522·793	·8	99·902	794·227
·9	62·517	311·026	·9	81·367	526·854	·9	100·216	799·230
20 ·0	62·832	314·160	26 ·0	81·681	530·930	32 ·0	100·531	804·249
·1	63·145	317·309	·1	81·995	535·022	·1	100·844	809·284
·2	63·459	320·474	·2	82·309	539·129	·2	101·158	814·334
·3	63·773	323·655	·3	82·623	543·253	·3	101·472	819·399
·4	64·088	326·852	·4	82·937	547·392	·4	101·787	824·481
·5	64·402	330·064	·5	83·257	551·547	·5	102·101	829·578
·6	64·716	333·292	·6	83·556	555·717	·6	102·415	834·691
·7	65·030	336·536	·7	83·880	559·903	·7	102·729	839·820
·8	65·344	339·795	·8	84·294	564·105	·8	103·043	844·964
·9	65·658	343·070	·9	84·508	568·323	·9	103·357	850·124

MECHANICAL POWERS.

The whole science of Mechanics is simply the application of *weight* and *power*, or *force* and *resistance*. The *weight* is the resistance to be overcome; the *power* is the force requisite to overcome that resistance. When they are equal, they are said to be in *equilibrio;* that is, no motion can take place; but when the force becomes greater than the resistance, they are not in a state of equilibrium, and motion takes place.

Power may be considered as a compound of *weight* multiplied by its *velocity*. It cannot be increased by mechanical means. When two forces act on each other by means of any machine, that which gives motion is called *power:* that which receives it, is called *weight.*

Mechanical powers are the most simple of mechanical applications to increase force and overcome resistance.

These powers are three in number, viz: *lever*, *inclined plane*, and *pulley*. The *wheel* and *axle* is a *revolving* lever; the *wedge* is a double inclined plane; and the *screw* is a *revolving* inclined plane.

THE LEVER.

To make the principle easily understood, we must suppose the Lever to be an inflexible *bar* or *rod*, either straight or bent, without weight, some part of which being supported by a *fulcrum* or *prop*, as a *center of motion;* the rod itself supposed capable of turning freely about that point.

CASE I.

When the Fulcrum of the Lever is between the Weight and the Power.

Rules.—1. Divide the weight to be raised by the power to be applied and the quotient is the difference of leverage, or the distance from the fulcrum at which the power supports the weight. Or,

2. Multiply the weight by its distance from the fulcrum, and the power by its distance from the same point, and the weight and power will be to each other as their products.

Examples.—1. A weight of 1440 lbs. is to be raised by a force of 70 lbs: required the length of the longer arm of the lever, the shorter being 1 foot.

$$\frac{1440\times1}{70}=20\tfrac{4}{7}\text{ feet to 1. Ans.}$$

Proof $\quad 1440\times1 \quad =1440$

$\qquad\qquad 70\times20\tfrac{4}{7}=1440$

2. A weight of 680 lbs. is placed 18 inches from the fulcrum of a lever: what force will raise it, the length of the other arm of the lever being 11 feet?

$$\frac{680\times18}{132}=92\cdot72 \text{ lbs. Ans.}$$

CASE II.

When the Fulcrum is at one extremity of the Lever, and the Power or Weight at the other.

Rule.—As the distance between the *power* and fulcrum is to the distance between the *weight* and fulcrum, so is the effect to the power.

Examples.—1. What power will raise 1500 lbs., the weight being 6 feet from it, and 3 feet from the fulcrum?

$$6+3=9$$
$$\text{Then, } 9 : 3 :: 1500 : 500 \text{ lbs. Ans.}$$

2. Suppose a weight of 2440 lbs. is to be raised with a lever 12 feet long, the fulcrum being fixed 1 foot from the weight, what power must be applied to the other end of the lever to effect a balance?

$$12-1=11.$$
$$11 : 1 :: 2440 \text{ lbs.} : 221\cdot818 \text{ lbs. Ans.}$$

The GENERAL RULE, therefore, for determining the relation of POWER to WEIGHT, in a lever, is, the power multiplied by its distance from the fulcrum is equal to the weight multiplied by its distance from the fulcrum.

Let P represent the power.
W " the weight.
p " the distance of P. from the fulcrum.
w " the " of W. from the fulcrum.

We shall then have the following formula:

$$P : W :: w : p, \text{ or } P\times p=W\times w.$$

$$\text{And } \frac{W\times w}{p}=P \qquad \frac{P\times p}{w}=W.$$

$$\frac{W\times w}{P}=p \qquad \frac{P\times p}{W}=w$$

If several weights or powers which tend to turn the lever round in one way be represented by P P′ P″, and their distances from the fulcrum be represented by $p\ p'\ p''$; and, again, if those which tend to turn it in an opposite way be represented by W W′ W″ W‴, and their distances from the fulcrum be $w\ w'\ w''\ w'''$, the condition of equilibrium is

$$P\times p+P'\times p'+P''\times p''=W\times w+W'\times w'+W''\times w''+W'''\times w'''..$$

If the arms of the lever be not straight, but equally bent or curved, the distances p and w are the perpendiculars drawn from the fulcrum

upon the directions of the power and weight, and must be measured on a line running horizontally through the fulcrums to perpendicular lines in the direction of the weight and power.

NOTE. In the above formula, the weights of the levers are not taken into consideration, the center of gravity being assumed to be over the fulcrum.

THE WHEEL AND AXLE, OR PERPETUAL LEVER.

The principle of the *wheel and axle* is so analogous to that of the *lever*, that it may be considered next in the order of mechanical powers.

General Principle.—As the radius of the wheel is to the radius of the axle, so is the weight to the power.

CASE I.

The radius of a wheel, and radius of a barrel or axle to which it is attached being given, to find the weight that a given power will raise by means of such application.

Rule.—Multiply the power applied at the periphery of the wheel by its radius, and divide the product by the radius of the axle. The quotient is the weight that the power will raise, *friction* not being taken into account.

Example.—Required the weight that can be raised by a power of 50 lbs. applied at the circumference of a wheel of $2\frac{1}{2}$ feet radius; the weight to be attached to the end of the rope, which is to be wound round a barrel or axle 12 inches in diameter.

$2\frac{1}{2}$ feet $=30$ inches, and $50\times30\div6=250$lbs. weight.

CASE II.

The weight to be raised, the diameter of the axle and diameter of the wheel being given, to find the amount of power required to raise the weight.

Rule.—Multiply the weight to be raised by the radius of the axle, and divide the product by the radius of the wheel. The quotient is the power required.

Example.—Required the power necessary to raise a weight of 500 lbs. by an axle of 12 and wheel of 50 inches diameter.

$500\times6\div25=120$lbs. power.

INCLINED PLANE.

When a power acts on a body on an inclined plane, so as to keep that body at rest; then the weight, the power, and the pressure on the plane, will be as the length, the height, and the base of the plane when the power acts parallel to the plane; that is

The *weight* will be as the *plane*;
The *power* will be as the *height*;
The *pressure* on the plane will be as the *base*.

These properties give rise to the following formula:

Let W represent weight.
P " power.
l " length of the plane.
h " height of plane.
p " pressure on the plane.
b " base of plane.

Then,
$$P=\frac{W\times h}{l}$$
$$W=\frac{P\times l}{h}$$
And,
$$p=\frac{W\times b}{l}$$

General Principle.—As the length of the plane is to the height or angle of inclination, so is the weight to the power.

CASE I.

The length and height of an inclined plane being known, to find the weight that a given power will support upon the plane,

Rule.—Multiply the power by the length of the plane, and divide the product by the height; the quotient is the weight that the power will support.

Examples.—1. Let the length of an inclined plane be 30 yards and its height 4 yards: required the weight that a power of 50 lbs. will support upon the plane.

$$50\times 30\div 4=375 \text{ lbs.}$$

2. Required the power necessary to raise 1280 lbs. up an inclined plane 8 feet long and 5 feet high,

$$8 : 5 : : 1280 : 800. \quad \text{Ans.}$$

To find the Length of the Plane, the Length of the Base or Height, when any two of them are given.

Rules.—1. *For the length of the base*, subtract the square of the height from the square of the length of the plane, and the square root of the remainder will be the length of the base.

2. *For the length of the plane*, add the squares of the two other dimensions together, and the square root of their sum will be the length required.

3. *For the height*, subtract the square of the base from the square of the length of the plane, and the square root of the remainder will be the height required.

Example.—If the height of an inclined plane be 18 feet, and its length 85 feet, what is the length of the base, and the pressure of 800 lbs. upon the plane?

$$\sqrt{\overline{85}^2-\overline{18}^2}=\sqrt{8901}=94{\cdot}03=\text{length of the base.}$$

And $\dfrac{800\times 94{\cdot}03}{85}=885$ lbs. the pressure upon the plane.

CASE II.

If two bodies on two inclined planes sustain each other by the aid of a cord over a pulley, their weights are directly as the lengths of the planes.

Example.—If a body, weighing 60 lbs., resting upon an inclined plane of 8 feet rise in 80 feet, be sustained by another weight on an opposite plane of 9 feet rise to 81 feet of inclination, what is the weight of the latter.

As 80 : 81 : : 60 : $60\frac{3}{4}$. Ans.

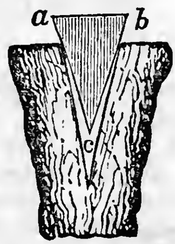
THE WEDGE.

The *wedge* is a solid figure which is called in geometry a *triangular* prism. It is formed either of wood or metal, and is very generally used in cleaving timber or spliting rocks, in which case its edge is introduced into a cleft already made to receive it.

The circumstances in which it is applied are such that it is not easy to devise a general rule to determine the amount of its action. The wedge has a great advantage over all other mechanical powers, in consequence of the way in which the power is applied to it, namely, by percussion, or a stroke, so that, by the blow of a hammer, almost any constant pressure may be overcome.

When two bodies are forced from one another by means of a wedge in a direction parallel to its head.

Rule.—As the length of the wedge is to half its back, so is the resistance to the force. Or, multiply the resisting power by half the thickness of the head or back of the wedge, and divide the product by the length of one of its inclined sides, the quotient is the force equal to the resistance.



Example.—The length of the back of a double wedge, *a b c*, is 6 inches, and the length of it through the middle, 10 inches : what is the power necessary to separate a substance having a resistance of 150 lbs. ?

As 10 : 3 : : 150 : 45 lbs. Ans.

When only one of the Bodies is movable.

Rule.—As the length of the wedge is to its back, so is the resistance to the power.

Example.—What power, applied to the back of the wedge, will raise a weight of 18000 lbs., the wedge being 8 inches deep and 125 long on its base ?

As 125 : 8 : : 18000 : 1152 lbs. Ans.

NOTE. As the power of the wedge in practice depends upon the split or rift in the wood to be cleft, or the body to be raised, the above rules are only theoretical where a rift exists

THE SCREW.

The *screw* is a spiral thread or groove, cut round a cylinder, and every where making the same angle with the length of it. So that if the surface of a cylinder, with this spiral thread on it, were unfolded and stretched into a plane, the spiral thread would form a straight inclined plane, whose length would be to its height as the circumference of the cylinder is to the distance between two threads of the screw. This will appear evident by considering that in making one round, the spiral rises along the cylinder the distance between the two threads.

It is manifest, therefore, that the *screw* is an *inclined plane*, wound round a cylinder. The *length* of the plane is found by adding the square of the circumference of the screw to the square of the distance between the threads, and extracting the square root of the sum. The *height* of the plane is the distance between any two contiguous threads.

Rule.—As the length of an inclined plane is to the pitch or height of it, so is the weight to the power.

When a wheel or capstan is applied to turn the screw, the length of the lever is the radius of the circle described by the handle of the wheel or capstan bar.

Let P	represent	power.
" R	"	length of lever.
" W	"	weight.
" l	"	length of the inclined plane.
" p	"	pitch of screw or height of plane.
" x	"	effect of power at circumference of screw.
" r	"	radius of screw.

Then, by the above rules, we have the following formula:

$$\begin{array}{l}
\text{As } l : p :: W : P, \\
l : W :: p : P, \\
W : l :: P : p, \\
P : W :: p : l, \\
r : R :: P : x, \\
P : x :: r : R, \\
R : r :: x : P.
\end{array}$$

Example.—What is the power requisite to raise a weight of 12000 lbs. by a screw of 10 inches circumference and 1 inch pitch?

$$\overline{10}^2+\overline{1}^2=101, \text{ and } \sqrt{101}=10{\cdot}049875$$

Then, $10{\cdot}049875 : 1 :: 12000 : 1194{\cdot}0447$ lbs. Ans.

When the circumference described by the power is used (C representing the circumference), we have the following formula:

$$\begin{array}{l}
P : W :: p : C, \\
C : p :: W : P, \\
P \times C = W \times p.
\end{array}$$

When a hollow screw revolves upon one of less diameter and

pitch, the effect is the same as that of a single screw, in which the distance between the threads is equal to the difference of the distances between the threads of the two screws.

If one screw has 10 threads in an inch pitch, and the other 12, the power is to the weight as the difference between $\frac{1}{10}$ and $\frac{1}{12}$ or $\frac{1}{60} = 1$ to 60.

If a complex machine, composed of the screw, and wheel and axle, the relation between the weight and power is thus:

Let x represent the effect of the power on the wheel.
" R " the radius of the wheel.
" p " the pitch of the screw.
" r " the radius of the axle.
" C " the circum. described by the power.

Then we shall have the following formula, from the properties of the screw already laid down:

$P \times C = x \times p$; and of the wheel and axle, $x \times R = W \times r$.
Hence, we have, $P \times C \times x \times R = x \times p \times W \times r$.

We may expunge the common multiplier x, without changing the value of the equation, and we shall then have,

$$P \times C \times R = W \times p \times r;$$
$$\text{Or, } P : W :: p \times r : C \times R.$$

Example.—What weight can be raised with a power of 15 lbs. when applied to a crank 38 inches long, turning on an endless screw of $4\frac{1}{2}$ inches diameter and $1\frac{1}{2}$ inch pitch, applied to a wheel and axle of 24 and 6 inches diameter respectively?

$38 \times 2 = 76$ = diamater of circumference.
By the table of circumferences $76 = 238{\cdot}7$.
$1\frac{1}{2} : 238{\cdot}7 :: 15 : 2387$.
Radii of wheel and axle, 12 and 3.
$3 : 12 :: 2387 : 9548$ lbs. Ans.

When a series of wheels and axles act upon each other, the weight will be to the power as the continued product of the radii of the wheels to the continued product of the radii of the axles;

$$\text{Thus, } W : P :: R^3 : r^3;$$
$$\text{Or, } r^3 : R^3 :: P : W,$$

there being three wheels and axles of the same proportion to each other.

Example.—If a man can draw a weight of 150 lbs. up the side of a perpendicular wall 20 feet high, what weight will he be able to raise along a smooth plank, 30 feet long, laid sloping from the top of the wall?

(See formula under the head "Inclined Plane.")

$$\frac{P \times l}{h} = W$$

$$\text{Then, } \frac{150 \times 30}{20} = 225. \quad \text{Ans.}$$

THE PULLEY.

A pulley is a small *grooved* wheel, movable about a pivot, the pivot itself being at the same time either *fixed* or *movable*.

A *single pulley* that only turns on its axis, and does not move out of its place, serves only to change the direction of the power, but gives no mechanical advantage, since it is evident that the power and weight are equal. It is, however, very convenient in accommodating the direction of the power to that of the resistance. Thus, by *pulling downwards*, we are able to draw a weight *upwards*. The advantage gained is always as twice the number of movable pulleys, without taking any notice of the fixed pulleys necessary to compose the system of pulleys.

The principle upon which the weight is sustained by means of a pulley or system of pulleys is very simple, and may be readily understood from the following rules:

CASE I.

To find the Weight that may be raised by a known power and a given number of fixed and movable or stationary and rising pulleys.

Rule.—Multiply the power by twice the number of movable pulleys, and the product is the weight the power is equal to.

Example.—Required the weight that a power of 180 lbs. will raise by a block and tackle, the bottom or movable block consisting of four *sheaves* or pulleys.

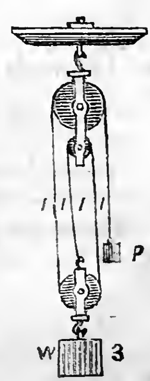
$$180\times 8=1440 \text{ lbs. weight.}$$

When only one Cord or Rope is used.

Rule.—Divide the weight to be raised by the number of parts of the rope engaged in supporting the lever or movable block.

Example.—Required the power that will raise a weight of 3300 lbs. with a single movable pulley, as shown in the margin.

$$3300\div 3=1100. \quad \text{Ans.}$$



The single movable pulley may be so constructed, as shown in the marginal figure, that the weight will be three times the power, the *number* placed at each rope expresses the part of the weight it sustains.

Example.—What power will it require to raise 900 lbs. when the lower block contains six *sheaves*, and the end of the rope is fastened to the upper block, and what power when fastened to the lower block?

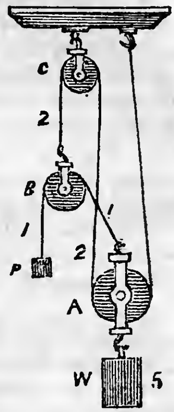
$$\frac{900}{6\times 2}=75 \text{ lbs.} \quad \text{1st Ans.}$$

$$\frac{900}{6\times 2+1}=68\tfrac{3}{13} \text{ lbs.} \quad \text{2d Ans.}$$

Or, if n, represents the number of parts of the rope which sustains the lower block, we shall have the following formula:

$$W = n \times P.$$

CASE II.



In the system represented in the adjoining figure, called the *Spanish Burton*, the tension of the rope, P B, is equal to the power; and this rope, being finally attached to the pulley which sustains the weight, supports a part of the weight equal to the power. The rope from C to B balances the united tensions of both parts of the rope, extending from B to the weight and power, and therefore its tension is twice the power; and being brought under the pulley which sustains the weight, and finally attached to the fixed point, it sustains a part of the weight equal to four times the power: thus the whole weight must be equal to *five* times the power. The power being taken as the unit, the number placed at each rope expresses the part of the weight it sustains, *i. e.* as 5 to 1.

Rule.—Divide the weight to be raised by the sum of the numbers placed at each rope, and the quotient will be the power.

Example.—Required the power necessary to raise 4500 lbs. with a single movable pulley, and with *two* ropes, as shown in the above figure.

$$4500 \div 5 = 900 \text{ lbs.}$$

NOTE. The first cord, B P, is obviously not taken into the account.

When more than one Rope is used.

In a system of pulleys where the ends of one rope are fastened to the *support* and *power*, and the ends of the other to the lower and upper blocks, the weight is to the power as 4 to 1, and with any number of ropes, the ends being fastened to the support,

$$W = 2^n \times P.$$

Example.—What weight will a power of 3 lbs. sustain in a system of 4 movable pulleys and 4 ropes?

$$3 \times 2 \times 2 \times 2 \times 2 = 48 \text{ lbs.} \quad \text{Ans.}$$

MECHANICAL CENTERS.

These are the centers of gravity, oscillation, percussion, and gyration.

CENTER OF GRAVITY.

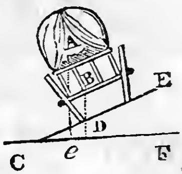
The center of gravity of a body, or any system of bodies connected, is a point about which, if suspended, all the parts are in equilibrio.

Hence, if a body be suspended or supported by this point, the body will rest in any position into which it is put. We may, therefore, consider the whole weight of a body as centered in this point.

The common center of gravity of two or more bodies, is the point about which they would equiponderate, or rest in any position. If the centers of gravity of two bodies, A and B, be connected by a right line, the distances, A C and B C, from C, the common center of gravity, are reciprocally as the weights or bodies A and B; that is, as A C : B C : : B : A.

If a line be drawn from the center of gravity of a body, perpendicular to the horizon, it is called *the line of direction*, because it is the line the center of gravity would describe if the body fell freely.

Thus, the adjoining figure represents a loaded wagon on the declivity of a hill. The line C F represents the horizon; D E the base of the wagon. If the wagon be loaded in such a manner that the center of gravity be at B, the perpendicular, B D, will fall within the base, and the wagon will stand. But if the load be altered, so that the center of gravity be raised to A, the perpendicular, A *e*, will fall outside of the base, and the wagon will be overset. From this it follows that a wagon, or any carriage, will be most firmly supported when the center of gravity falls exactly between the wheels; and that is the case on a level road. The center of gravity, in the human body, is between the hips, and the base is the feet.



The method of determining the center of gravity mechanically in various bodies of uniform density is as follows:

OF SURFACES.

To find the Center of Gravity in the Surface of any Parallelogram, Rhombus, Rhomboid, Regular Polygon, Ellipse, Circle, or Lune.

Rule.—Draw diagonals, diameters, or radii, intersecting one another; the point of intersection is the center of gravity; *i. e.*, the center of gravity is the geometrical center of these figures.

To find the Center of Gravity of a Triangle.

Rule.—Draw a line from any angle to the middle of the opposite side, then $\frac{2}{3}$ of this line from the angle will be the position of the center of gravity.

To find the Center of Gravity of a Trapezium.

Rule.—Draw the two diagonals, and find the centers of gravity of each of the four triangles thus formed, then join each opposite pair of these centers of gravity, and the two joining lines will cut each other in the center of gravity of the figure.

To find the Center of Gravity of a Circular Arc.

Rule.—$\frac{\text{Radius of circle} \times \text{chord of arc}}{\text{length of arc}}$ = distance of the center of gravity from the center of the circle.

To find the Center of Gravity of the Sector of a Circle.

Rule.—$\frac{2 \times \text{chord of arc} \times \text{radius of circle}}{3 \times \text{length of arc}}$ = distance of the center of gravity from the center of the circle.

To find the Center of Gravity of a Semicircle.

Rule.—$\frac{4 \times \text{radius of circle}}{3 \times 3{\cdot}14159}$ = distance from the center

To find the Center of Gravity of a Segment of a Circle.

Rule.—$\frac{\text{Chord of the segment cubed}}{12 \times \text{area of segment}}$ = distance from the center.

To find the Center of Gravity of a Parabola.

Rule.—The distance of the center of gravity from the vertex is $\frac{3}{5}$ of the axis.

To find the Center of Gravity of a Cylinder Cone, Frustrum of a Cone, Pyramid, or Frustrum of a Pyramid.

Rule.—The center of gravity is at the same distance from the base as that of the parallelogram, triangle, or trapezoid, which is a right section of either of the above figures.

OF SOLIDS.

To find the Center of Gravity of a Sphere, Spherical Segment, or Zone.

Rule.—At the middle of their height, is their center of gravity.

To find the Center of Gravity of a given Pyramid or Cone.

Rule.—Draw a straight line from the vertex to the center of gravity of the base, and divide it in the ratio of 3 to 1, the greatest segment being next the vertex. The point so found is the center of gravity of the pyramid. This construction applies to a pyramid of any number of sides, and therefore also to a cone.

Or, $\frac{1}{4}$ of the line joining the vertex and center of gravity of the base.

To find the Center of Gravity of a Frustrum of a Cone or Pyramid.

Rule.—$\frac{1}{4} h \times \frac{(R+r)^2+2R^2}{(R+r)^2-Rr}$ where h the height, and R and r the radii, the greater and lesser ends in a cone and the sides of a pyramid.

To find the Center of Gravity of a Paraboloid.

Rule.—The distance from the vertex is $\frac{2}{3}$ of the axis or abscissa.

To find the Center of Gravity of the Frustrum of a Paraboloid.

Rule.—$\frac{1}{3}h \times \frac{2R^2+r^2}{R^2+r^2}$ where h the height, and R and r the radii of the greater and lesser ends.

To find the Center of Gravity of a Spherical Segment.

Rule.—The distance from the center$=\frac{3{\cdot}1416\, v^2(r-v)^2}{s}$ where v is the versed sine, s the solidity of the segment, and r the radius of the sphere.

To find the Center of Gravity of a Spherical Sector.

Rule.—The distance from the center$=\frac{3}{4}(r-\frac{v}{2})$.

To find the Center of Gravity of any System of Bodies.

Rule.—The distance of the common center of gravity from a given plane $=\frac{B\ D+B'\ D'+B''\ D''+, \text{\&c.}}{B+B'+B''+, \text{\&c.}}$; $B\ B'\ B''$ being the masses or solid contents of the bodies, and $D.\ D'\ D''$ the distances of their respective centers of gravity, from the given plane.

FRAUDULENT BALANCES.

Commercial balances are frequently misconstructed for fraudulent purposes; but the end to be obtained by the use of such a balance, may be readily defeated by the following

Rule.—First establish an equilibrium between the substance to be weighed and the weights; then transpose them, and the weight will preponderate, if the article weighed be lighter than the weight, and contrawise. Then,

To ascertain the True Weight,

Find the weight that will produce equilibrium after transposition, and let this and the former counterpoise be reduced to the same denomination of weight; then multiply together the two weights thus expressed, and extract the square root of their product; the result will be the true weight.

Example.—If one counterpoise weigh 7 lbs., and the other $9\frac{1}{7}$, the product is 64, the square root of which is 8. Hence, 8 lbs. is the true weight; and in this way the most precious metals may be accurately weighed on the most fraudulent balances, which may be proved by the following formula:

Let a and b be the arms of the two balances, A B the two counterpoises, and x the true weight.

We have

$$x : A :: a : b$$
$$B : x :: a : b.$$

Hence we may infer, that

$$x : A :: B : x$$
$$\therefore x^2 = A\,B.\ x = \sqrt{A\,B}.$$

GRAVITATION, OR THE FALLING OF BODIES.

Gravitating bodies attract each other with forces varying inversely as the squares of their distances.

In bodies descending freely by their own weight, the velocities are as the times, and the spaces as the square of the times; therefore, if the times be as the numbers, 1, 2, 3, 4, &c., the velocities will be also, 1, 2, 3, 4, &c.; the spaces passed through, as 1, 4, 9, 16, &c.; and the spaces for each time as the series of odd numbers, 1, 3, 5, 7, &c.

It has been ascertained, by experiment, that a body falling freely from rest, will descend $16\frac{1}{12}$ feet in the first second of time, and will then have acquired a velocity, which, being continued uniformly, will carry it through $32\frac{1}{6}$ feet in the next second. Therefore, if the first series of numbers be expressed in seconds, 1″, 2″, 3″, &c., the velocities in feet will be $32\frac{1}{6}$, $64\frac{1}{3}$, $96\frac{1}{2}$, &c.; the spaces passed through as $16\frac{1}{12}$, $64\frac{1}{3}$, $144\frac{3}{4}$, &c., and the spaces for each second, $16\frac{1}{12}$, $48\frac{1}{4}$, $80\frac{5}{12}$, &c.

CASE I.

To find the Velocity a Falling Body will acquire in any given time.

Rule.—Multiply the time, in seconds, by $32\frac{1}{6}$, and it will give the velocity acquired in feet, per second.

Example.—Required the velocity in 7 seconds.

$$32\tfrac{1}{6} \times 7 = 225\tfrac{1}{6} \text{ feet. Ans.}$$

TABLE,

Showing the Relation of Time, Space, and Velocity.

Time in seconds of the bodies' fall.	Velocity acquired at the end of that time.	Squares.	Space fallen through in that time.	Space.	Whole space fallen through in the last second of the fall.
1	32·16	1	16·08	1	16·08
2	64·33	4	64·33	3	48·25
3	96·5′	9	144·75	5	80·41
4	128·66	16	257·33	7	112·58
5	160·83	25	402·08	9	144·75
6	193·	36	579·	11	176·91
7	225·17	49	788·08	13	209·08
8	257·33	64	1029·33	15	241·25
9	289·5	81	1302·75	17	273·42
10	321·66	100	1946·08	19	305·58

CASE II.

To find the Velocity a Body will acquire by falling from any given height.

Rule.—Multiply the space, in feet, by $64\frac{1}{3}$, the square root of the product will be the velocity acquired, in feet, per second.

Example.—Required the velocity which a ball has acquired in descending through 201 feet.

$$64\tfrac{1}{3} \times 201 = 12931 ; \ \sqrt{12931} = 1137 \text{ feet. Ans.}$$

CASE III.

To find the Space through which a Body will fall in any given time.

Rule.—Multiply the square of the time, in seconds, by $16\frac{1}{12}$, and it will give the space in feet.

Example.—Required the space fallen through in seven seconds.

$$16\tfrac{1}{12} \times 7^2 = 788\tfrac{1}{12} \text{ feet. Ans.}$$

CASE IV.

To find the Time which a Body will be in falling through a given Space.

Rule.—Divide the *square root* of the space fallen through by 4, and the quotient will be the time in which it was falling.

Example.—Required the time a body will be in falling through 402·08 feet of space.

$$\sqrt{402{\cdot}08} = 20{\cdot}049, \text{ and } 20{\cdot}049 \div 4 = 5{\cdot}012. \text{ Ans.}$$

CASE V.

The Velocity being given, to find the Space fallen through.

Rule.—Divide the velocity by 8, and the square of the quotient will be the distance fallen through to acquire that velocity.

Example.—If the velocity of a cannon ball be 660 feet per second, from what height must a body fall to acquire the same velocity?

$$660 \div 8 = 82{\cdot}5^2 = 6806{\cdot}25 \text{ feet. Ans.}$$

CASE VI.

To find the Time, the Velocity per second being given.

Rule.—Divide the given velocity by 8, and one-fourth part of the quotient will be the answer.

Example.—How long must a bullet be falling to acquire a velocity of 480 feet?

$$480 \div 8 = 60 \div 4 = 15 \text{ seconds. Ans.}$$

Bodies ascending, are retarded in the same ratio that descending bodies are accelerated; consequently, the following rules will be found useful:

CASE VII.

To find the Space moved through by a Body projected Upward or Downward with a given velocity.

Rule.—Multiply the square of the time in seconds by 16·083 (as per table), the velocity of the projection in feet by the number of seconds the body is in motion, and the *difference* of these products is the answer.

NOTE. If projected *upward*, the *difference* of the above products will give the distance of the body from the point of projection ; if *downward*, their *sum*.

Examples.—1. If a rocket, projected upward, return to the earth in 12 seconds, how high did it ascend before it began to return?

The rocket was half the time in ascending.

$$12 \div 2 = 6, \text{ and } 6^2 \times 16{\cdot}03^3 = 579 \text{ feet. Ans.}$$

$$\text{Or } 6^2 \times 16{\cdot}033 - 193 \times 6 = 579 \quad \text{“} \quad \text{“}$$

2. If a body be projected upward with a velocity of 30 feet per second, through what space will it ascend before it begins to return?

$$30^2 \div 64{\cdot}33 = 13{\cdot}9 \text{ feet. Ans.}$$

CASE VIII.

To find the Velocity of a falling stream of Water per second at the end of any given second, the perpendicular distance being given.

NOTE. The space through which a body will descend, on an inclined plane, is to the space through which it would fall freely in the same time as the height of the plane to its length.

Examples.—1. What is the velocity of the stream per second on a sluice, one end of which is 30 inches lower than the other.

By Case ii.

$$30 \text{ inches} = 2{\cdot}5 \text{ feet; and } 2{\cdot}5 \times 6433 = 160{\cdot}82$$

$$\sqrt{160{\cdot}82} = 12{\cdot}65 \text{ feet. Ans.}$$

2. What is the distance a stream of water will descend on an inclined plane, 10 feet high and 100 feet long at the base, in 5 seconds?

$$5^2 \times 16{\cdot}083 = 402{\cdot}08$$

$$\text{Then, } 100 : 10 :: 402{\cdot}08 : 40{\cdot}20 \text{ feet. Ans.}$$

NOTE. The momentum with which a falling body strikes, is equal to its weight multiplied by its velocity.

If a weight of 2800 lbs. fall through 9 feet, with what force does it strike?

$$\sqrt{9 \times 64{\cdot}33} = 24 \times 2800 = 67200 \text{ lbs. Ans.}$$

THE GRAVITIES OF BODIES.

The *gravity* of a body, or its weight above the earth's surface, decreases as the square of its distance from the earth's center in semi-diameters of the earth.

Example.—Suppose a body to weigh 400 lbs. at 2000 miles above the earth's surface, what would it weigh at the surface, estimating the earth's semi-diameter at 4000 miles?

$$4000+2000=1\tfrac{1}{2} \text{ semi-diameters.}$$
$$\text{Then } 1{\cdot}5\times1{\cdot}5\times400=900 \text{ lbs. Ans.}$$

Rule.—Multiply the square of the semi-diameters by the weight, and the product will be the answer.

Example.—If a body weigh 900 lbs. at the surface of the earth, what will it weigh 200 miles above the surface?

Reverse the above rule.

$$900\div1{\cdot}5\times1{\cdot}5=400 \text{ lbs. Ans.}$$

THE PENDULUM.

The force of gravity is the cause of a pendulum's motion; consequently, its motion at any place is dependent upon the energy of the force of gravity at that place.

Pendulums of the same length vibrate slower the nearer they are brought to the equator, because of the earth's spheroidal form, its polar axis being about 26 miles longer than its equatorial diameter; for which reason, also, gravity is lessened $\frac{1}{289}$ part, the centrifugal force arising from the diurnal motion of the earth being greater at the equator than at the poles.

The measure of the force of gravity in feet per second at any place is equal to the product of the length of a pendulum that beats seconds: these, multiplied by 9·8696; or, if any length of a pendulum be taken in *feet*, and the time in *seconds* observed between each of its oscillations, then the length of the pendulum, divided by the square of the time in seconds, and the quotient multiplied by 9·8696, the product will equal the number of feet by which gravity will at that place increase the velocity of the descent of a falling body in each second of time.

The simple pendulum is a ready means by which to obtain the center of oscillation in a compound pendulum. Thus: suspend a small ball by a fine thread in front of that of which the center of oscillation is required, and lengthen or shorten the thread until the vibrations of each are alike; then stop both, and let them hang freely; and opposite the center of the ball is the center of oscillation.

The length of a pendulum to vibrate seconds, or 60, in the latitude of London is 39·1393 inches; in the latitude of Edinburgh, 39·1555; in that of Paris, 39·1286; and in that of New York, 39·1013. Hence, the length of the seconds pendulum being known, the length of any other pendulum to perform a given number of vibrations in the same time may be obtained by the following

Rule.—Multiply the square root of the given length by 60, and divide the product by the given number of vibrations per minute. The square of the quotient will equal the length of pendulum required.

Example.—Required the length of a pendulum to make 70 vibrations in a minute in the latitude of New York.

$$\frac{\sqrt{39 \cdot 1013} \times 60}{70} = 5 \cdot 360 \text{ inches, the length required.}$$

NOTE. The vibration of pendulums are as the square roots of their lengths. The length of one vibrating seconds in the latitude of New York is 39·1013 inches; consequently, $\sqrt{39 \cdot 1013} \times 60 = 375 \cdot 23$, a constant number for a divisor.

Hence, the following general

Rule.—Divide the constant number, 375·23, by the number of vibrations to be made per minute, and the square of the quotient is the length of the pendulum in inches.

Example.—Required the length of a pendulum to make 80 vibrations per minute.

$$\frac{375 \cdot 23}{80} = 4 \cdot 690, \text{ and } \overline{4 \cdot 690}^2 = 21 \cdot 996 \text{ inches. Ans.}$$

NOTE. The length of pendulums for less or greater times is as the square of the times: thus, for $\frac{1}{2}$ seconds it would be the square of $\frac{1}{2}$, or $\frac{39 \cdot 1013}{4} = 9 \cdot 7753$ inches, the length of a half-second pendulum.

The Length of a Pendulum being given, to find the space through which a body will fall in the time the Pendulum makes one Vibration.

Rule.—Multiply the length of the pendulum by 4·9348; the product will be the time.

Example.—Required the space through which a body will descend in *vacuo*, in one second.

$$39 \cdot 1013 \times 4 \cdot 9348 = 192 \cdot 957. \text{ Ans.}$$

To find the Length of a Pendulum, the Vibrations of which shall be the same number as the inches in its length.

Rule.—Extract the *cube root* of the constant number, 375·234. and the square of the root thus found will be the length of the pendulum required.

$$\text{Thus, } \sqrt[3]{375 \cdot 234} = 7 \cdot 21, \text{ and } \overline{7 \cdot 21}^2 = 51 \cdot 98 \text{ inches. Ans.}$$

SPECIFIC GRAVITIES.

Specific gravity is the relative weight of any body of a certain bulk, compared with the weight of some body taken as a standard of the same bulk. The standard of comparison is water; one cubic foot of which is found to weigh 1000 ounces avoir. at a temperature of 60° of Fahrenheit; so that the weight expressed in ounces of a cubic foot of any body, will be its *specific gravity*, that of water being 1000.

To find the Specific Gravity of a body Heavier than Water.

Rule.—Weigh it both in and out of water, and take the difference; then, as the weight lost in water is to the whole weight, so is 1000 to the specific gravity of the body.

NOTE. The most convenient way to obtain the specific gravity, will be to reduce the weight, both in and out of water, to grains. This will be easily understood by reference to the notes and rules under Apothecaries' Weight, in the Table of Weights and Measures, page 15, *which see.* Apothecaries' or jewelers' scales should in all cases be used, as great accuracy is requisite.

Example.—What is the specific gravity of a stone which weighs 15 lbs., but in water only 10 lbs.?

$15-10=5. \quad 5 : 15 :: 1000 : 3000.$ Ans.

If the body be Lighter than Water.

Rule.—Annex to it a piece of metal, so that the compound may sink in water. Weigh the piece added and the compound mass separately, both in and out of water; then find how much each loses in water, by subtracting its weight in water from its weight in air, and subtract the less of these differences from the greater; then,

As the last remainder is to the weight of the light body in air, so is 1000 to the specific gravity of the body.

Example.—What is the specific gravity of a piece of elm which weighs in air 25 lbs.? Annex to it a piece of metal that weighs 18 lbs. in air and 15 in water, and the two pieces in water weigh 6 lbs.

$$\begin{array}{ll} 25+18-6=37 & \quad 34 : 25 :: 1000 : \cdot 735. \text{ Ans.} \\ 18-15 \quad = \underline{\ 3} & \\ \qquad\qquad 34 & \end{array}$$

If the body be a Fluid.

Rule.—Take a piece of a body whose specific gravity is known, and weigh it both in and out of the fluid: then take the difference of these weights, and say,

As the weight of the body is to the loss of weight, so is the specific gravity of the body to that of the fluid.

Example.—What is the specific gravity of a fluid in which a piece of copper (*s. g.*$=9000$) weighs 75 lbs. in, and 81 lbs. out of it?

$81-75=6.$ Then $80 : 6 :: 9000 : \cdot 675.$ Ans.

To determine the Quantity of two Ingredients in a Compound which they form.

Let H be the weight of the heavy body.
h, its specific gravity.
L, the weight of the lighter body
l, its specific gravity.
C, the weight of the compound
c, its specific gravity.

Then,

$$\frac{(c-l)\times h}{(h-l)\times c}\times C=H.$$

Also,

$$\frac{(h-c)\times l}{(h-l)\times c}\times C=L.$$

Example.—A mixture of gold and silver weighed 170 lbs. and its specific gravity was 15630; hence,

$$h \text{ (by the table)}=19326. \quad l=10474$$
$$c=15630 \quad C=170 \text{ lbs.}; \text{ wherefore, by the rule,}$$
$$\frac{(19326-15630)\times 10744}{(19326-10744)\times 15630}\times 170=\frac{39709824}{134136660}\times 170$$
$$=\cdot 296\times 170=50\cdot 32 \text{ lbs. of gold};$$

consequently, there will be $170-50\cdot 32=119\cdot 68$ lbs. of silver.

TABLE OF SPECIFIC GRAVITIES.

Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.	Weight of a cub. foot in	W't. of a cub. in. in	W't. of cub. in.
METALS.		LBS.	OZ.	LBS.
Antimony,	6·712	419·50	3·883	·244
Arsenic,	5·763	360·19	3·335	·208
Bismuth,	9·823	614·00	5·684	·355
Brass, common,	7·820	489·	4·525	·282
Bronze, gun metal,	8·700	543·75	5·034	·315
Copper, cast,	8·788	549·25	5·085	·317
wire drawn,	8·878	543·90	5·137	·320
Gold, pure, cast,	19·258	1203·60	11·145	·697
hammered,	19·361	1210·10	11·204	·700
22 carats fine,	17·486	1093·	10·118	·633
20 carats fine,	15·709	982·	9·091	·568
Iron, cast,	7·207	450·44	4·170	·260
bars,	7·788	486·81	4·507	·281
Lead, cast,	11·352	709·50	6·568	·410
Mercury, 32°,	13·598	849·87	7·869	·492
60°,	13·580	848·75	7·858	·491
Platinum, rolled,	22·069	1379·31	12·771	·798
hammered,	20·337	1271·1	11·769	·736
Silver, pure, cast,	10·474	654·62	6·061	·379
hammered,	10·511	657·0	6·082	·381
Steel, soft,	7·833	489·56	4·533	·283
temp'd and hardened,	7·818	488·62	4·524	·283
Tin, Cornish,	7·291	455·62	4·219	·263
Zink, cast,	6·861	428·81	4·970	·248
STONES AND EARTHS.				
Alabaster, white,	2·730	170·62	1·580	·099
yellow,	2·699	167·44	1·562	·098
Amber,	1·078	67·37	·623	·039
Asbestos, starry,	3·073	192·06	1·778	·111
Borax,	1·714	106·50	·992	·062
Brick,	1·900	118·75	1·099	·069
Chalk,	2·784	174·	1·611	·100
Charcoal,	·441	27·56	·025	·016
triturated,	1·380	86·25	·798	·050

Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.	Weight of a cub. foot in	W't. of a cub. in. in	W't. of cub. in.
STONES AND EARTHS.		LBS.	OZ.	LBS.
Clay,	1·930	120·62	1·116	·070
common soil,	1·984	124·	1·148	·071
Coral, red,	2·700	169·	1·562	·098
Coal, bituminous,	1·270	79·37	·735	·046
Newcastle,	1·270	79·37	·735	·046
Scotch,	1·300	81·25	·752	·047
Maryland,	1·355	84·69	·784	·049
Anthracite,	1·436	89·75	·831	·052
	1·640	102·50	·949	·059
Diamond,	3·521	220·06	2·037	·127
Earth, loose,	1·500	93·75	·868	·054
Emery,	4·000	250·	2·314	·144
Flint, black,	2·582	161·37	1·494	·094
white,	2·594	162·12	1·501	·094
Glass, flint,	2·933	183·31	1·694	·099
white,	2·892	180·75	1·673	·098
bottle,	2·732	170 75	1·581	·099
green,	2·642	165·12	1·529	·096
Granite, Scotch,	2·625	164·06	1·519	·095
Susquehanna,	2·704	169·	1·565	·098
Quincy,	2·652	165·75	1·534	·097
Patapsco,	2·640	165·	1·527	·096
Lockport,	2·655	166·	1·566	·098
Grindstone,	2·143	133·93	1·240	·077
Gypsum, opaque,	2·168	135·50	1·254	·077
Hone, white, razor,	2·876	179·75	1·664	·104
Ivory,	1·822	113·87	1·054	·066
Limestone, green,	3·180	198·75	1·840	·115
white,	3·156	197·25	1·826	·114
Lime, compact,	2·720	170·0	1·574	·098
foliated,	2·837	177·4	1.642	·102
Lime, quick,	·804	50·43	·465	·029
Manganese,	7·000	437·50	4·051	·252
Marble, African,	2·708	169·25	1·567	·098
Egyptian,	2·668	166·75	1·544	·097
Parian,	2·838	177·37	1·642	·103
common,	2·686	166·75	1·554	·097
French,	2·649	165·56	1·533	·096
white Italian,	2·708	169·25	1·567	·098
Rutland, Vt.,*	2·708	169·30	1·567	·098
Castleton,	2·673	167·	1·546	·097
Stockbridge,	2·669	166·86	1·545	·097
Mica,	2·800	175·	1·620	·100
Millstone,	2·484	155·25	1·431	·090
Nitre,	1·900	118·75	1·1	·069
Porcelain, China,	2·385	149·	1·322	·086
Pearl, Oriental,	2·650	165·62	1·533	·097

* From Samuel Porter's quarry, West Rutland, Vt. The grain of this marble is very fine and hard, and susceptible of a very high polish. Its specific gravity is just equal to the best white Italian marble, and a little superior to any in America.

Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.	Weight of a cub. foot in	W't of a cub. in. in	W't of cub. in.
STONES AND EARTHS.		LBS.	OZ.	LBS.
Phosphorus,	1·770	110·62	1·024	·065
Pumice Stone,	·915	57·18	·530	·033
Paving Stone,	2·416	151·	1·398	·088
Porphyry, red,	2·765	172·81	1·600	·099
Rotten Stone,	1·981	123·81	1·146	·071
Salt, common,	2·130	133·12	1·232	·077
Saltpetre,	2·090	130·62	1·151	·076
Sand,	1·800	112·50	1·041	·065
Shale,	2·600	162·50	1·504	·095
Slate,	2·672	167·	1·546	·097
Stone, Bristol,	2·510	156·87	1·452	·091
" common,	2·520	157·50	1·458	·091
Sulphur, native,	2·033	127·06	1·176	·075
Talc, black,	2·900	181·25	1·678	·105
WOODS, (dry.)				
Apple,	·793	49·56	·459	·029
Alder,	·800	50·	·463	·029
Ash,	·845	52·81	·489	·031
Beech,	·852	53·25	·493	·031
Birch,	·720	45·	·416	·026
Box, Dutch,	·912	50·75	·528	·033
" French,	1·328	83·	·768	·048
" Brazilian,	1·031	64·44	·60	·037
Campeachy,	·913	57·02	·528	·033
Cherry,	·715	44·68	·413	·026
Cocoa,	1·040	65·	·601	·037
Cork,	·240	15·	·139	·009
Cypress,	·644	40·	·315	·023
Ebony, American,	1·331	83·18	·770	·048
Elder,	·695	43·44	·402	·025
Elm,	·671	42·	·388	·024
Fir, yellow,	·657	41·	·380	.023
" white,	·569	35·56	·329	·021
Hacmetac,	·592	37·	·342	·021
Lignum vitæ,	1·333	83·31	·771	·048
Live Oak,	1·120	70·	·648	·040
Larch,	·544	34·	·315	·020
Logwood,	·913	57·02	·528	·033
Mahogany,	1·063	66·44	·615	·038
Maple,	·750	46·87	·434	·027
Mulberry,	·897	56·01	·519	·032
Oak, English,	·932	58·25	·539	·033
" heart, 60 years,	1·170	73·12	·677	·043
Orange,	·705	44·06	·408	·025
Pine, yellow,	·660	41·25	·382	·024
" white,	·554	34·62	·321	·020
Poplar,	·383	24·25	·221	·014
" white,	·529	33·06	·306	·019

Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.	Weight of a cub. foot in	W't of a cub. in. in	W't of cub. in.
WOODS, (dry.)		LBS.	OZ.	LBS.
Pear tree, - - - - - - - - - -	·661	41·31	·382	·024
Plum, - - - - - - - - - - - -	·785	49·06	·454	·029
Quince, - - - - - - - - - - -	·705	44·	·407	·025
Sassafras, - - - - - - - - - -	·482	30·12	·279	·017
Walnut, - - - - - - - - - - -	·671	42·	·388	·024
Willow, - - - - - - - - - - -	·585	36·56	·338	·021
Yew, Dutch, - - - - - - - - -	·788	49·25	·455	·028
" Spanish, - - - - - - - -	·807	50·44	·467	·029
* *Well-seasoned American*, 1839.				
Ash, - - - - - - - - - - - - -	·722	45·12	·418	·026
Beech, - - - - - - - - - - - -	·624	39·	·361	·023
Cherry, - - - - - - - - - - - -	·606	38·	·351	·022
Cypress, - - - - - - - - - - -	·441	27·56	·255	·016
Hickory, red, - - - - - - - - -	·838	52·37	·479	·030
Mahogany, St. Domingo, - - -	·720	45·	·416	·026
White Oak, (upland,) - - - - -	·687	43·	·397	·025
" (James River,) - -	·759	47·44	·439	·027
Pine, yellow, - - - - - - - - -	·541	34·	·261	·020
" white, - - - - - - - - -	·473	30·	·273	·017
Poplar, (tulip tree,) - - - - - -	·587	36·70	·399	·021

MISCELLANEOUS.	Specific Gravity.	W't of cub. in.	MISCELLANEOUS.	Specific Gravity.	W't of cub. in.
		LBS.			LBS.
Asphaltum, - -	·905	.033	Gunpowd'r, shak'n	1·000	·036
	1·650	·058	" solid,	1·550	·056
Beeswax, - -	·965	·035		1·800	·065
Butter, - - -	·942	·034	Gum Arabic, -	1·452	·051
Camphor, - -	·988	·362	Indigo, - - -	1·009	·037
India Rubber, -	·933	·033	Lard, - - - -	·947	·034
Fat of Beef, -	·923	·033	Mastic, - - -	1·074	·038
" Hogs, -	·936	·034	Spermaceti, - -	·943	·034
" Mutton,	·923	·033	Sugar, - - -	1·606	·058
Gamboge, - -	1·222	·044	Tallow, - - -	·941	·034
Gunpowder, loose,	·900	·032	Atmospheric Air,	0·012	·007

Comparative Weight of Timber in a Green and Seasoned State.

TIMBER.	Weight of a cubic foot. Green.	Weight of a cubic foot. Seasoned.
	LBS. OZ.	LBS. OZ.
English Oak, - - - - - - - - - - - - - - - -	71·10	43· 8
Cedar, - - - - - - - - - - - - - - - - - -	32·	28· 4
Riga Fir, - - - - - - - - - - - - - - - - -	48·12	35· 8
American Fir, - - - - - - - - - - - - - - -	44·12	30·11
Elm, - - - - - - - - - - - - - - - - - - -	66· 8	37· 5
Beech, - - - - - - - - - - - - - - - - - -	58·	50·
Ash, - - - - - - - - - - - - - - - - - - -	58·	50·

* Ordnance Manual, 1841.

LIQUIDS. Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.	ELASTIC FLUIDS. Divide the Specific Gravity by 16, and the quotient is the weight of a cubic foot in lbs.	Specific Gravity.
Acid, acetic,	1·062	1 cubic ft. of atmospheric air	
nitric,	1·217	weighs 527·04 troy grains.	
sulphuric,	1·841	Its assumed gravity of 1 is	
muriatic,	1·200	the unit of elastic fluids.	1·000
Alcohol, pure,	·792	Ammoniacal gas,	·597
of commerce,	·835	Azote,	·976
Oil, essential, turpentine,	·870	Carbonic acid,	1·524
olive,	·915	Carburetted hydrogen,	·555
whale,	·923	Chlorine,	2·470
linseed,	·932	Chloro-carbonic,	3·389
Proof spirit,	·925	Hydrogen,	·070
Vinegar,	1·080	Oxygen,	1·104
Water, distilled,	1·000	Sulphuretted hydrogen,	1·777
Ether, sulphuric,	·715	*Steam, 212°	·490
Honey,	1·450	Nitrogen,	·972
Human blood,	1·054	Vapor of alcohol,	1·613
Milk,	1·032	turpentine spirits,	5·013
Water, sea,	1·026	water,	·623
Dead sea,	1·240	Smoke of bituminous coal,	·102
Wine,	·992	wood,	·90
port,	·997		
champagne,	·997		

A gallon of oil weighs	7·45	lbs. avoirdupois.
" proof spirits,	7·40	"
" vinegar	8·64	"
" wine	7·93	"
" water	8·00	"

APPLICATION OF THE FOREGOING.

The first, second, and third columns require no farther explanation than the titles they bear. The second column, under the head *Miscellany*, is a multiplier, to find the weight of any number of cubic inches in avoirdupois pounds of any of the different bodies required.

When the Weight of a Body is required.

Rule.—Find the contents of the body in cubic feet or inches, and multiply it by the weight of a cubic foot or inch in the table; the product will be the answer in avoirdupois weight.

Examples.—1. What is the weight of a piece of cast iron $56\frac{3}{4}$ inches long, $16\frac{1}{2}$ inches broad, and $\frac{3}{4}$ of an inch in thickness?

$$56·75 \times 16·50 \times ·75 = 702·28 \text{ cubic inches.}$$

$$702·28 \times 4·191 \div 16 = 183·94 \text{ lbs. Ans.}$$

2. What is the weight of a cube of Italian marble, the side being 3 feet?

$$3^3 \times 169·25 = 4569·75 \text{ lbs. Ans.}$$

* Weight of a cubic foot, 258·3 grains.

Given the Diameter of a Balloon to find what weight it will raise.

Rule.—As 1 cubic foot of the balloon is to the specific difference between atmospheric air and the gas used to inflate the balloon, so is the whole capacity of the balloon to the weight it will raise.

Example.—The diameter of a balloon is 26·6 feet, and the gas used to inflate it is hydrogen: what weight will it raise?

sp. gr. of air. grains. sp. gr. of hydr.
1·000 : 527·04 : : 070 : 36·89 weight of 1 cubic foot of hydrogen.

Then $1\cdot : 527\cdot04 - 36\cdot89 :: 26\cdot6^3 \times \cdot5236 : 4830293$ grains ÷ 7000 (7000 in a pound) = 690·04 lbs. Ans.

ON THE STRENGTH OF MATERIALS OF CONSTRUCTION.

Materials are exposed to four different kinds of strain:

1. They may be torn asunder, as in the case of ropes and stretchers. The strength of a body to resist this kind of strain is called its resistance to tension, or absolute strength.

2. They may be crushed or compressed in the direction of their length, as in the case of columns, truss beams, &c.

3. They may be broken across, as in the case of joists, rafters, &c. The strength of a body to resist this kind of strain is called its lateral strength.

4. They may be twisted or wrenched, as in the case of axles, screws, &c.

Extensive and accurate experiments are necessary to determine the several measures of these strengths in the different materials; and when this is done, the subsequent calculations become comparatively easy We shall, therefore, in the first place, lay down the results of the experiments of practical men.

DIRECT COHESION.

The power of cohesion is that resistance which bodies exhibit when force or weight is applied to tear asunder, in the direction of their length, the fibres or particles of which they are composed. The strength to resist force or weight that produce fracture, is as the number of particles that are in the area of the cross section acted upon. Hence, the following

Rule.—Multiply the area of the section in inches by the power in lbs. (as in the following table,) opposite the name of the material, and the product will equal the weight in lbs. the rod, bar, or piece, will just support; but the greatest constant load should never exceed ¼.

TABLE OF THE COHESIVE FORCE OF METALS, &c.

Weight or Force necessary to tear asunder 1 Square Inch, in Avoirdupois Pounds.

METALS.

	LBS.		LBS.
Copper, cast - - - -	22·500	Iron, medium bar - -	60·000
wire - - - - -	61·200	" inferior bar - -	30·000
Gold, cast - - - - -	20·450	Lead, cast - - - - -	·880
wire - - - - -	30·888	" milled - - - - -	3·320
Iron, cast; grey, 2 fusion	30·680	Platinum, wire - - -	53·000
" English - - -	52·000	Silver, cast - - - - -	41·000
" French - - -	70·000	" wire - - - - -	38·257
" " soft - -	63·600	Steel, soft - - - - -	120·000
" German - - -	68·300	" fine - - - - -	135·000
Iron, wrought - - -	60·000	" razor, tempered	150·000
" Swedish - - -	72·000	Tin, cast block - - -	5·000
" English - - -	55·872	Zink, cast - - - - -	2·600
" German - - -	69·000	" sheet - - - - -	16·000
Iron, wire - - - -	85·700 / 113·		

Strength of Alloys when pulled in the direction of their Length.

PARTS.	PARTS.	LBS.	PARTS.	PARTS.	LBS.
Gold 5,	Copper 1	50·000	Silver 5,	Copper 1	48·000
Brass, - - - - - -		45·000	" 4,	Tin 1 -	41·000
Copper 10,	Tin 1	32·000	Tin 10,	Antimony 1 -	11·000
" 8,	" 1	36·000	" 10,	Zink 1 - - -	12·914
" 4,	" 1	35·000	" 10,	Lead 1 - - -	6·800
Bronze (gun metal), -		30·000			

WOODS.

	LBS.		LBS.
Ash, white, seasoned -	14·000	Mahogany - - - - - -	21·800
" red, seasoned - -	17·800	" Spanish - -	12·000
Birch - - - - - - -	15·000	Maple - - - - - - -	10·500
Bay - - - - - - -	14·500	Oak, American, white -	11·500
Beech - - - - - - -	11·500	" English - - - -	10·000
Box - - - - - - -	20·000	" seasoned - - -	13·600
Cedar - - - - - - -	11·400	Pine, pitch - - - -	12·000
Chesnut, sweet - - -	10·500	" Norway - - -	13·000
Cypress - - - - - -	6·000	Poplar - - - - - - -	7·000
Deal, Christiana - - -	12·400	Quince - - - - - - -	6·000
Elm - - - - - - - -	13·400	Sycamore - - - - - -	13·000
Fir, strongest . - - -	12·000	Teak, Java - - - - -	14·000
" American - - -	8·800	Walnut - - - - - -	17·800
Lance wood - - - -	24·000	Willow - - - - - -	13·000
Lignum vitæ - - - -	11·800	Yew - - - - -	8·000
Locust - - - - - - -	20.500		

MISCELLANEOUS SUBSTANCES.

Brick	·290	Mortar, 20 years	·52
Glass plate	9·400	Slate	12·000
Hemp fibres glued tog'er	92·000	Stone, fine grain	·200
Ivory	16·000	Whalebone	7·600
Marble	9·000		

ON THE RESISTANCE TO CRUSHING WOOD.

According to the experiments of Rondelet, made on a hydrostatic press, on cubes of an inch in length, it required from 5000 to 6000 pounds per square inch to crush *oak*, and under this pressure its length was diminished more than $\frac{1}{3}$. To crush *fir*, it required from 6000 to 7000 pounds per square inch, and the length was reduced $\frac{1}{2}$. Mr. Rennes' trials, which are considered the most precise on this subject, afforded results considerably lower than those of Rondelet. The following are the results of his experiments:

Base 1 inch square;	height 1 inch,	Elm was crushed by	1284	lbs.
" " "	" "	White deal " "	1928	"
" " "	" "	Americ'n pine " "	1606	"
" " "	" "	English oak " "	3860	"
" " "	" "	African teak " "	8480	"
	Length 4 inches	" " " "	5147	"
Base 3 inches square;	length 6 to 9,	African oak " "	60480	"

The load a piece of timber will bear, when pressed in the direction of its length, without risk of being crushed, may be found by the following rule, when the pressure is exactly in the axis of the piece:

Rule.—Multiply the area of the piece in inches by the weight that has been found capable of crushing a square inch of the same kind of wood. (*See the preceding experiments.*) Then *one-fourth* of the product will give the greatest load in pounds that the piece would bear with safety.

Example.—Required the weight that a piece of oak, 3 inches by 2 inches, would support with safety.

$3\times2=6=$area of one end, and by the table above oak was crushed by 3860. Consequently, $\frac{6\times3860}{4}=5790$ lbs. Ans.

If the *area* that would support a given weight be *required*, divide 4 times the weight by the number of pounds that would crush a square inch, and the quotient is the area in inches.

If the *force* that would crush a square inch be represented by f, the depth of the piece by d, and the breadth by b, we shall have the following formula:

$$\frac{d\times b\times f}{4}=\text{weight.}$$

STONE, &c.

The Force necessary to Crush one Cubic Inch.

	LBS.		LBS.
Aberdeen granite, blue	24·556	Dundee sandstone - -	14·919
Very hard freestone -	21·254	Yorkshire paving stone	15·856
Black Limerick limestone	19·924	Red brick - - - - - -	1·817
Compact limestone - -	17·354	Pale red brick - - -	1·265
Craigleith stone - -	15·568	Chalk - - - - - - - -	1·127

Cubes of One-fourth of an Inch.

	LBS.		LBS.
Iron, cast, vertically -	11·140	Cast tin - - - - - - -	·966
horizontally -	10·110	Cast lead - - - - - - -	·483
Cast copper - - - -	7·318		

Transverse Strength, or Resistance to Lateral Pressure.

The strength of bodies to resist fracture in this direction is as the breadth and square of the depth, and inversely as the length.

Divisors for beams of uniform breadth and thickness :—

Wrought iron - -	952	Yellow Pine - - -	255
Cast do. - -	850	Oak - - - - - -	212

1. *The length of a Beam being given, of uniform breadth and thickness, and also the weight it is to support in the middle of its length, to determine the breadth and depth required.*

Rule.—Multiply the length between the bearings in feet by the weight to be supported in pounds, and by any proportionate breadth to the depth; divide the product by the proper divisor, as above, and the cube root of the quotient equal the depth in inches; and the depth divided by the proportional breadth equal the breadth in inches.

Example.—Suppose a uniform beam of cast iron, 18 feet in length, be required to carry a weight of 20,000 pounds on the middle, between the supports, what must be the breadth and depth, in inches, when the breadth is $\frac{1}{5}$ of the depth?

$\frac{18\times20000\times5}{850}=\sqrt[3]{2117·6}=12·8$ inches in depth, and $\frac{12·8}{5}=2·56$ inches in breadth or thickness.

2. *Given the length and breadth of a uniform Beam, and the weight it is to support in the middle, to find the required depth; or, the depth given, to find the breadth required.*

Rule.—Multiply the distance between the bearings in feet by the weight to be supported in pounds, and divide the product by the tabular number, as above, multiplied by the square of the depth in inches, the quotient equal the breadth in inches; or, divide the product by the tabular number multiplied by the breadth in inches, and the square root of the quotient equal the depth.

Example.—Let it be required to find the breadth of a uniform beam

of oak to sustain a weight of 6000 pounds in the middle of its length, the distance between the supports being 20 feet, and the depth of the beam 9 inches.

$\frac{6000\times 20}{212\times 9^2}=7$ inches, the breadth; and $\sqrt[2]{\frac{6000\times 20}{7}}=3$ inches, the depth.

NOTE 1. When the load is not on the middle of the beam, but placed nearer to one end, divide four times the product of the distance of the weight in feet from each bearing by the whole distance in feet, and the quotient equal the length of the beam to be taken into account.

Example.—Suppose a beam 30 feet in length, with a load placed 9 feet from one end, required the length to be taken into calculation as effected by the load.

$30-9=21$, and $\frac{21\times 9\times 4}{30}=25{\cdot}2$ feet effective length.

NOTE 2. When the load is distributed over the whole length of a beam, it will bear double the assumed load as above; hence, in such cases, the divisors must be doubled.

NOTE 3. When a beam is to be fixed at one end, and the weight placed on the other, take only one-fourth of the tabular number for a divisor: but if the weight is to be laid uniformly along its whole length, use one-half.

Example to Rule 1.—Let the length of the arm to the center of motion of a scale beam or balance equal $1\frac{1}{2}$ feet, and the extreme weight to be weighed 336 pounds, the thickness to be $\frac{1}{10}$th of the depth; required the depth and thickness in inches of wrought iron.

$$\frac{952}{4}=238 \text{ divisor.}$$

Then $\frac{1{\cdot}5\times 336\times 10}{238}=\sqrt[3]{21{\cdot}2}=2{\cdot}77$ inches in depth, and $\frac{2{\cdot}77}{10}={\cdot}277$ inches in thickness.

Example to Rule 2.—Required the depth for the cantilevers of a balcony of cast iron, to project 4 feet, and to be placed 5 feet apart, the weight of the stone part being 1000 pounds, the breadth of each cantilever 2 inches, and the greatest possible load that can be collected upon 5 feet in length of the balcony 2200 pounds.

$1000+2200=3200$ lbs.; and $850\div 2=425$, the divisor.

hence, $\sqrt[2]{\frac{3200\times 4}{425\times 2}}=3{\cdot}8$ inches, the depth required.

DEFLECTION OF RECTANGULAR BEAMS.

To ascertain the amount of Deflection of a uniform Beam of cast iron, supported at both ends, and loaded in the middle to the extent of its elastic force.

Rule.—Multiply the square of the length in feet by ·02, and the product divided by the depth in inches, equal the deflection.

Example.—Required the deflection of a cast iron beam, 18 feet

long between the supports, 12·8 inches deep, 2·56 inches in breadth, and bearing a weight of 20·000 lbs. in the middle of its length.

$$\frac{18^2 \times \cdot 02}{12 \cdot 8} = \cdot 506 \text{ inches from a straight line in the middle.}$$

NOTE. For beams of a similar description, loaded uniformly, the rule is the same only multiply by ·025 in place of ·02.

To find the Deflection of a Beam when fixed at one end and loaded at the other.

Rule.—Divide the length in feet of the fixed part of the beam by the length in feet of the part which yields to the force, and add 1 to the quotient; then multiply the square of the length in feet by the quotient so increased, and also by ·13; divide this product by the middle depth in inches, and the quotient will be the deflection, in inches also.

Multiply the deflection so obtained for cast iron by ·86, the product equal the deflection for wrought iron; for oak, multiply by 2·8; and for fir, 2·4.

TABLE

Of the Strength, Extensibility, and Stiffness of Woods and Metals, Cast Iron being 1, or Unity.

Metals.	Strength.	Exten.	Stiffn.	Woods.	Strength.	Exten.	Stiffn.
Iron, wrought	1·12	0·86	1·3	Oak	0·25	2·8	0·093
Gun metal	0·65	1·25	0·535	Ash	0·23	2·6	0·089
Brass	0·435	0·9	0·49	Elm	0·21	2·9	0·073
Zink	0·365	0·5	0·76	Pine, yellow	0·3	2·6	0·1154
Tin	0·182	0·75	0·25	Beech	0·15	2·1	0·073
Lead	0·096	2·5	0·0385	Mahogany, Honduras	0·24	2·9	0 487

RULES AND FORMULAS

For the Deflection of Rectangular Beams, according to MR. BARLOW.

1. The deflection of the same beam, resting on props at each end, and loaded in the middle with weights, are as those weights.

2. The deflection is inversely as the cube of the depth; also, the depth being the same, the deflection is inversely as the breadth.

3. The deflection is directly as the cube of the length.

When the Beam is Fixed at one end, and Loaded at the other end.

$$\frac{l^3 W}{b\, d^3\, e} = C, \text{ a constant quantity.}$$

When l=the length of the beam.
b=the breadth.
d=the depth.
W=the weight with which it is loaded.
e=the deflection.

When the Beam is uniformly Loaded we shall have the following formula.

$$\frac{3\, l^3 W}{8\, b\, d^3\, e} = C.$$

When supported at both ends and loaded in the middle.

$$\frac{l^3 W}{3^2\, b\, d^3\, e} = C.$$

When uniformly loaded.

$$\frac{5}{8} \times \frac{l^3 W}{32\, b\, d^3} = C.$$

TABLE

Of the Depths of Square Beams or Bars of Cast Iron, calculated to support from 1 Cwt. to 14 Tons in the middle, the Deflexion not to exceed $\frac{1}{40}$th of an Inch for each Foot in Length.

Lengths in Feet.		4	6	8	10	12	14	16	18	20	22	24	26	28
Weight in cwt.	Weight in lbs.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.	Depth.
		in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
1	,112	1·2	1·4	1·7	1·9	2·0	2·2	2·4	2·5	2·6	2·7	2·9	3·0	3·1
2	,224	1·4	1·7	2·0	2·2	2·4	2·6	2·8	3·0	3·1	3·3	3·4	3·6	3·7
3	,336	1·6	1·9	2·2	2·4	2·7	2·9	3·1	3·3	3·4	3·6	3·8	3·9	4·1
4	,448	1·7	2·0	2·4	2·6	2·9	3·1	3·3	3·5	3·7	3·9	4·0	4·2	4·3
5	,560	1·8	2·2	2·5	2·8	3·0	3·3	3·5	3·7	3·9	4·1	4·3	4·4	4·6
6	,672	1·8	2·2	2·6	2·9	3·2	3·4	3·7	3·9	4·1	4·3	4·5	4·6	4·8
7	,784	1·9	2·3	2·7	3·0	3·3	3·6	3·8	4 1	4·2	4·4	4·6	4·8	5·0
8	,896	2·0	2 4	2·8	3·1	3·4	3·7	3·9	4·2	4·4	4·6	4·8	5·0	5·2
9	1,008	2·0	2·5	2·9	3·2	3·5	3·8	4·0	4·3	4·5	4·7	4·9	5·1	5·3
10	1,120	2·1	2·6	3·0	3·3	3·6	3·9	4·2	4·4	4·7	4·9	5·2	5·3	5·4
11	1,232	2·1	2·6	3·0	3·4	3·7	4·0	4·3	4·5	4·8	5·0	5·3	5·4	5·6
12	1,344	2·2	2·7	3·1	3·5	3·8	4·1	4·4	4·7	4·9	5·1	5·3	5·5	5·7
13	1,456	2·2	2·7	3·1	3·5	3·8	4·2	4·4	4·7	4·9	5·2	5·4	5·6	5·9
14	1,568	2·3	2·8	3·2	3·6	3·9	4·2	4·5	4·8	5·0	5·3	5·5	5·7	6·0
15	1,680	2·3	2·8	3·2	3·6	4·0	4·3	4·6	4·9	5·2	5·4	5·6	5·8	6·1
16	1,792	2·4	2·9	3·3	3·7	4·0	4·4	4·7	5·0	5·2	5·5	5·7	5·9	6·2
17	1,904	2·4	2·9	3·4	3·8	4·1	4·4	4·7	5·0	5·3	5·5	5·8	6·0	6·2
18	2,016	2·4	3·0	3·4	3·8	4·2	4·5	4·8	5·2	5·4	5·6	5·9	6·1	6·4
19	2,128	2·5	3·0	3·5	3·9	4·2	4·6	4·9	5·2	5·4	5·7	6·0	6·2	6·5
1ton.	2,240	2·5	3·0	3·5	3·9	4·3	4·6	4·9	5·1	5·5	5·8	6·0	6·3	6·5
1¼	2,800	2·6	3·2	3·7	4·1	4·5	4·9	5·2	5·5	5·8	6·1	6·4	6·6	6·9
1½	3,360	2·8	3·4	3·9	4·3	4·7	5·1	5·5	5·8	6·1	6·4	6·7	7·0	7·2
1¾	3,920	2·9	3·5	4·0	4·5	4·9	5·3	5·7	6·0	6·3	6·7	6·9	7·2	7·5
2	4,480	2·9	3·5	4·1	4·7	5·1	5·5	5·9	6·2	6·5	6·8	7·2	7·6	7·7
2½	5,600	3·1	3·8	4·4	4·9	5·5	5·8	6·2	6·6	6·9	7·3	7·6	7·9	8·2
3	6,720	3·3	4·0	4·6	5·1	5·7	6·1	6·5	6·9	7·3	7·6	7·9	8·3	8·6
3½	7,840	3·4	4·1	4·8	5·3	5·8	6·3	6·7	7·1	7·5	7·9	8·2	8·6	8·9
4	8,960	3·5	4·3	4·9	5·5	6·0	6·5	7·0	7·4	7·8	8·2	8·5	8·9	9·2
4½	10,080		4·4	5·1	5·7	6·2	6 7	7·2	7·6	8·0	8·4	8·8	9·1	9·5
5	11,200		4·5	5·2	5·8	6·4	6·9	7·4	7·8	8·2	8·6	9·0	9·4	9·7
6	13,440			5·5	6·1	6·7	7·2	7·7	8·2	8·6	9·0	9·4	9·8	10·2
7	15,680			5·7	6·3	6·9	7·5	8·0	8·5	8·9	9·4	9·8	10·2	10·6
8	17,920			5·9	6·6	7·2	7·8	8·3	8·8	9·3	9·7	10·1	10·6	10·9
9	20,160			6·0	6·8	7·4	8·0	8·5	9·0	9·5	10·0	10·4	10·9	11·3
10	22,400				6·9	7·6	8·2	8·8	9·3	9·8	10·3	10·7	11·2	11·6
11	24,640				7·1	7·8	8·4	9·0	9·5	10·0	10·5	11·0	11·5	11·9
12	26,880				7·2	7·9	8·6	9·2	9·7	10·2	10·8	11·2	11·7	12·1
13	29,120				7·4	8·1	8·8	9·4	9·9	10·4	11·0	11·5	11·9	12·4
14	31,360				7·5	8·3	8·9	9·5	10·1	10·6	11·1	11·7	12·1	12·6
Deflex in inches.		·1	·15	.2	·25	·3	·35	·4	·45	·5	·55	·6	·65	·7

EXPERIMENTS

On rectangular Bars of Malleable Iron by Mr. Barlow, for the purpose of determining the Point of Neutral Axis, the Center of Compression, and the greatest Deflexion to which Railroad Bars, or Lines of Rail, might be submitted without causing permanent Injury to the Properties of the Iron.

Weight in Tons.	Deflexion in Inches.	Deflexion per Half Ton.	Remarks.	Weight in Tons.	Deflexion in Inches.	Deflexion per Half Ton.	Remarks.
·125	·043			·50	·003		
·500	·059			1·00	·050	·020	The depth of this bar only $2\frac{1}{2}$ inches. Mean, ·0173.
1·00	·074	·015	Mean, ·0103. $w = 4\frac{1}{4}$. Neutral axis, 1 : 4·9.	1·50	·060	·010	
1·50	·083	·009		2·00	·074	·014	
2·00	·095	·012		2·50	·093	·019	
2·50	·101	·006		3·00	·110	·017	
3·00	·109	·008	Elasticity preserved at $4\frac{1}{4}$ tons.	3·50	·149		w=3. Neutral axis, 1 : 4·9. Elasticity preserved, 3 tons.
3·50	·120	·011					
4·00	·131	·011					
4·50	·148	·017					
·50	·017						
1·00	·037			7·50			Bent 8 inches.
1·50	·052	·015	Mean, .0108. $w = 4\frac{1}{4}$. Neutral axis, 1 : 4·9. Elasticity injured.				
2·00	·061	·009					
2·50	·064	·003					
3·00	·078	·014					
3·50	·089	·011					
4·00	·102	·013					
4·50	·124	·022					

NOTE.—Distance between the bearings, 33 inches; breadth of bar, $1\frac{1}{2}$ inches; depth, 3 inches.

The neutral axis was found to be $\frac{1}{5}$th of the depth from the top of the bar; the center of compression, $\frac{2}{3}$ds of that fifth above the neutral axis; and the rule for obtaining the utmost degree of deflexion as follows:

Divide ·22 by $\frac{4}{5}$ths the depth of the bar in inches, and the quotient is the utmost deflexion that can be subjected with safety on bearings 33 inches apart.

To find the weight that railroad bars will support.—Observe, that whatever figure may be given to the transverse section, the head, or top portion of the rail, is generally supposed to occupy $\frac{2}{5}$ths of the whole section; or, in the larger description, to have two inches section, and to be one inch deep; that the lower web be the same depth as the head.

Resistance of the Head or Upper Portion of the Rail.

Rule.—Subtract the thickness of the middle rib from 2 inches, and multiply the remainder by 10.—Again, subtract $\frac{1}{2}$ an inch from

the whole depth, and multiply the remainder by 12; then divide the former product by the latter, and the quotient equal the resistance, in tons, due to the head, not including the continuation of the middle rib.

Resistance of the Center Rib.

Rule.—Multiply the whole depth of the rail in inches by the whole depth minus ½ an inch, and that product by 10 times the thickness of the rib; ⅓d of the last product equal the resistance, in tons, of the middle rib continued through the whole depth.

Resistance of the Lower Web.

Rule.—Multiply the whole depth of the rail, minus 1 inch, by the breadth of the bottom web, minus the thickness of the rib, and that product by 10.—Again: from the whole depth of the rail subtract 1 inch, and to 12 times the square of the remainder add 6 times the remainder, and call this the first number. From this subtract twice the remainder, and add 1, and call this the second number. Then say, as the first number is to the second, so is the product obtained in the former part of the rule to the resistance of the lower web, not including the continuation of the middle rib.

Then, the sum of these three resistances multiplied by 4, and divided by the clear bearing length, will be the weight, in tons, that the rail will sustain without injury.

Example 1.—Let the depth of the rail be 5 inches, with a plain rib, whose thickness is ·9 of an inch; required the greatest weight that it ought to be required to bear.

$$\text{Resistance of head} \left\{ \begin{array}{l} (2-{\cdot}9)\times 10=11 \\ (5-\tfrac{1}{2})\times 12=54 \end{array} \right\} \frac{11}{54}=0{\cdot}2.$$

$$\text{Resistance of rib } \frac{4\tfrac{1}{2}\times 5\times {\cdot}9\times 10}{3}=\frac{67{\cdot}5}{67{\cdot}7};\ \text{and, } \frac{4\times 67{\cdot}7}{33}=8{\cdot}21$$

tons, the greatest weight; and the deflexion with this weight $\frac{{\cdot}22}{4{\cdot}5}$ $={\cdot}05$ of an inch nearly.

Example 2.—Suppose a rail with bottom web, the depth of rail being 5 inches, the thickness of the rib ·6 of an inch, breadth of section of lower web 1·32, and weight 50 lbs.; required the greatest load.

$$\text{Resistance of head} \left\{ \begin{array}{l} (2-{\cdot}6)\times 10=14 \\ (5-{\cdot}5)\times 12=54 \end{array} \right\} \frac{14}{54}\ \text{- - - -} = 0{\cdot}26 \text{ Tons.}$$

$$\text{Resistance of rib } \frac{4\tfrac{1}{2}\times 5\times {\cdot}6\times 10}{3}\ \text{- - - - - - - -} = 45{\cdot}00$$

$$\text{Lower web} \left\{ \begin{array}{l} (5-1)\times{\cdot}72\times 10\times 28{\cdot}8 \\ 12(5-1)^2+24=216,\ \text{or 1st number} \\ 216-7=209,\ \text{or 2d number} \end{array} \right\}$$

$$\text{Then } 216 : 209 :: 28{\cdot}8 : 27{\cdot}94\ \text{- - - - - - - -} = 27{\cdot}94$$

$$73{\cdot}20$$

$$\text{A} \left\{ \frac{73{\cdot}2\times 4}{33}=8.75 \text{ tons, the greatest weight.} \right.$$

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TABLE

OF

PRACTICAL FORMULA,

By which to determine the Amount of Weight a Column of given dimensions will support in lbs.

For a rectangular column of cast iron, . . $W = \frac{15300\, l\, b^3}{4\, b^2 + \cdot 18\, l^2}$.

For a rectangular column of malleable iron, $W = \frac{17800\, l\, b^3}{4\, b^2 + \cdot 16\, l^2}$.

For a rectangular column of oak, . . . $W = \frac{3960\, l\, b^3}{4\, b^2 + \cdot 5\, l^2}$.

For a solid cylinder of cast iron, ... $W = \frac{9562\, d^4}{4\, d^2 + \cdot 18\, l^2}$.

For a solid cylinder of malleable iron, . . $W = \frac{11125\, d^4}{4\, d^2 + \cdot 16\, l^2}$.

For a solid cylinder of oak, ... $W = \frac{2470\, d^4}{4\, d^2 + \cdot 5\, l^2}$.

NOTE —W = the weight the column will support, in lbs.
b = the breadth in inches.
l = the length in feet.
d = the diameter in inches.

Example 1.—A rectangular column of oak, 6 inches on the side, and 12 feet in length.—What weight will it support?

$$\frac{3960 \times 12 \times 6^3}{4 \times 6^2 + \cdot 5 \times 12^2} = \frac{10264320}{216} = 47520 \text{ lbs.}$$

Example 2.—What weight will a cast iron cylinder support, whose diameter is 4 inches, and length 10 feet?

$$\frac{9562 \times 5^4}{4 \times 5^2 + \cdot 18 \times 10^2} = \frac{5976250}{118} = 50646 \text{ lbs.}$$

TABLE,

TO SHOW THE WEIGHT OR PRESSURE

A Column of Cast Iron will sustain with safety.

Length or Height in feet.	8	10	12	14	16	18	20	22	24
Diameter.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.	Weight in cwts.
$2\frac{1}{2}$ inches.	91	77	65	55	47	40	34	29	25
3 —	145	128	111	97	84	73	64	56	49
$3\frac{1}{2}$ —	214	191	172	156	135	119	106	94	83
4 —	288	266	242	220	198	178	160	144	130
$4\frac{1}{2}$ —	379	354	327	301	275	251	229	208	189
5 —	479	452	427	394	365	337	310	285	262
6 —	573	550	525	497	469	440	413	386	360
7 —	989	959	924	887	848	808	765	725	686
8 —	1289	1259	1224	1185	1142	1097	1052	1005	959
9 —	1672	1640	1603	1561	1515	1467	1416	1364	1311
10 —	2077	2045	2007	1964	1916	1865	1811	1755	1697
11 —	2520	2490	2450	2410	2358	2305	2248	2189	2127
12 —	3020	2970	2930	2900	2830	2780	2730	2670	2600

TABLE.

Mr. Barlow *furnishes the following as some of the results obtained by him, upon the Deflection of Beams.*

	Length.	Depth.	Breadth.	Pounds.	Deflection.
	feet.	inches.	inches.		
Fir.	6	2	$1\frac{1}{2}$	180	1·
Fir.	3	2	$1\frac{1}{2}$	120	·10
Fir.	6	$1\frac{1}{2}$	2	180	2·

HYDRODYNAMICS.

The science of Hydrodynamics embraces *Hydrostatics* and *Hydraulics*, the former of which treats of the properties and equilibrium of liquids in a state of rest, and the latter of liquids in motion, as conducting water in pipes, raising liquids by pumps, &c.

1. The peculiar distinguishing properties of liquids or fluids in general are, capability of flowing, and constant tendency to press outwards in every direction.

2. Fluids are of two kinds, aëriform and liquid, or elastic and non-elastic; that is, bodies which are easily compressed into a smaller bulk, and bodies which are scarcely susceptible of compression. Atmospheric air, steam, or vapor of water, and all other gaseous bodies, are of the first kind; and water, alcohol, mercury, &c., are of the second.

Compression of Liquids, in Millionth Parts per Atmosphere.

$$\left.\begin{array}{ll} \text{Mercury,} & 2{\cdot}65 \\ \text{Alcohol,} & 21{\cdot}60 \\ \text{Water,} & 46{\cdot}63 \\ \text{Ether,} & 61{\cdot}58 \end{array}\right\} \text{of their original bulk.}$$

3. The weight of water or other fluid is as the quantity, but the pressure exerted is as the vertical height.

4. Fluids press equally in all directions; hence, any vessel containing a fluid sustains a pressure equal to as many times the weight of the column of greatest height of that fluid, as the area of the vessel is to the sectional area of the column.

5. The hydraulic press is of this principle. A jet of water is thrown into a cavity by means of a force pump; the action and non-compressible property of the liquid repels a piston or ram, the force of which equals the product of the effective power or pressure exerted on the fluid in the pump, multiplied by the number of times the area of the base of the ram exceeds the sectional area of the pump.

Example.—Required the repulsive force of a six-inch ram, when power of 50 lbs. is applied to the end of the lever, which is as

12 to 1, and the diameter of the pump, or plunger, $\frac{7}{8}$ths of an inch.

$$\frac{\text{Area of ram}}{\text{Area of pump}} = \frac{28{\cdot}2744}{{\cdot}6013} = 47;$$

and $50 \times 12 \times 47 = 28200$ lbs., or 12 tons, nearly.

6. The lateral pressure of a fluid on the sides of any vessel in which it is contained is equal to the product of the length multiplied by half the square of the depth, and by the weight of the fluid in cubic unity of dimensions.

Example.—A cistern 12 feet square and 8 feet deep is filled with water: required the whole amount of lateral pressure.

(Weight of a cubic foot of water, 62·5 lbs.)

$12 \times 4 = 48$ feet, the whole length of sides,

and $\frac{8^2}{2} = 32$; then $\frac{48 \times 32 \times 62{\cdot}5}{2000} = 48$ tons net.

7. Fluids always tend to a natural level, or curve similar to the earth's convexity, every point of which is equally distant from the center of the earth, the apparent level, or level taken by any instrument for that purpose, being only a tangent to the earth's circumference: hence, in leveling for canals, &c., the difference caused by the earth's curvature must be deducted from apparent level to obtain the true level.

To find the Difference between True and Apparent Level.

When the distance is	Feet,	the square of that distance multiplied by	·000000287	equal the difference in inches when refraction is not taken into account.
	Yards,		·000002583	
	Chains,		·00125	

If the distance is considerable, and refraction must be attended to, diminish the distance in respect to calculation by $\frac{1}{12}$th.

Example.—What is the difference between true and apparent level at a distance of 18 chains, when refraction is taken into account?

$$\frac{18}{12} = 1{\cdot}5, \text{ and } 18 - 1{\cdot}5 = 16{\cdot}5^2 \times {\cdot}00125 = {\cdot}3403 \text{ inches.}$$

8. When a body is partly or wholly immersed in a fluid, the vertical pressure of the fluid tends to raise the body with a force equal to the weight of the fluid displaced: hence, the weight of any displaced quantity of a fluid by a buoyant body equal the weight of that body.

9. The center of pressure, and also the center of percussion in a fluid, is two-thirds the depth from the surface.

10. The resistance by which a moving body is opposed in passing through a liquid is as the square of its velocity: hence, if a body be propelled at a certain velocity by a known power, to double that velocity will require four times the power; to triple it, nine times the power, &c.

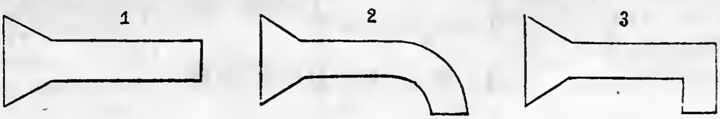
Weight of Water at its Common Temperature.

1	cubic inch	=	·03617 lbs.
12	" inches	=	·434 "
1	" foot	=	62·5 "
1	" "	=	7·81 gallons.
1·6	" feet	=	1 lb.
32	" "	=	1 ton.
1	cylindrical inch	=	·02842 lbs.
12	" inches	=	·341 "
1	" foot	=	49·1
1	" "	=	6·136 gallons.
2·036	" feet	=	1 cwt.
40·73	" "	=	1 ton.
12·5	gallons	=	1 cwt.
250	"	=	1 ton.

OF LIQUIDS IN MOTION.

The flowing of water through pipes, or in natural channels, is liable to be materially affected by friction. Water flows smoothly and with least retardation when the course is perfectly smooth and straight. Every little inequality which is presented to the liquid tends to retard its motion, and so likewise does every bend or angle in its path.

Thus, suppose equal quantities of water to be discharged through pipes of equal diameters and lengths, but of the following forms:



and the time that the quantity discharged through the first is 1; the time that will be required to discharge an equal quantity through the second is 1·11; and the time for the same quantity through the third, 1·55. Hence, the necessity of avoiding as much as possible any bends or angles in pipes or channels for the conduction of fluids.

1. When water issues out of a circular aperture in a thin plate on the bottom or side of a reservoir, the issuing stream tends to converge to a point at the distance of about half its diameter outside the orifice, and this contraction of the stream reduces the area of its section from 1 to ·666, according to Bossut; to ·631, according to Venturi; and to ·64, according to Eytelwein. But, from more accurate experiments, it is found that the quantity discharged is not sufficient to fill this section with the velocity due or corresponding to the height, and that the orifice must be diminished to ·619, or nearly $\frac{5}{8}$ths.

2. When water issues through a short tube, the vein of the stream is less contracted than in the former case, in the proportion of 16 to 13; and if it issues through an aperture which is the frustrum of a cone, whose greater base is the aperture, the height

of the frustrum, half the diameter of the aperture, and the area of the small end to the area of the large end as 10 to 16, there will be no contraction of the vein. Hence, when the greatest possible supply of water is required, this form of orifice ought to be employed.

3. The quantity of water that flows out of a vertical rectangular aperture that reaches as high as the surface is $\frac{2}{3}$ds of the quantity that would flow out of the same aperture placed horizontally at the depth of the base.

To determine the quantity of water discharged by a small vertical or horizontal orifice, the time of discharge and height of the fluid in the vessel being known:

Let A represent the area of the orifice, Q the quantity of water discharged, T the time of discharge, H the height of fluid in the vessel, and $g=16{\cdot}087$ feet per second; then

$$Q=2\times A\times T\sqrt{g\times H}.$$

$$A=\frac{Q}{2\times T\times\sqrt{g\times H}}.$$

$$T=\frac{Q}{2\times A\times\sqrt{g\times H}}.$$

$$H=\frac{Q^2}{4\times g\times T^2\times A^2}.$$

By means of these formulæ, the quantity of water discharged in the same time from any other vessel, in which A is the area of the orifice, and H the altitude of the fluid; for, since T and g are constant, we shall have

$$Q : Q' = A\sqrt{H} : A'\sqrt{H}.$$

TABLE,

Showing the Quantity of Water discharged per Minute by Experiments with Orifices differing in Form and Position.

Constant Weight of the Fluid above the Center of the Orifice. FT.	IN.	LINES.	Form.	Position.	Diameter of the Orifice.	Number of Cubic Inches discharged per Minute.
11	8	10	Circular.	Horizontal.	6 lines.	2·311
			Ditto.	Ditto.	12 "	9·281
			Ditto.	Ditto.	24 "	37·203
			Rectangular.	Ditto.	12 by 3.	2·933
			Square.	Ditto.	12 side.	11·817
			Ditto.	Ditto.	24 "	47·361
9	0	0	Circular.	Vertical.	6 lines.	2·018
			Ditto.	Ditto.	12 "	8·135
4	0	0	Ditto.	Ditto.	6 "	1·353
			Ditto.	Ditto.	12 "	5·436
5	0	7	Ditto.	Ditto.	12 "	·628

Deductions from the preceding Experiments.

1. That the quantities of water discharged in equal times by the same orifice, from the same head of water, are nearly as the areas of the orifices.

2. That the quantities of water discharged in equal times by the same orifices, under different heads, are nearly as the square roots of the corresponding heights of the water in the reservoir, above the surface of the orifices.

3. That the quantities of water discharged during the same time by different apertures, under different heights of water in the reservoir, are to one another in the compound ratio of the areas of the apertures, and of the square roots of the heights in the reservoir.

4. That, on account of the friction, small orifices discharge proportionally less fluid than those which are larger and of similar figure, under the same altitude of fluid in the reservoir.

5. That, in consequence of a slight augmentation which the contraction of the fluid vein undergoes, in proportion as the height of the fluid in the reservoir increases, the expenditure ought to be a little diminished.

6. That circular apertures are most advantageous, as they have less rubbing surface under the same area.

7. That the discharge of a fluid through a cylindrical horizontal tube, the diameter and length of which are equal to one another, is the same as through a simple orifice.

8. That if the cylindrical horizontal tube be of greater length than the extent of the diameter, the discharge of water is much increased, and may be increased with advantage to four times the diameter of the orifice.

TABLE

Of Comparison of the Theoretic with the Real Discharges per Minute through an Orifice One Inch in Diameter; also through a Tube of a Cylindrical Form, One Inch in Diameter and Two Inches long.

Constant altitude of the water in the reservoir above the center of the orifice.	Theoretical discharges through a circular orifice one inch in diameter.	Real discharges in the same time, and through the same orifice.	Real discharges in the same time by a cylindrical tube, one inch in diameter, and two inches long.
Paris Feet.	Cubic Inches.	Cubic Inches.	Cubic Inches.
1	4,381	2,722	3,539
2	6,196	3,846	5,002
3	7,589	4,710	6,126
4	8,763	5,436	7,070
5	9,797	6,075	7,900
6	10,732	6,654	8,654
7	11,592	7,183	9,340
8	12,392	7,672	9,975
9	13,144	8,135	10,579
10	13,855	8,574	11,151
11	14,530	8,990	11,693
12	15,180	9,384	12,205
13	15,797	9,764	12,699
14	16,393	10,130	13,177
15	16,968	10,472	13,620

NOTE.—The Paris foot = 12·7977 English inches.
The inch = 1·0659 " "
The line = ·0074 " "

TABLE

Of the Heights corresponding to different Velocities, in French Metres, per Second.

Velocity in Metres.	Corresponding Height in Metres.	Velocity in Metres.	Corresponding Height in Metres.	Velocity in Metres.	Corresponding Height in Metres.
·1	·00051	3·5	·6244	6·8	2·3571
·2	·00204	3·6	·6606	6·9	2·4269
·3	·00459	3·7	·6978	7·0	2·4978
·4	·00816	3·8	·7361	7·1	2·5696
·5	·0127	3·9	·7753	7·2	2·6425
·6	·0184	4·0	·8156	7·3	2·7164
·7	·0250	4·1	·8569	7·4	2·7914
·8	·0326	4·2	·8992	7·5	2·8673
·9	·0413	4·3	·9425	7·6	2·9443
1·0	·0510	4·4	·9869	7·7	3·0223
1·1	·0617	4·5	1·0322	7·8	3·1013
1·2	·0734	4·6	1·0786	7·9	3·1813
1·3	·0861	4·7	1·1260	8·0	3·2624
1·4	·0999	4·8	1·1744	8·1	3·3445
1·5	·1147	4·9	1·2239	8·2	3·4275
1·6	·1305	5·0	1·2744	8·3	3·5116
1·7	·1473	5·1	1·3258	8·4	3·5968
1·8	·1651	5·2	1·3784	8·5	3·6829
1·9	·1840	5·3	1·4319	8·6	3·7701
2·0	·2039	5·4	1·4864	8·7	3·8583
2·1	·2248	5·5	1·5420	8·8	3·9475
2·2	·2467	5·6	1·5986	8·9	4·0377
2·3	·2696	5·7	1·6562	9·0	4·1290
2·4	·2936	5·8	1·7148	9·1	4·2212
2·5	·3186	5·9	1·7744	9·2	4·3145
2·6	·3446	6·0	1·8351	9·3	4·4088
2·7	·3716	6·1	1·8968	9·4	4·5041
2·8	·3996	6·2	1·9595	9·5	4·6005
2·9	·4287	6·3	2·0232	9·6	4·6978
3·0	·4588	6·4	2·0879	9·7	4·7956
3·1	·4899	6·5	2·1537	9·8	4·8950
3·2	·5220	6·6	2·2205	9·9	4·9954
3·3	·5551	6·7	2 2883	10·0	5·1000
3·4	·5893	·	·	·	·

NOTE.—The metre equals 39·37023 inches, or 3·281 English feet.

Practical Rules, according to the Formula of Torricelli,
BY M. MORIN.

To obtain the velocity due to a given height of water, above the center or middle of an orifice, or the height due to a given velocity,

Rule.—Multiply the height of the water above the center of the

orifice by 19·62, and the square root of the product equal the velocity due to that height. Or: Divide the square of the velocity by 19·62, and the quotient equals the height, each being expressed in French metres.

Practical Formulas, by M. PRONY, *as deduced from the Experiments of Bossut, on Simple Orifices and Additional Tubes.*

Let A = the area of the orifice, in square feet.
T = the time, in seconds.
H = the altitude above the middle of the orifice, in feet.
Q = the quantity of water discharged, in cubic feet.

$$1.\ A=\frac{Q}{4{\cdot}818\,T\sqrt{H}}.\quad 2.\ T=\frac{Q}{4{\cdot}818\,A\sqrt{H}}.\quad 3.\ H=\frac{Q}{(4{\cdot}818\,A\,T)^2}.$$

$$1.\ d=\frac{Q}{\sqrt{4{\cdot}9438\,T\sqrt{H}}}.\quad 2.\ T=\frac{Q}{4{\cdot}9438\,T\sqrt{H}}.\quad 3.\ H=\frac{Q}{(4{\cdot}9438\,d^2\,T)^2}.$$

d, being the diameter of the tube.

On the Discharge of Water by Horizontal Conduit, or Conducting Pipes. From the Experiments of M. BOSSUT.

1. The less the diameter of the pipe, the less, proportionally, is the discharge of fluid.
2. The greater the length of conduit pipe, the greater the diminution of discharge.
3. The discharges made in equal times, by horizontal pipes of different lengths, but of the same diameter, and under the same altitude of water, are to one another in the inverse ratio of the square roots of the lengths.
4. That, in order to have a perceptible and continuous discharge of fluid, the altitude of the water in the reservoir, above the axis of the conduit pipe, must not be less than ·148 inches for every 180 feet of the length of the pipe.
5. That, in the construction of hydraulic machines, it is not enough that elbows and contractions be avoided, but also any intermediate enlargements, the bad effects of which are proportionate, as in the following

TABLE.

Head of Water, in Cubic Inches.	Number of enlarged Par.s.	Seconds in which 4 Cubic Feet were discharged.
32·5	0	109
32·5	1	147
32·5	3	192
32·5	5	240

Practical Rules and Examples for ascertaining the Diameters of Pipes, and Quantity of Water discharged in any given time.

MR. TREDGOLD'S RULE, FROM EYTELWEIN'S EQUATION.

Rule.—Multiply 2500 times the diameter of the pipe, in feet, by

the height in feet, and divide the product by the length in feet, added to 50 times the diameter, and the square root of the quotient will be the velocity of discharge, in feet, per second.

Example.—Suppose the diameter of a pipe to be ·375 feet; the height of the water in the reservoir, above the point of discharge, 51·5 feet, and the length of pipe 14637 feet. Required the velocity of the water, and the quantity discharged in cubic feet, per second?

$$\frac{2500 \times \cdot375 \times 51\cdot5}{14637 + (50 \times \cdot375)} = \frac{48281\cdot25}{14655\cdot75} = \sqrt{3\cdot3} = 1\cdot816 \text{ feet per second,}$$

velocity;

And $\cdot375^2 \times \cdot7854 \times 1\cdot816 = \cdot20057$ cubic feet per second.

Or, multiply 1542, 133 times the fifth power of the diameter of the pipe, in feet, by the height, in feet, and divide the product by the length, in feet, added to 50 times the diameter, in feet, and the square root of the quotient equals the discharge, in cubic feet, per second.

These rules apply only to the case where the pipe is straight. If there be bends in it, the velocity (found by the rule as above,) must be diminished, by taking the product of its square, multiplied by the sum of the sines of the several angles of inflection, and by ·0038, which will give the degree of pressure required to overcome the resistance occasioned by the bends, and deducting this height from the height corresponding to the velocity, the corrected velocity is obtained.

If power is to be obtained by the issuing stream of water through a pipe, the whole of the power due to the height which is necessary to send the water through the pipe, at the required velocity, is not lost, for the power due to this velocity is still in the water. Thus, suppose five tenths or half a foot of head of water be given to maintain a required velocity of water through a pipe, and this velocity be found, as per rule, to be 2·988 feet per second: then $\left(\frac{2\cdot988}{8}\right)^2 = \cdot1395$, or the height that would give that water a velocity of 2·988 per second. Hence, $\cdot5 - \cdot1395 = \cdot3605$ of a foot, the positive height lost by the resistance in passing through the pipe.

A pressure of ·75 of a foot in height, will give a velocity of discharge equal to 3·5355 feet per second, at the lower end of a straight pipe of 2 feet diameter and 200 feet in length; but if there be a bend, or number of bends, in the pipe, the velocity of the water will not be so great. Say that there are three bends, one of 90°, and the other two of 150° each, the sum of the sines of those angles equal 2. Hence, $3\cdot5355^2 \times 2 \times \cdot0038 = \cdot095$, and $\cdot75 - \cdot095 = \cdot655$, that must be used or taken instead of ·75 in the rule for obtaining the velocity of discharge at the lower end of the pipe.

As an approximate rule for determining the diameters of pipes capable of conducting any required quantity of water, in cubic feet, per minute:—

Multiply the square of the quantity in cubic feet per minute by ·96, and the product equals the diameter of the pipe in inches.

Example.—Required the diameter of a pipe large enough to conduct 625 cubic feet of water per minute?

$$\sqrt{625}=25, \text{ and } 25\times{\cdot}96=24 \text{ inches diameter.}$$

TABLE

Of the Diameters of Pipes, sufficient in Size to discharge a required Quantity of Water per Minute.

Cubic feet.	Diameter in inches.	Cubic feet.	Diameter in inches.	Cubic feet.	Diameter in inches.
1	·96	18	4·07	130	10·94
2	1·36	20	4·29	140	11·35
3	1·66	25	4·80	150	11·75
4	1·92	30	5·25	160	12·14
5	2·15	35	5·67	170	12·51
6	2·35	40	6·07	180	12·67
7	2·60	45	6·53	190	13·23
8	2·72	50	6·80	200	13·57
9	2·88	55	7·12	225	14·40
10	3·04	60	7·43	250	15·17
11	3·18	70	8·03	275	15·91
12	3·33	80	8·60	300	16·62
13	3·46	90	9·10	350	17·95
14	3·60	100	9·60	400	19·20
15	3·72	110	10·06	500	20·46
16	3·84	120	10·51	600	23·51

Of the Discharge of Water by Rectangular Orifices in the Side of a Reservoir Extending to the Surface.

The velocity of the water varies nearly as the square root of the height, and the quantity discharged per second two thirds of the velocity due to the mean height, allowing for the contraction of the fluid, according to the form of the opening, which renders the co-efficient, in this case, equal to 5·1. Hence:—

Suppose, in the side of a lake or in the side of a reservoir, a rectangular opening is made, without any oblique lateral walls, 3 feet wide, and extending 2 feet below the surface of the water. Required the quantity that will be discharged, in cubic feet, per second?

The area of the opening$=3\times2=6$ feet; and $\frac{2}{3}$ of $\sqrt{2\times5{\cdot}1\times6}=$ 28·85 cubic feet.

The same co-efficient is applicable to the finding of the dimensions of a weir in the side of a lake, or in the side of a still river. For example:—

In the side of a lake is a wier of 3 feet in breadth, and the surface of the water stands 5 feet above it. It is required how much the weir must be widened, in order that the water may be a foot lower, and an equal quantity discharged?

Here, the velocity is $\frac{2}{3}$ the $\sqrt{5}\times 5\cdot 1$, and the quantity of water $\frac{2}{3}$ $\sqrt{5}\times 5\cdot 1\times 3\times 5$; but the velocity must be reduced to $\frac{2}{3}$ of $\sqrt{4}\times 5\cdot 1$, then the section will be $\dfrac{\frac{2}{3}\sqrt{5}\times 5\cdot 1\times 3\times 5}{\frac{2}{3}\sqrt{4}\times 5\cdot 1}=\dfrac{\sqrt{5}\times 3\times 5}{\sqrt{4}}=7\cdot 5$; and $\dfrac{7\cdot 5}{4}\times\sqrt{5}=4\cdot 19$ feet in breadth.

EXTRACT FROM A TABLE

By M. MORIN, *relative to the Motion of Water, and Quantities Discharged, by Pipes of Different Diameters.*

Quantity of Water discharged, in Litres, per Second	Mean Velocity, in Metres, per Second.	Height due to the Velocity, in Metres.	Mean Velocity, in Metres, per Second.	Height due to the Velocity, in Metres.	Mean Velocity, in Metres, per Second.	Height due to the Velocity, in Metres.
Litres.	Diameter of Pipe, ·05 Metres, or 1·9685 inches.		Diameter of Pipe, ·06 Metres, or 2·3622 inches.		Diameter of Pipe, ·08 Metres, or 3·1496 inches.	
1	·5093	·007933	·3537	·003343	·1989	·000862
2	1·0186	·030318	·7073	·012431	·3979	·003101
3	1·5279	·067153	1·0610	·027364	·5968	·006720
4	2·0372	·118450	1·4147	·048101	·7958	·011716
5	2·5465	·184195	1·7684	·074648	·9947	·018091
6	3·0558	·264393	2·1221	·107002	1·1937	·025845
7	- -	- -	2·4757	·145165	1·3926	·034976
8	- -	- -	2·8294	·189137	1·5916	·045486
9	- -	- -	- -	- -	1·7905	·057375
10	- -	- -	- -	- -	1·9894	·070642
Litres.	Diameter of Pipe, ·10 Metres, or 3·937 inches.		Diameter of Pipe, ·20 Metres, or 7·874 inches.		Diameter of Pipe, ·30 Metres, or 11·811 inches.	
3	·3820	·002297				
4	·5093	·003966	·1273	·000157		
5	·6366	·006087	·1592	·000232		
6	·7639	·008660	·1910	·000320		
7	·8913	·011684	·2228	·000423	·0990	·000068
8	1·0186	·015160	·2546	·000540	·1132	·000086
9	1·1459	·019087	·2865	·000671	·1273	·000105
10	1·2732	·023466	·3483	·000816	·1415	·000126
11	1·4006	·028296	·3501	·000975	·1556	·000148
12	1·5279	·033579	·3820	·001149	·1698	·000173
14	1·7825	·045499	·4456	·001538	·1981	·000228
16	2·0372	·059225	·5093	·001983	·2264	·000290
18	2·2918	·074758	·5730	·002485	·2546	·000360
20	2·5465	·092098	·6366	·003044	·2829	·000137

NOTE.—The French litre equals 61·02442 English cubic inches.

Of the Motion of Water in Rivers.

The velocity of the water in a river decreases from or near the surface downwards; so that the mean velocity under any point of

the surface will, on this account, be less than that obtained by a float, or light body swimming on the surface of the fluid. But, after having, by means of a float, determined the velocity at the surface, at any particular point of the width of the stream, the mean velocity of the water, passing under that point, and also the velocity at the bottom, will be obtained by the following rules:—

1. Take 1·03 from the square root of the velocity at the surface, and the square of the remainder will be the velocity at the bottom.

2. Add the surface and bottom velocities together, and half the sum is the mean velocity.

TABLE,

Containing the Quantities of Water, in Cubic Feet, that will be discharged over a Weir per Minute, for every Inch in its Breadth, when the Depths of the Water from the Surface to the Top edge of the Wasteboard do not exceed 18 *inches.*

Depth of the Water, in Inches.	Cubic Feet per Minute, according to Du Buat's Formula.	Cubic Feet per Minute, according to Experiments made in Scotland.	Depth of the Water, in Inches.	Cubic Feet per Minute, according to Du Buat's Formula.	Cubic Feet per Minute, according to Experiments made in Scotland.
1	0·403	0·428	10	12·748	13·535
2	1·140	1·211	11	14·707	15·632
3	2·095	2·226	12	16·758	17·805
4	3·225	3·427	13	18·895	20·076
5	4·507	4·789	14	21·117	22·437
6	5·925	6·295	15	23·419	24·883
7	7·466	7·933	16	25·800	27·413
8	9·122	9·692	17	28·258	30·024
9	10·884	11·564	18	30·786	32·710

To find the velocity of water issuing through a circular orifice at any given depth from the surface.

Rule.—Multiply the square root of the height or depth to the center of the orifice by 8·1, and the product is the velocity of the issuing fluid in feet per second.

Example.—Required the velocity of water issuing through an orifice under a head of 11 feet from the surface.

$\sqrt{11} = 3{\cdot}3166 \times 8{\cdot}1 = 26{\cdot}864$ feet, velocity per second.

In the discharge of water by a rectangular aperture in the side of a reservoir, and extending to the surface, the velocity varies nearly as the square root of the height, and the quantity discharged per second equal $\frac{2}{3}$ of the velocity due to the mean height, allowing for the contraction of the fluid according to the form of the opening, which renders the co-efficient in this case equal to 5·1; whence the following general rules:

1. *When the aperture extends to the surface of the fluid.*—Multiply the area of the opening in feet by the square root of its depth

also in feet, and that product by 5·1; then $\frac{2}{3}$ of the last product equal the quantity discharged in cubic feet per second.

2. *When the aperture is under a given head.*—Multiply the area of the aperture in feet by the square root of the depth also in feet, and by 5·1; the product is the quantity discharged in cubic feet per second.

Example 1.—Required the quantity of water in cubic feet per second discharged through an opening in the side of a dam or weir, the width or length of the opening being $6\frac{1}{2}$ feet, and the depth 9 inches, or ·75 of a foot.

Square root of $\cdot75 = \cdot866$.

Then $\dfrac{6\cdot5 \times \cdot75 \times \cdot866 \times 5\cdot1 \times 2}{3} = 14\cdot3839$ cubic feet.

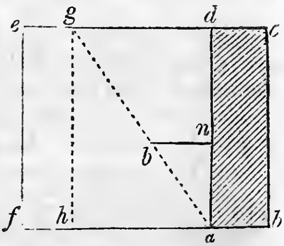
Example 2.—What would be the quantity discharged through the above opening if under a head of water 4 feet in height?

Square root of $4 = 2$, and $2 \times 5\cdot1 = 10\cdot2$ feet velocity of the water per second.

And $6\cdot5 \times \cdot75 \times 2 \times 5\cdot1 = 49\cdot725$ cubic feet discharged in the same time.

STATICS.

PRESSURE OF EARTH AGAINST WALLS.



Let *a b c d* be the vertical section of a wall, behind which is a bank or terrace of earth, of which a prism whose section is represented by *d a g* would detach itself, and fall down, were it not prevented by the wall. Let *a g* be the line of *rupture* or the *natural declivity* which the earth would assume, were it not for the resistance of the wall.

In sandy or loose earth the angle *h a g* seldom exceeds 30°, in firmer earth it becomes 37°, and in some favorable cases more than 45°

It has been found, however, theoretically, and confirmed experimentally, that the angle formed with the vertical by the prism of earth that exerts the greatest horizontal stress against the wall, is *half* the angle which the natural slope of the earth makes with the vertical.

Then the resultant, $l\,n$, of the pressure of the bank behind a vertical wall is at a distance, $a\,n$, from the bottom of the wall $=\frac{1}{3}$ of $d\,a$.

Of the experimental results, the best which we have seen are those of M. Mayniel, from which the following are selected, viz.:

That the *friction*, in vegetable earth, is $\frac{1}{2}$ the pressure; in sands $\frac{4}{10}$.

The *line of rupture*, $a\,g$, in a bank of vegetable earth, is $=\cdot 618$ of $d\,a$.

When the bank is of sand, then $d\,g=\cdot 677$ of $d\,a$.

When the bank is of vegetable earth, mixed with small gravel, then $d\,g=\cdot 646$ of $d\,a$.

If it be of rubbles, then $d\,g=\cdot 414$ of $d\,a$.

THICKNESS OF WALLS, BOTH FACES VERTICAL.

Brick.—Weight of a cubic foot $=108$ lbs. avoirdupois, bank of vegetable earth carefully laid course by course, $d\,e=\cdot 16\,a\,d$.

Unhewn Stone.—135 lbs. per cubic foot, bank as before $d\,e=\cdot 15\,a\,d$.

Brick.—Bank clay, well rammed, $d\,e=\cdot 17\,a\,d$.

Hewn Freestone.—170 lbs. per cubic foot, bank vegetable earth, $d\,e=\cdot 14\,a\,d$.

Bricks.—Bank of sand, $d\,e=\cdot 33\,a\,d$.

Unhewn Stone.—Bank of sand, $d\,e=\cdot 30\,a\,d$.

When the earth of the bank or terrace is liable to be much saturated with water, the proportional thickness of wall must be at least double.

(See Gregory's Mathematics for further particulars, page 224.)

MOTION.

Motion, generally, is the effect of impulsive force, or the act of changing place; in Mechanics, it is understood as the act of transmitting power, or the means by which power is distributed. Equality or inequality of motion is as the diameters of the wheels or pulleys by which it is transmitted.

THE VELOCITY OF WHEELS.

The relative velocity of wheels is as the number of their teeth.

To find the Velocity or Number of Turns of the last Wheel to one of the first.

Rule.—Divide the product of the teeth of the wheels that act as drivers by the product of the driven, and the quotient is the number.

Example.—If a wheel of 32 teeth drive a pinion of 10, on the axis of which there is one of 30 teeth, acting on a pinion of 8, what is the number of turns of the last?

$$\frac{32}{10}\times\frac{30}{8}=\frac{960}{80}=12.\quad \text{Ans.}$$

To find the Proportion that the Velocities of the Wheels in a Train should bear to one another.

Rule.—Subtract the less velocity from the greater, and divide the remainder by one less than the number of wheels in the train; the quotient is the number, rising in arithmetical progression from the less to the greater velocity.

Example.—What are the velocities of three wheels to produce 18 revolutions per minute, the driver making 3 revolutions per minute?

$$\frac{18-3=15}{3-1=\ 2}=7{\cdot}5;\ \text{then } 3+7{\cdot}5=10{\cdot}5,$$

and $10{\cdot}5+7{\cdot}5=18$; thus, 3, 10·5 and 18 are the velocities of the three wheels.

To find the Number of Teeth required in a Train of Wheels to produce a certain Velocity.

Rule.—As the velocity required is to the number of teeth in the driver, so is the velocity of the driver to the number of teeth in the driven.

Example.—If the driver has 90 teeth, makes 2 revolutions, and the velocities required are 2·10· and 18· what are the number of teeth in each of the other two?

2d wheel, 10 : 90 : : 2 : 18 teeth.
3d wheel, 18 : 90 : : 2 : 10 teeth.

THE TEETH OF WHEELS.

To Construct a Tooth.

Divide the pitch into 10 parts. Let 3·5 of these parts be below the pitch line, and 3·0 of them above.

The thickness should be 4·7 of the pitch.
The length should be 6·5 of the pitch.

The Diameter of a Wheel is measured from the Pitch line.

The wood used for teeth is about ¼ the strength of cast iron; therefore they should be twice the depth to be of equal strength.

To find the Diameter of a Wheel, the Pitch and Number of Teeth being given.

$$\frac{\text{Pitch}\times\text{number of teeth}}{3{\cdot}1416}=\text{diameter.}$$

NOTE. The pitch, as found by this rule, is the arc of a circle; the true pitch required is a straight line, and must be measured from the centres of two contiguous teeth.

To find the Pitch, the Diameter and Number of Teeth being given

$$\frac{\text{Diameter}\times 3{\cdot}1416}{\text{number of teeth}}=\text{pitch}$$

To find the Radius.

$$\frac{\text{Pitch}\times\text{number of teeth}}{3{\cdot}1416}\div 2=\text{radius.}$$

To find the Number of Teeth.

$$\frac{2\times \text{radius}\times 3\cdot 1416}{\text{pitch.}}=\text{number of teeth.}$$

The Chord and Versed Sine being given, to find the Diameter.

Rule.—Add the square of half the chord of the arc to the square of the versed sine, and divide by the versed sine.

Example.—Let x=diameter,
" b=versed sine,
" a=half the chord,

Then $(x-b)b=a^2$

$$bx-b^2=a^2$$

$$x=\frac{a^2+b^2}{b}$$

Given the velocities of two Wheels or Pulleys, and the number of teeth or diameter of the one, to find the number of teeth or diameter of the other, as required for the given velocity.

Rule.—Multiply the number of teeth or diameter given by its velocity, and divide the product by the other's velocity; the quotient is the number of teeth or diameter required.

Examples.—1. The velocity of a wheel containing 120 teeth is 30 revolutions per minute: required the number of teeth in another to make 300 revolutions in the same time.

$$\frac{120\times 30}{300}=12 \text{ teeth.}$$

2. The diameter of a pulley is 37 inches, and making 45 revolutions per minute: what must be the diameter of another to make 28 in the same time?

$$\frac{37\times 45}{28}=59\cdot 46 \text{ inches.}$$

To determine the diameters of a Pair of Wheels in contact with each other, their velocities and the distance of their centers apart being given.

Rule.—Divide the greatest velocity by the least; the quotient is the ratio of diameter the wheels must bear to each other. Hence, divide the distance between the centers by the ratio, plus 1; the quotient equal the radius of the smaller wheel; and subtract the radius thus obtained from the distance between the centers; the remainder equal the radius of the other.

Example.—The distance of two shafts from center to center is 50 inches, and the velocity of the one is 25 revolutions per minute; the other is to make 80 in the same time: required the diameters of the wheels at the pitch lines suitable for the purpose.

$80\div 25=3\cdot 2$, ratio of velocity, and $\frac{50}{3\cdot 2+1}=11\cdot 9$, the radius of smaller wheel; then $50-11\cdot 9=38\cdot 1$, radius of larger: hence, their diameters equal $23\cdot 8$ and $76\cdot 2$ inches.

TABLE,

Whereby to find the Diameter of a Wheel for a given Pitch of Teeth, or the Pitch of the Teeth to a given Diameter.

No. Teeth.	Diameter.	No. Teeth.	Diameter.	No. Teeth.	Diameter.	No. Teeth.	Diameter.
10	3·2360	55	17·5165	100	31·8362	145	46·1585
11	3·5494	56	17·8347	101	32·1544	146	46·4768
12	3·8637	57	18·1528	102	32·4727	147	46·7951
13	4·1785	58	18·4710	103	32·7910	148	47·1134
14	4·4939	59	18·7891	104	33·1092	149	47·4316
15	4·8097	60	19·1073	105	33·4275	150	47·7499
16	5·1258	61	19.4254	106	33·7457	151	48·0682
17	5·4421	62	19·7436	107	34·0640	152	48·3865
18	5·7587	63	20·0618	108	34·3823	153	48·7048
19	6·0755	64	20·3800	109	34·7005	154	49·0231
20	6·3924	65	20·6982	110	35·0188	155	49·3414
21	6·7095	66	21·0163	111	35·3371	156	49·6596
22	7·0266	67	21·3345	112	35·6553	157	49·9779
23	7·3439	68	21·6527	113	35·9736	158	50·2962
24	7·6612	69	21·9709	114	36·2919	159	50·6145
25	7·9787	70	22·2891	115	36.6101	160	50·9328
26	8·2962	71	22·6073	116	36·9284	161	51·2511
27	8·6137	72	22·9255	117	37·2467	162	51·5694
28	8·9314	73	23·2437	118	37·5650	163	51·8877
29	9·2490	74	23·5620	119	37·8832	164	52·2060
30	9·5667	75	23·8802	120	38·2015	165	52·5243
31	9·8845	76	24·1984	121	38·5198	166	52.8425
32	10·2022	77	24·5166	122	38·8380	167	53·1608
33	10·5201	78	24·8348	123	39·1563	168	53·4791
34	10·8379	79	25·1531	124	39.4746	169	53·7974
35	11·1558	80	25·4713	125	39·7929	170	54·1157
36	11·4737	81	25·7895	126	40·1112	171	54·4340
37	11·7916	82	26·1077	127	40·4294	172	54·7523
38	12·1095	83	26·4260	128	40·7477	173	55·0706
39	12·4275	84	26·7442	129	41·0660	174	55·3889
40	12·7454	85	27·0625	130	41·3843	175	55·7072
41	13·0634	86	27·3807	131	41·7025	176	56·0255
42	13·3814	87	27·6989	132	42·0208	177	56·3438
43	13·6995	88	28·0172	133	42·3391	178	56·6621
44	14·0175	89	28·3354	134	42·6574	179	56·9803
45	14.3355	90	28·6537	135	42·9757	180	57·2986
46	14·6536	91	28·9719	136	43·2939	181	57·6169
47	14·9717	92	29·2902	137	43·6122	182	57·9352
48	15·2897	93	29·6084	138	43·9305	183	58.2535
59	15·6078	94	29·9267	139	44·2488	184	58·5718
50	15·9259	95	30·2449	140	44·5671	185	58·8901
51	16·2440	96	30·5632	141	44·8854	186	59·2084
52	16·5621	97	30.8814	142	45·2036	187	59·5267
53	16·8803	98	31·1997	143	45·5219	188	59·8450
54	17·1984	99	31·5179	144	45·8402	189	60·1633

Rules.—1. Multiply the diameter in the table by the given pitch the product is the diameter of the wheel at the pitch circle.

2. Divide the given diameter by the diameter in the table, and the quotient is the pitch of the teeth.

Examples.—1. Required the diameter of a wheel at the pitch circle to contain 48 teeth of $2\frac{1}{2}$ inches pitch.

$$15{\cdot}2897 \times 2{\cdot}5 = 38{\cdot}224 \text{ inches.}$$

2. The diameter of a wheel at the pitch line to be 86 inches, and is required to contain 90 teeth: what will be the pitch of the teeth?

$$\frac{86{\cdot}0000}{28{\cdot}6537} = 3 \text{ inches, nearly.}$$

TABLE,

By which to determine the Number of Teeth or Pitch of small Wheels on the Manchester Principle.

Diametral Pitch.	Circular Pitch.	Diametral Pitch.	Circular Pitch.
3	1·047	9	·349
4	·785	10	·314
5	·628	12	·262
6	·524	14	·224
7	·449	16	·196
8	·393	20	·157

NOTE. The pitch is reckoned on the diameter of the wheel in place of the circumference, and distinguished as wheels of 8 pitch, 12 pitch, &c.

Example.—To find the number of teeth that a wheel of 16 inches diameter will contain of a 10 pitch.

$16 \times 10 = 160$ teeth, and the circular pitch $=$ ·314 inches.

The diameter of a wheel for a 9 pitch of $126 = \frac{126}{9} = 14$ inches diameter, the circular pitch of which is ·349 inches.

TABLE

Of Dimensions of the Teeth of Wheels, that will safely transmit a given Number of Horses' Power at a given Velocity.

Pitches, in Inches.	Thickness of Teeth, in Inches.	Horse Power at 3 Feet per Second.	Horse Power at 4 Feet per Second.	Horse Power at 5 Feet per Second.	Horse Power at 7 Feet per Second.	Horse Power at 11 Feet per Second.
4	1·9	19	25·5	32	45	70·5
$3\frac{1}{2}$	1·6	14·75	19·5	24·5	34·25	54
3	1·4	11	14·5	18	2·5	39·5
$2\frac{1}{2}$	1·2	7·5	10	12·5	17·5	27·5
2	·95	4·75	6·25	8	11	17·25
$1\frac{3}{4}$	·83	3·5	5	6·25	8·5	13·5
$1\frac{1}{2}$	·71	2·75	3·5	4·5	6·25	10
$1\frac{1}{4}$	·59	2	2·5	3·125	4·4	6·8
$1\frac{1}{8}$	·53	1·5	2·25	2·5	3·5	5·5
1	·48	1·2	1·6	2	2·8	4·4
$\frac{7}{8}$	·41	1	1·4	1·75	2·5	3·8
$\frac{3}{4}$	·36	.7	·9	1·125	1·5	2·5
$\frac{5}{8}$	·33	·5	·625	·75	1	1.7
$\frac{1}{2}$	·24	.3	·4	·5	·7	1·1
$\frac{3}{8}$	·18	2	·25	·3	·4	·6
$\frac{1}{4}$	·12	0225	·1	·125	·175	·275

The Diameter of the great Wheel of a Clock or other Wheel-work being 3·2 Inches and 96 Teeth, and the Pinion which it is to drive has 8 Leaves: Required the Distance their Centers should have so as they may pitch properly together.

Rule.—As 98·25 is to 3200, the diameter of the wheel; so is half the sum of the wheel and pinion added together to a number which shall be the distance required.

Example.—98+4=102, and 102÷2=52.

Then 98·25 : 3200 : : 52 : 1693·6, the distance which the centers ought to have, which is 1·6 inches, and a very small fraction ($\frac{936}{1000}$) besides.

Having the Distance of the Centers, it is required to find the Diameter of a Wheel of 96 Teeth and the Diameter of a Pinion of 8, which it is to drive, so that they may pitch properly together.

The half sum of the wheel and pinion is 52, the distance of the centers is 1693·6, and the wheel 96+2·25; then say as 52 : 1693·6 : : 98·25 : 3199·9, which number equals the distance of the wheel, 96.

To find the Diameter of the Pinion,

Say as 52 ; 1693·6 : : 9·5 : 309·4= the diameter of the pinion.

NOTE.—The above rules are equally correct for clock-work, mill-work, or any kind of wheel-work, and their extreme accuracy is rarely to be met with.

To divide the Circumference of a Circle into any given Number of Parts, whether Even or Odd.

Example.—As there are odd numbers of teeth in some wheels, it is proper to show how to divide the circumference of a circle into any given odd or even number of parts, so as that number may be laid upon the dividing-plate of a cutting engine.

There is no odd number but from which, if a certain number be subtracted, there will remain an even number easy to be subdivided. Thus, supposing the given number of equal divisions of a circle on the dividing-plate to be 69, subtracting 9, there will remain 60. Every circle is supposed to contain 360 degrees; therefore say, as the given number of parts in the circle which is 69 is to 360 degrees, so is 9 parts to the corresponding arc of the circle that will contain them; which arc, by the rule of Proportion, will be found to be $46\frac{95}{100}$.

Therefore, by the line of chords on a common scale, or rather on a sector, set off $46\frac{95}{100}$ (or $46\frac{9}{10}$) degrees with the compasses in the periphery of the circle, and divide that arc or portion of the circle into 9 equal parts, and the rest of the circle into 60, and the whole of the circle will be divided into 69 equal parts, as was required.

Again, suppose it were required to divide the circumference of a circle into 83 equal parts, subtract 3 and 80 will remain.

Thus, as 83 parts are to 360, so, *by the rule of proportion*, are 3 parts to 13 degrees and one-hundredth part of a degree, which small fraction may be neglected. Therefore, by the line of chords and compasses, set off 13 degrees in the periphery of the circle, and divide that portion or arc into 3 equal parts and the rest into 80, and the work will be done.

Having the Diameter of a Wheel which is proposed to be cut into a given Number of Teeth, to find the Thickness of a Cutter suitable, so as to give the Teeth and Space in due Proportion.

Example.—A wheel of 4·8 inches in diameter is wanted to have 144 teeth cut on it: suppose the depth of the teeth to be *one-tenth* of an inch, then the diameter of the wheel taken at the bottom of the teeth and space will be 4·6 inches; and it is intended to have the teeth and spaces alike at the bottom, which will allow the teeth to be broader from the bottom upwards. To find the circumference of a diameter for 4·6 inches, say as 113 is to 355 so is 4·6 to its circumference, which in this case is 14·4513 inches. Reduce this to thousand parts, and we shall have 14451 to divide by 288, the number of teeth and spaces taken together, the quotient will be 50·2 nearly; call it 50 hundredths of an inch or half a tenth, and this is what the thickness of the cutter ought to be.

TABLE,

By which to determine the Number of Teeth of small Wheels, at a given Number per Inch of the Diameter.

Diameter.	Teeth to the Inch.	No. of Teeth.	Diameter.	Teeth to the Inch.	No. of Teeth.
in.			in.		
2	10	18	3	20	58
2½	14	33	3½	25	85½
3	12	34	4	15	58
3½	16	54	5	18	88
4	9	34	5½	25	135½
4½	12	52	6	12	70
5½	10	53	7	16	110

Rule.—Multiply the given diameter by the number of teeth to the inch and subtract 2 for the pitch line the remainder will be the number of teeth.

To find the Breadth of the Teeth

Rule.—Divide the horses' power by the velocity of the wheel at the pitch circle, per second, in feet, and twice the quotient is the breadth in inches.

Example.—Required the breadth of the teeth for a first mover from a 15-horse engine, the velocity of the wheel being 6 feet per second

$$\frac{15}{6}=2{\cdot}5\times 2=5 \text{ inches.}$$

TEETH OF WHEELS.

To find the Horses-Power that the Teeth will transmit.

Rule.—Multiply one fourth of the square of the pitch in inches by the breadth of the teeth in inches; the product is the horses-power that the teeth will transmit when the pitch line passes through 4 feet per second.

In quick speeds, or fractional pitches, it may be more convenient to take the following

Rule.—Multiply the square root of the pitch in inches, by the breadth of the teeth in inches; the product is the horses-power at 16 feet per second.

A general rule to ascertain the length of the teeth is to take $\frac{1}{3}$ of the pitch for the distance from the root to the pitch line, and $\frac{1}{4}$ of the pitch for the distance from the pitch line to the top.

Hicks' Rule for Calculating the Strength of Shafts.

Rule.—Multiply the horses-power by the assumed number, (300) and divide the product by the revolutions per minute; the cube root of the quotient will be the diameter required.

OF WATER-WHEELS.

The properties of water, as a motive power, are gravity and impul sive force, each being rendered peculiarly available for the production of uniform circular motion through the medium of the water-wheel.

Water-wheels are necessarily and designedly of various modifications, so as to obtain the greatest amount of mechanical effect, from a known quantity of water flowing at a certain velocity, or from a given height, and generally ranked, by estimation of effect, into *first, second*, and *third-class* wheels.

1st class includes overshot wheels, pitch-back wheels, and turbines.

2d class consists of breast wheels, or those which receive the water below the level of the axis.

And 3d class is composed of undershot wheels, tub wheels, and flutter wheels.

The most modern and best-conducted experiments on each description, as known to the public at present, are those by Poncelot of America, and of Morin in France, the results of which are as follows:

Overshot wheels, &c.; ratio of power to effect, varies fm.				0·60 to 0·80
Breast wheels	"	"	"	0·45 to 0·50
Undershot wheels, &c.	"	"	"	0·27 to 0·30

The greatest effect is obtained by an overshot wheel when the diameter of the wheel is so proportioned to the height of the fall, that the water shall flow upon the wheel at a point about $52\frac{3}{4}$ degrees distant from the top of the wheel.

If r represent the radius of the wheel to the extreme part of the buckets, and h the effective height of the fall, then $h=r(1+\sin. 37\frac{1}{4})$ or $h=1\cdot605\,r$; for the sine $37\frac{1}{4}=\cdot605$. Also $\cdot623\,h=r$. Therefore, when the effective height of the fall is determined, the radius of the wheel is easily obtained. When the effective fall is $\frac{8}{9}$ths of the whole fall, if h is made the whole fall, $r=\cdot554\,h$, or $1\cdot108\,h=$ the diameter of the wheel. The effective height of the fall is less than the true height, by as much as is required to give the water a proper velocity before it flows upon the wheel. And modern practice dictates a velocity of about 6 feet per second for the velocity of wheels in general: hence, if the velocity of the wheel be given, divide twice the square of the circumference in feet, per second, by 64·38, and the quotient equal the height of the fall.

If the portion of the total descent passed through by the water be given, then the velocity of the circumference should be one-half of that, due to this height. Therefore, multiply the portion of fall, in feet, by 64·38, and the square root of the product equal the water's velocity, in feet, per second. Also,

If the area of cross section of the overflow be multiplied by the velocity at the end of the fall, the product equal the quantity, in cubic feet, per second.

Observations from Experiments on Overshot Wheels. By R. Mallet, M. In. C. E. *From the Transactions of the Institution of Civil Engineers.*

1. When the depth of water in the reservoir is invariable, the diameter of the wheel should never exceed the entire height of the fall, less so much as is requisite to generate a proper velocity on entering the buckets.

2. Where the depth of water in the reservoir varies considerably and unavoidably, an advantage may be obtained by applying a larger wheel, dependant upon the extent of fluctuation, and ratio in time, that the water is at its highest or lowest levels during a given prolonged period. If this be a ratio of equality, in time there will be no advantage; and hence, in practice, the cases will be rare when any advantage will be obtained by the use of an overshot wheel greater in diameter than the height of fall, minus the head due to the required velocity of the water reaching the wheel.

3. If the level of the water in the reservoir never fall below the mean depth of the reservoir, when at the highest and lowest, and the average depth be between an eighth and a tenth of the height of the fall, then the average mechanical force of the large wheel will be greater than that of the small one; and it will of course retain its increased advantage at periods of increased depths of the reservoir.

4. That a positive advantage is gained by a wheel revolving in a conduit, varying with the conditions of the wheel and fall of nearly 11 per cent. of the total power.

To ascertain the Power of a Water-wheel.

Rule.—Multiply the velocity of the wheel, in feet, per minute, by the weight of the water, in lbs., expended on the wheel in the same time; divide the product by the co-efficient of power to effect, and the quotient equal the mechanical effect of the wheel, expressed in horses' power.

$$\text{Co-efficients}\begin{cases}\text{1st class wheels} & 47190\\ \text{2d class do.} & 69300\\ \text{3d class do.} & 115500\end{cases}$$

Or multiply the product of the quantity of water expanded, in cubic feet, per minute, and the velocity of the wheel, in feet, in the same time, by the following decimal equivalents: the product will be the number of horses' power that the wheel is equal to in useful effect.

$$\text{Decimal equivalents}\begin{cases}\text{1st class wheels} & \cdot001325\\ \text{2d class do.} & \cdot000902\\ \text{3d class do.} & \cdot000541\end{cases}$$

Example—Suppose a stream of water flowing on an overshot wheel at the rate of 95 cubic feet per minute, and the velocity of the wheel's periphery equal 6 feet per second, or 360 feet per minute: required the effect of the wheel in horses' power?

$$\frac{360\times95\times62\cdot5}{47190}=45\cdot29 \text{ horses' power.}$$

$$\text{Or, } 360\times95\times\cdot001325=45\cdot29 \quad \text{"} \quad \text{"}$$

NOTE. Where the fall of water do not exceed 4 feet, an undershot wheel ought to be applied; from 4 to 10 feet, a breast wheel; and from 10 feet upwards, an overshot, or pitch-back wheel.

To ascertain the Power of a Stream.

Rule.—Multiply the weight of the water in lbs. discharged in one minute by the height of the fall in feet; divided by 33000, and the quotient is the answer.

Example.—What power is a stream of water equal to of the following dimensions, viz.: 1 foot deep by 22 inches broad, velocity 350 feet per minute, and fall 60 feet; and what should be the size of the wheel applied to it?

$$12\times22\times\overline{350\times12}\div1728\times62\tfrac{1}{2}\times60\text{ feet}\div33000=72\cdot9.\quad\text{Ans.}$$

Height of fall 60 feet, from which deduct for admission of water, and clearance below, 15 inches, which gives 58·9 feet for the diameter of the wheel.

$$\left.\begin{matrix}\text{Clearance above} & 3\\ \text{" below} & 12\end{matrix}\right\}\text{15 inches.}$$

The power of a stream, applied to an overshot wheel, produces effect as 10 to 6·6.

Then, as 10 : 6·6 : : 72·9 : 48 horses' power equal that of an overshot wheel of 60 feet applied to this stream.

When the fall exceeds 10 feet, the overshot wheel should be applied.

The higher the wheel is in proportion to the whole descent, the greater will be the effect.

The effect is as the quantity of water and its perpendicular height multiplied together.

The weight of the arch of loaded buckets, in pounds, is found by multiplying $\frac{4}{9}$ of their number, $\times$ the number of cubic feet in each, and that product by 40.

To ascertain the Power of an Undershot Wheel when the Stream is confined to the Wheel.

Rule.—Ascertain the weight of the water discharged against the floats of the wheel in one minute by the preceding rules, and divide it by 100000; the quotient is the number of horses' power.

NOTE. The 100000 is obtained thus: The power of a stream, applied to an undershot wheel, produces effect as 10 to 3·3, then 3·3 : 10 : : 33000 : 100000.

When the opening is above the center of the floats, multiply the weight of the water by the height, as in the rule for an overshot wheel.

Example.—What is the power of an undershot wheel, applied to a stream 2 by 80 inches, from a head of 25 feet?

$\sqrt{25 \times 6{\cdot}5} \times 60 = 1950$ feet velocity of water per minute, and $2 \times 80 = 160$ inches $\times 1950 \times 12 \times 1728 = 2166{\cdot}6$ cubic feet $\times 62{\cdot}5 = {}^{*}135412$ lbs. of water discharged in one minute; then $135412 \div 100000 = 1{\cdot}35$ horses' power.

NOTE. The maximum work is always obtained when the velocity of the wheel is half that of the stream. Let V represent velocity of float boards, and v velocity of water; then $\frac{(v-V)^2}{V^2} \times$ force of the water, will be the force of the effective stroke.

The effect of an undershot wheel to the power expended is, at a medium, one-half that of an overshot wheel.

The virtual or effective head being the same, the effect will be very nearly as the quantity of water expended.

When the fall is below 4 feet, an undershot wheel should be applied.

To find the Power of a Breast Wheel.

Rule.—Find the effect of an undershot wheel, the head of water of which is the difference of level between the surface and where it strikes the wheel (breast), and add to it the effect of that of an overshot wheel, the height of the head of which is equal to the difference between where the water strikes the wheel, and the tail water; the sum is the effective power.

Example.—What would be the power of a breast wheel applied to a stream 2×80 inches, 14 feet from the surface, the rest of the fall being 11 feet?

$\sqrt{14 \times 6{\cdot}5} \times 60 = 1458{\cdot}6$ feet velocity of water per minute.

And $2 \times 80 \times 1458 \times 12 \div 1728 = 1620$ cubic feet $\times 62{\cdot}5 = 101250$ lbs. of water discharged in one minute.

Then $101250 \div 100000 = 1{\cdot}012$ horses' power as an undershot.

$\sqrt{11 \times 6{\cdot}5} \times 60 = 1290$ feet velocity of water per minute.

And $2 \times 80 \times 1290 \times 12 \div 1728 = 1433$ cubic feet $\times 62{\cdot}5 = 89562$ lbs of water discharged in one minute.

* Equal $160 \times 12 \div 1728 \times 62{\cdot}5 \times 1950 =$ momentum of water and its velocity.

$\times 11$ height of fall $\div 50000 = 19{\cdot}703$ horses, which, added to the above, $= 20{\cdot}715$. Ans.

NOTE. When the fall exceeds 10 feet, it may be divided into two, and two breast wheels applied to it

When the fall is between 4 and 10 feet, a breast wheel should be applied.

The power of a water-wheel ought to be taken off opposite to the point where the water is producing its greatest action upon the wheel.

Remarks on Reaction Water-wheels. From the Journal of the Franklin Institute, and other sources.

Reaction water-wheels are a very numerous family, of which the well-known hydraulic motor, called Barker's mill, is the parent: those used in various parts of the United States have usually vertical axes of rotation, and curved buckets, or vanes, against which the impulsive force of the water (spouting from within the wheel by adjutages, of which the curved vanes form the sides) acts indirectly, or rather reacts, thus producing (in reference to the affluent water) a backward rotary motion, similar, in character and effect, to the forward rotary motion produced by direct impulse in the case of undershot wheels.

In the American Philosophical Transactions for 1793, it is stated that the principles of reaction wheels had been fully investigated analytically in examining the merits of Rumsey's improvements on Barker's mill; and the conclusion come to, after a train of reasoning based upon scientific principles, was, that "action and reaction are equal;" that the undershot wheel is propelled by the action; and Barker's mill by the reaction of the same agent, or momentum: therefore their mechanical effects must be equal.

This conclusion no doubt tended to retard any effort at improvement of wheels on that principle for a considerable length of time; for it is only, comparatively speaking, quite recently, that reaction water-wheels, of the form at present in use, have occupied a prominent position before the public.

In 1830, Calvin Wing, of the United States, took out a patent for a reaction water-wheel with curved vanes or buckets, the vanes of which lapped over, or rather on to each other, in the ratio of $1\frac{1}{2}$ inches, for each inch of the width of the adjutage, or shortest horizontal distance between any two adjacent vanes.

In this wheel the water has free entrance to a circular space within, and, spouting out by the openings between the curved vanes, impels the wheel around in a backward direction, by its reaction against the vanes, in issuing with velocity from within the wheel.

But this species of wheel, so far, seems not to realize the amount of effect as anticipated; for, according to recent experiments, it appears that, with 788 cubic feet of water, at the rate of one foot per minute, applied on an overshot wheel, will grind and dress one bushel of wheat per hour; whereas to do the same by means of the reaction wheel required 1600.

Some of later date, as the turbine of France, by M. Fourneyron, and the recently-patented water-mill, by Whitelaw and Stirrat, Scotland, seem much improved hydraulic motors; for, according to the experiments of M. Morin, and others of high authority, they rank, in effect to power, equal to first-class wheels.

The chief objection to the common overshot wheel, is its great size and formidable cost, to which might be added, the loss of power consequent on the friction of the gearing requisite for bringing up the speed of the prime mover to the velocity indispensable to most ordinary mechanical operations. These objections do not apply to this species of water power, as the machine occupies but a very small space in comparison with a water-wheel of the same power; its speed is high, and the expense of its construction greatly inferior to that of any other effectual mechanism we are at present acquainted with for deriving a rotary motion from a head of water.

The arms of the machine by Whitelaw and Stirrat are bent in the form of an Archimedes spiral, so as to obviate the communication of a centrifugal force to the water, which, if the arms were straight, it would necessarily acquire to the diminution of the useful effect. Any number of arms may be used, but two is the common number. The machine revolves horizontally, and the affluence of the water at the orifices of the arms is regulated by means of valves of a peculiar description, governed by the centrifugal force of the machine, or, in other words, by its velocity.

To find the Center of Gyration of a Water-wheel.

Rule.—Take the radius of the wheel, the weight of its arms, and the weight of its rim, as composed of floats, shrouding, &c.

Let R represent the weight of rim,
" r " the radius of the wheel,
" A " the weight of arms,
" W " the weight of the water in action when the buckets are filled, as in operation.

Then $\sqrt{(R\times r^2\times 2+A\times r^2\times 2+W\times r^2 \div R+A+W\times 2)}=$ center of gyration.

Example.—In a wheel 20 feet diameter, the weight of the rim is 3 tons, the weight of the arms 2 tons, and the weight of the water 1 ton: what is the distance of the center of gyration from the center of the wheel?

$$
\begin{aligned}
R &= 3 \text{ tons} \times 10^2\times 2=600\\
A &= 2 \text{ "} \times 10^2\times 2=400\\
W &= 1 \text{ "} \times 10^2 \;..=\underline{100}
\end{aligned}
$$

$3+2+1=6\times 2=\frac{1100}{12}=91{\cdot}6$, the square root of which is 9·5, or $9\frac{1}{2}$ feet. Ans.

The mill of Mr. Harvey Ely, at Rochester, N. Y., has 3 water-wheels; two of which are 18 feet diameter, and one 16 feet: length of each 11 feet.

The water is applied at a point about 3 feet below the top of the wheel; opening $2\frac{1}{2}$ inches by 120 inches; making $5\frac{1}{2}$ revolutions per minute, propelling 9 pair of stones, 5 feet diameter, 140 revolutions

per minute. One large wheat elevator, 48 feet high, elevating 500 bushels of wheat per hour; three screens, three fanning mills, two smut machines, and six bolting chests.

The ordinary work of this mill, when running day and night, is 360 to 400 barrels of flour, or from 15 to 17 barrels per hour. The maximum is 630 barrels in 24 hours, or 26 barrels per hour. Some other mills in Western New York are said to exceed this.

NOTE.* At the mill of Mr. Samuel Newlin, at Fishkill Creek, N. Y., 5 barrels of flour can be ground, and 400 bushels of grain elevated 36 feet per hour with a stream and overshot wheel of the following dimensions, viz.:

Height of head to center of opening, $24\frac{7}{8}$ inches; opening, $1\frac{3}{4}$ by 80 inches; wheel, 22 feet diameter by 8 feet face; 52 buckets, each 1 foot in depth.

The wheel making $3\frac{1}{2}$ revolutions, driving 3 run of $5\frac{1}{2}$ feet stones 130 turns in a minute, with all the attendant machinery.

This is a case of maximum effect, in consequence of the gearing being well set up, and kept in good order.

MAXIMS.

The following Maxims have been deduced from the Experiments of MR. SMEATON.

Maxim 1. The virtual or effective head* of water being the same, the effect will be nearly as the quantity expended. That is, if a mill, driven by a fall of water whose virtual fall is 10 feet, and which discharges 30 cubic feet of water in a second, grind 4 bolls of corn in an hour; another mill having the same virtual head, but which discharges 60 cubic feet of water, will grind 8 bolls of corn in an hour.

Maxim 2. The expense of water being the same, the expense will be nearly as the height of the virtual or effective head.

Maxim 3. The quantity of water expended being the same, the effect is nearly as the square of its velocity. That is, if a mill, driven by a certain quantity of water, moving with the velocity of 4 feet per second, grind 3 bolls of corn in an hour; another mill, driven by the same quantity of water, moving with the velocity of five feet per second, will grind nearly $4\frac{7}{10}$ bolls in the hour, because $3 : 4\frac{7}{10} :: 4^2 : 5^2$ nearly.

Maxim 4. The aperture being the same, the effect will be nearly as the cube of the velocity of the water. That is, if a mill driven by water, moving through a certain aperture, with the velocity of 4 feet per second, grind 3 bolls of corn in an hour; another mill, driven by water, moving through the same aperture with the velocity of 5 feet per second, will grind $5\frac{43}{50}$ bolls nearly in an hour; for as $3 : 5\frac{43}{50} :: 4^3 : 5^3$ nearly.

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* The *virtual* or *effective head* of water moving with a certain velocity, is equal to the height from which a heavy body must fall, in order to acquire the same velocity. The height of the virtual head, therefore, may be easily determined from the water's velocity, for the heights are as the squares of the velocities, and, consequently, the velocities are as the square roots of the height.

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* Haswell's Engineers' and Mechanics' Pocket Book.

TABLE,

Containing the Velocity and Force of the Wind.—By Mr. Rouse.

Velocity of the Wind. Miles in an hour.	Velocity of the Wind. Feet in a second.	Perpendicular force on 1 foot area in pounds Avoirdupois.	Common Appellations of the Force of the Winds.
1	1·47	·005	Hardly perceptible.
2	2·93	·020	Just perceptible.
3	4·40	·044	
4	5·87	·079	Gentle, pleasant wind.
5	7·33	·123	
10	14·67	·492	Pleasant, brisk gale.
15	22·	1·107	
20	29·34	1·968	Very brisk.
25	36·67	3·075	
30	44·01	4·429	High winds.
35	51·34	6·027	
40	58·68	7·873	Very high.
45	66·01	9·963	
50	73·35	12·300	A storm, or tempest.
60	88·02	17·715	A great storm.
80	117·36	31·490	A hurricane.
100	146·70	49·200	A hurricane, that tears up trees, carries buildings before it, &c.

To find the Force of Wind acting perpendicularly upon a Surface.

Rule.—Multiply the surface in feet by the square of the velocity in feet, and the product by ·002288, the result is the force in avoirdupois pounds.

RESISTANCE TO TORSION OR TWISTING.

It is obvious that the strength of revolving shafts are directly as the cubes of their diameters and revolutions; and inversely, as the resistance they have to overcome.

Mr. Buchanan, in his Essay on the Strength of Shafts, gives the following data, deduced from several experiments, viz.: That the fly-wheel shaft of a 50 horse power engine, at 50 revolutions per minute, requires to be 7½ inches diameter, and therefore, the cube of this diameter, which is =421·875, serves as a multiplier to all other shafts in the same proportion: and taking this as a standard, he gives the following multipliers, viz.:

For the shaft of a steam engine, water-wheel, or any shaft connected with a first power, - - - - - -	400
For shafts in inside of mills, to drive smaller machinery, or connected with the shafts above, - - - - - - -	200
For the small shafts of a mill or machinery, - - - - -	100

From the foregoing, we derive the following

Rule.—The number of horses' power a shaft is equal to, is directly as the cube of the diameter and number of revolutions; and inversely, as the above multipliers.

NOTE. Shafts here are understood as the journals of shafts, the bodies of shafts being generally made square.

For cylindrical shafts, we have

$\sqrt[3]{\left(\frac{240 \times \text{No. of horses' power}}{\text{No. of revolutions in a minute}}\right)}$ = the diameter of the shaft in inches.

This rule is for cast iron, and it may be used for wrought iron by multiplying the result by ·963; or, for oak, by 2·238; or, for fir, by 2·06.

If the shaft belong to a 7-horse-power engine, and the strap turns $11\frac{1}{2}$ times in a minute,

$\sqrt[3]{\left(\frac{240 \times 7}{11 \cdot 5}\right)} = 5 \cdot 267$ inches diameter for cast iron.

For fir, $5 \cdot 267 \times 2 \cdot 06 = 10 \cdot 85$.
For oak, $5 \cdot 267 \times 2 \cdot 38 = 12 \cdot 535$.
And for wrought iron, $5 \cdot 267 \times \cdot 963 = 5 \cdot 0719$.

NOTE. This rule comes from the best authority, and gives perfectly safe results, though some employ 340, instead of 240, as a multiplier, which gives a greater diameter to the shaft.

It is to be remembered that these rules relate to the shafts of first movers, or the shafts immediately connected with the moving power. But these shafts may communicate motion to other shafts, called second movers, and these again to others, called third movers, and so on. The diameters of the second movers may be found from those of the first by multiplying by ·8, and those of the third movers by multiplying by ·793, respectively.

One material may resist, much better than another, one kind of strain; but expose both to a different kind of strain, and that which was weakest before may now be the strongest. This may be illustrated in the case of cast and wrought iron. The cast iron is stronger than the wrought iron when exposed to twisting or torsional strain; but the malleable iron is the stronger of the two when they are exposed to lateral pressure.

We shall subjoin a few results of experiments on the weight which was necessary to twist bars ¼ close to the bearings:

	LBS.	OZ.		LBS.	OZ.
Cast steel, - - - - -	19	9	Swedish iron, wrought,	9	8
Cast metal, - - - - -	9	7	Hard gun-metal, - - -	5	0
Blister steel, - - - - -	16	11	Brass, bent, - - - - -	4	11
Wrought iron, - - - -	10	2	Copper, cast, - - - - -	4	5

The above rules will often find their application in the practice of the engineer. Much of the beauty of any mechanical structure

depends on the proper proportioning of the magnitude of materials to the stress they have to bear; and, what is of far greater moment, its absolute security.

It is a well-known fact, that a cast iron rod will sustain more torsional pressure than a malleable iron rod of the same dimensions: that is, a malleable iron rod will be twisted by a less weight than what is required to wrench a cast iron rod of the same dimensions.

When the strength of malleable iron is less than that of cast iron to resist torsion, it is stronger than cast iron to resist lateral pressure, and that strength is in proportion as 9 is to 14.

From the foregoing, it is easy for the mill-wright to make his shafts of the iron best suited to overcome the resistance to which they will be subject, and the proportion of the diameters of their journals, according to the iron of which they are made.

Example.—What will be the diameter of a malleable iron journal, to sustain an equal weight with a cast iron journal of 7 inches diameter?

$$7^3 = 343;$$

$$14 : 343 :: 9 : 220\tfrac{1}{2}; \text{ now } \sqrt[3]{220{\cdot}5} = 6{\cdot}04 \text{ inches diameter.}$$

*Additional Results of Experiments on Torsional Strain.**

Square bars, with a journal 1 inch in diameter, and ¼ inch in length.

Wrought iron (Ulster Iron Co.), twisted with 326 lbs., and broke with 570 lbs. applied at the end of a lever 30 inches in length.

Wrought iron (Swedes), same length of lever, twisted with 367 lbs., and broke with 615 lbs.

Cast iron (foundry), journal 1 inch long, same length of lever, broke with 436 lbs.

The diameters for second and third movers are found by multiplying the diameters, ascertained by the above rules, by ·8 and ·793, respectively.

Grier, in his Mechanics' Calculator, gives the following rule for cast iron shafts:

$$\sqrt[3]{\left(\frac{240 \times \text{number of horses' power}}{\text{number revolutions per minute}}\right)} = \text{diameter in inches.}$$

For wrought iron, multiply result by ·963; for oak, by 2·238; and for pine, by 2·06.

* Haswell's Engineers' and Mechanics' Pocket-book.

The following is a table of the diameters of shafts, being the first movers, or having 400 for their multipliers:

TABLE

Of Diameters of the Journals of First Movers.

Horses' Power.	REVOLUTIONS. 10	15	20	25	30	35	40	45	50	55
4	5·5	4·8	4·5	4·	3·7	3·8	3·5	3·3	3·2	3·1
5	5·9	5·1	4·7	4·4	4·1	3·9	3·7	3·6	3·5	3·3
6	6·3	5·5	5·	4·6	4·4	4·1	4·	3·8	3·7	3·6
7	6·6	5·8	5·2	4·9	4·6	4·4	4·2	4·	3·9	3·7
8	6·9	6.	5·5	5·1	4·8	4·6	4·4	4·2	4·1	4·
9	7·2	6 3	5·7	5·5	5·	4·8	4·5	4·4	4·2	4·1
10	7·4	6·6	5·9	5·6	5·2	4·9	4·7	4·6	4·4	4·2
12	7·9	6·9	6·3	5·8	5·6	5·4	5·2	5·	4·8	4·6
14	8·3	7·2	6·7	6·2	5·9	5·6	5·4	5·2	5·	4·7
16	8·7	7·6	7·1	6·6	6·1	5·8	5·6	5·4	5·2	5·
18	9·	7·9	7·5	7·	6·6	6·2	5·8	5·6	5·4	5·2
20	9·3	8·1	7·4	7·2	6·6	6·4	5·9	5·7	5·6	5·4
25	10·	8·5	8·	7·4	7·1	6·8	6·3	6·	5·9	5·6
30	10·7	9·3	8·4	7·9	7·4	7·1	6·9	6·7	6·5	6·3
35	11·4	9·8	8·9	8.4	7·9	7·4	7·1	6·9	6·6	6·5
40	11·7	10·5	9·3	8·8	8·3	7·8	7·4	7·2	6·9	6·7
45	12·	10·6	9·7	9·2	8·7	8·1	7·6	7·4	7·	6·8
50	12·6	11·	10·	9·3	9·	8·5	8·	7·8	7·4	7·3
55	13·4	11·4	10·4	9·8	9·1	8·8	8·4	8·	7·5	7·4
60	13·6	12·	10·8	10·	9·3	9·	8·6	8·2	7·7	7·6
	INCHES DIAMETER.									

TABLE

Of Diameters of the Journals of First Movers.—CONTINUED.

Horses' Power.	REVOLUTIONS.									
	60	65	70	75	80	85	90	95	100	105
4	3·	2·9	2·9	2·8	2·7	2·7	2·6	2·6	2·6	2·5
5	3·3	3·2	3·1	3·	3·	2·9	2·9	2·8	2·8	2·7
6	3·5	3·5	3·4	3·3	3·2	3·2	3·	3·	2·9	2·9
7	3·6	3·6	3·5	3·4	3·4	3·3	3·3	3·2	3·1	3·1
8	3·9	3.8	3·7	3·6	3·5	3·5	3·4	3·4	3·3	3·2
9	4·	3 8	3·7	3·7	3·6	3·6	3·5	3·5	3·4	3;3
10	4·1	4·	3·9	3·8	3·7	3·7	3·6	3·6	3·5	3·4
12	4·4	4·3	4·2	4·1	4.	3·9	3·8	3·8	3·7	3·6
14	4·5	4·4	4·4	4·3	4·2	4·1	4·	4·	3·9	3·8
16	4·8	4·7	4·6	4·5	4·4	4·4	4·3	4·2	4·1	4·
18	5·	4·9	4·8	4·7	4·6	4·5	4·4	4·3	4·2	4·2
20	5·2	5·1	5·	4·8	4·6	4·6	4·5	4·5	4·4	4·4
25	5·5	5·4	5·3	5·2	5·1	4·9	4·8	4·7	4·6	4·6
30	5·9	5·8	5·7	5·6	5·5	5·3	5·2	5·1	5·	4·9
35	6·3	6·1	5·9	5.7	5·6	5·5	5·4	5·3	5·2	5·2
40	6·6	6·4	6·2	6·	5.9	5·8	5·7	5·6	5·6	5·5
45	6·7	6·5	6·4	6·2	6·1	6·	5·9	5·8	5·7	5·6
50	7·2	6·9	6·8	6·6	6·5	6·4	6·2	6·	5·9	5·8
55	7·3	7·2	7·	6·7	6·6	6·5	6·3	6·2	6·1	6·
60	7·4	7·3	7·2	6·9	6·8	6·8	6·7	6·6	6·4	6·2
	INCHES DIAMETER.									

TABLE,

Exhibiting the velocity of motion, for boring cast iron Cylinders, Pumps, &c., and heavy turning, with fixed cutters.

It will be observed that the surface bored is constantly the same, 78·54 feet per minute; this velocity is found to be the most advantageous: a velocity greater than this, not only takes the temper out of the cutters, but also causing more heat, expands the metal; and if the machine stops but for a few seconds, a mark is left from the contraction of the metal.

NOTE. Turning has a velocity double to that of boring.

BORING.		TURNING.	
Inches diameter.	Revolutions of Bar per Minute.	Inches diameter.	Revolutions of Shaft per Minute.
1	25·	1	50·
2	12·5	2	25·
3	8·33	3	16·67
4	6·25	4	12·50
5	5·	5	10.
6	4·16	6	8·32
7	3·57	7	7·15
8	3·125	8	6·25
9	2·77	9	5·55
10	2·5	10	5·
15	1·66	15	3·33
20	1·25	20	2·50
25	1·	25	2·
30	0·833	30	1·667
35	0·714	35	1·430
40	0·625	40	1·250
45	0·56	45	1·12
50	0·5	50	1·
60	0·417	60	0·834
70	0·358	70	0·716
80	0·313	80	0·626
90	0·278	90	0·556
100	0·25	100	0·50

N. B. The progression of the cutters may be $\frac{1}{16}$th of an inch for the first cut, and for the last, $\frac{1}{24}$th.

If hand-tools are employed in turning, the velocity may be considerably increased.

CENTER OF PERCUSSION AND OSCILLATION.

The center of percussion and oscillation, is the point in a body revolving around a fixed axis, so taken, that when it is stopped by any force, the whole motion, and tendency to motion, of the revolving body, is stopped at the same time.

It is also that point of a revolving body which would strike any obstacle with the greatest effect; and, from this property, it has received the name of percussion.

The centers of oscillation and percussion are generally treated separately; but the two centers are in the same point, and therefore their properties are the same.

As in bodies at rest, the whole weight may be considered as collected in the center of gravity, so in bodies in motion, the whole force may be considered as concentrated in the center of percussion; therefore, the weight of the rod multiplied by the distance of the center of gravity from the point of suspension, will be equal to the force of the rod divided by the distance of the center of percussion from the same point.

Example.—The length of a rod being 20 feet, and the weight of a foot in length equal to 100 oz.; also, a weight or ball, fixed at under end, weighing 1000 oz.: at what point of the rod, from the point of suspension, will be the center of percussion?*

The weight of the rod is $20\times100=2000$ oz., which, multiplied by half its length, $2000\times10=20000$, gives the momentum of the rod. The weight of the ball$=1000$ oz., multiplied by the length of rod, $=1000\times20$, gives the momentum of the ball. Now, the weight of the rod, multiplied by the square of the length, and divided by $3=\frac{2000\times20^2}{3}=266666$, =the force of the rod, and the weight of the ball, multiplied by the square of the length of the rod, $1000\times20^2=400000$, is the force of the ball: therefore, the center of percussion $=\frac{266666+400000}{20000+20000}=\frac{666666}{40000}=16{\cdot}66$ feet.

Example.—Suppose a rod 12 feet long, and 2 lbs. each foot in length, with 2 balls of 3 lbs. each, one fixed 6 feet from the point of suspension, and the other at the end of the rod, what is the distance between the points of suspension and percussion?

$12\times2\times6$	$=144$,	momentum of rod.	
3×6	$=18$,	do.	of 1st ball.
3×12	$=36$,	do.	of 2d do.
	198		

* $a=20$ feet long.
$b=100$ oz.; weight of a foot in length.
$c=1000$ oz.; fixed at end.

$$\frac{\frac{1}{3}ab\times a^2+ca^2}{\frac{1}{2}ab\times a+ac}=\text{center of percussion.}$$

$$\frac{24\times144}{3}=1152, \text{ force of rod.}$$
$$3\times\ 36=\ 108, \text{ do. of 1st ball.}$$
$$3\times144=\ \underline{432}, \text{ do. of 2d ball.}$$
$$1692.$$

Therefore, the center of percussion $=\frac{1692}{198}=8\cdot5\dot{4}\dot{5}$ ft. from the point of suspension.

As the center of percussion is the same with the center of oscillation, in the non-application to practical purposes, the following is the easiest and simplest mode of finding it in any beam, bar, &c.:

Suspend the body very freely by a fixed point, and make it vibrate in small arcs, counting the number of vibrations it makes in any time, as a minute, and let the number of vibrations made in a minute be called n; then shall the distance of the center of oscillation from the point of suspension be $\frac{140850}{n^2}=$inches. For the length of the pendulum, vibrating seconds, or 60 times in a minute, being $39\frac{1}{8}$ inches; and the lengths of the pendulums being reciprocally as the square of the number of vibrations made in the same time: therefore, $n^2 : 60^2 :: 39\frac{1}{8} : \frac{60^2\times39\frac{1}{8}}{n^2}=\frac{140850}{n^2}$ being the length of the pendulum, which vibrates n times in a minute, or the distance of the center of oscillation below the axis of motion.

There are many situations in which bodies are placed, that prevent the application of the above rules; and, for this reason, the following data are given, which will be found useful when the bodies and forms here given correspond:

1. If the body is a heavy straight line, of uniform density, and is suspended by one extremity, the distance of its center of percussion is $\frac{2}{3}$ of its length.

2. In a slender rod, of a cylindrical or prismatic shape, the breadth of which is very small, compared with its length, the distance of its center of percussion is nearly $\frac{2}{3}$ of its length from the axis of suspension. If these rods were formed so that all the points of their transverse sections were equi-distant from the axis of suspension, the distance of the center of percussion would be exactly $\frac{2}{3}$ of their length.

3. In an isosceles triangle, suspended by its apex, and vibrating in a plane perpendicular to itself, the distance of the center of percussion is $\frac{3}{4}$ of its altitude. A line, or rod, whose density varies as the distance from its extremity, or the point of suspension; also, *fly-wheels*, or *wheels in general*, have the same relation as the isosceles triangle; *i. e.* the center of percussion is distant from the center of suspension $\frac{3}{4}$ of its length.

4. In a very slender cone, or pyramid, vibrating about its apex, the distance of its center of percussion is nearly $\frac{4}{5}$ of its length.

The distance of either of these centers from the axis of motion is found thus:

If the axis of motion be in the vertex of the figure, and the motion be flatwise; then

In a right line, it is $=\frac{2}{3}$ of its length;
In an isosceles triangle, $=\frac{3}{4}$ of its height;
In a circle, $=\frac{5}{4}$ of its radius;
In a parabola, $=\frac{5}{7}$ of its height.

But if the bodies move sidewise, we have it

In a circle, $=\frac{3}{4}$ of the diameter;
In a rectangle, suspended by one angle $=\frac{2}{3}$ of the diagonal.
In a parabola, suspended by its vertex, $=\frac{5}{7}$ axis $+\frac{1}{3}$ perimeter; but if suspended by the middle of its base, $=\frac{4}{7}$ axis $+\frac{1}{2}$ perimeter.

In the sector of a circle, $=\dfrac{3\times \text{arc}\times \text{radius}}{4\times \text{chord}}$.

In a cone, $=\frac{4}{5}$ axis $+\dfrac{(\text{radius of base})^2}{5\times \text{axis}}$.

In a sphere, $=\dfrac{2\times \text{radius}^2}{5\,(d+\text{radius})}+\text{radius}+d$, where d is the length of the thread by which it is suspended.

We have given these rules for the sake of reference, but we shall illustrate, by examples, the most useful:

Examples.—1. What must be the length of a rod, without a weight, so that, when hung by one end, it shall vibrate seconds?

To vibrate seconds, the center of oscillation must be 39·1393 inches from that of suspension; hence, as this must be $\frac{2}{3}$ of the rod, 2 : 3 : : 39·1393 : 58·7089 inches, = the length of the rod.

2. What is the center of percussion of a rod 46 inches long

$$\tfrac{2}{3}\times 46=30\tfrac{2}{3} \text{ inches from the axis of motion.}$$

3. In an isosceles triangle, suspended by one angle, and oscillating flatwise, the height is 24 feet: what is the distance of the center of percussion from the axis of motion?

$$\tfrac{3}{4}\times 24=18 \text{ feet.}$$

4. In a sphere, the diameter is 14, and the string by which the sphere is suspended is 20 inches: where is the center of percussion or oscillation?

$$\frac{2\times 5^2}{5\,(20+5)}+5+20=\frac{50}{125}+25=25{\cdot}4. \quad \textbf{Ans.}$$

THE CENTER OF GYRATION.

The center of gyration is that point in any revolving body, or system of bodies, that if the whole quantity of matter were collected in it, the angular velocity would be the same; *i. e.*, the momentum of the body, or system of bodies, is centered at this point.

To find the Center of Gyration.

Rules.—1. Multiply the weight of the several particles by the squares of their distances in feet, from the center of motion, and divide the sum of the products by the weight of the whole mass; the square root of the quotient will be the distance of the center of gyration from the center of motion.

Example.—Suppose 3 weights of 3 lbs., 4 lbs., and 5 lbs., respectively, be fixed on a lever, which is assumed to be without weight, at the respective distances of 1, 2, and 3 feet: required the distance of the center of gyration from the center of motion.

$$3 \text{ lbs.} \times 1^2 = 3;\ 4 \text{ lbs.} \times 2^2 = 16;\ \text{and } 5 \text{ lbs.} \times 3^2 = 45.$$

Hence, $\frac{3+16+45}{3+\ 4+5} = 5{\cdot}33$; and $\sqrt{5{\cdot}33} = 2{\cdot}3$ feet distance from the center of motion.

Therefore, a single weight of 12 lbs., placed at 2·3 feet from the center of motion, and revolving in the same time, would have the same impetus as the 3 weights in their respective places.

2. Multiply the distance of the center of oscillation from the center of the system, or point of suspension, by the distance of the center of gravity from the same point; the square root of the product will be the center of gyration.

Example.—The center of gravity being 4 feet from the point of suspension, and that of oscillation 9 feet: required at what distance the center of gyration is from the point of suspension.

$$4 \times 9 = 36;\ \text{and } \sqrt{36} = 6 \text{ feet. Ans.}$$

Mr. Farey has given the following as the distances of the centers of gyration from the center of motion, in different revolving bodies:

In a straight uniform rod, revolving about 1 end, the length of the rod $\times$ by ·5773.

In a circular plate, revolving on its center, the radius of the circle $\times$ ·7071.

In a circular plate, revolving about 1 of its diameters as an axis, the radius $\times$ ·5.

In a thin circular ring, revolving about 1 of its diameters as an axis, the radius $\times$ ·7071.

In a solid sphere, revolving about 1 of its diameters as an axis, the radius $\times$ ·6325.

In a thin hollow sphere, revolving about 1 of its diameters as an axis, the radius $\times$ ·8164.

In a cone, revolving about its axis, the radius of the circular base $\times$ ·5477.

In a right-angled cone, revolving about its vertex, the height of the cone $\times$ ·866.

In a paraboloid, revolving about its axis, the radius of the circular base $\times$ ·5773.

In a straight lever, the arms being R and r, the distance of the center of gyration from the center of motion $=\sqrt{\dfrac{R^3+r^3}{3(R-r)}}$.

NOTE. The weight of the revolving body, multiplied into the height due to the velocity with which the center of gyration moves in its circle, is the energy of the body, or the mechanical power which must be communicated to it to give it that motion.

CENTRAL FORCES.

All bodies moving round a central point, have a tendency to fly off in a straight line: this tendency is called the centrifugal force: it is opposed to the centripetal force, or that power which maintains the body in its curvilinear path.

The centrifugal force of a body, moving with different velocities, in the same circle, is proportional to the square of the velocity, or to the square of the number of revolutions performed in a given time. Thus, the centrifugal force of a body making 40 revolutions per minute, is 4 times as great as the centrifugal force of the same body when making 20 revolutions per minute.

To find the Centrifugal Force of any Body.

Rules.—1. Divide the velocity in feet, per second, by 4·01, and the square of the quotient by the diameter of the circle; the quotient is the centrifugal force when the weight of the body is 1. Hence, the quotient multiplied by the weight of the body, is the centrifugal force.

Example.—Required the centrifugal force of the rim of a fly-wheel, 20 feet in diameter, moving with the velocity of $32\frac{1}{6}$ feet in a second.

$32\frac{1}{6} \div 4{\cdot}01 = 8{\cdot}02$; $8{\cdot}02^2 \div 20 = 3{\cdot}216$ times the weight of the rim.

2. Multiply the square of the number of revolutions in a minute by the diameter of the circle in feet, and divide the product by the constant number 5870; the quotient is the centrifugal force when the weight of the body is 1. Hence, as in the 1st rule, the quotient multiplied by the weight of the body, is the centrifugal force.

Example.—Required the centrifugal force of a stone, weighing 2 lbs., revolving in a circle 4 feet in diameter, at the rate of 120 revolutions in a minute.

$$120^2 \times 4 = 57600\text{; and } \frac{57600}{5870} = 9{\cdot}81.$$

Hence, $9{\cdot}81 \times 2 = 19{\cdot}62$ centrifugal force

Dr. Brewster has summed up the whole doctrine of centrifugal forces in the following propositions:

1. The centrifugal forces of two unequal bodies, moving with the same velocity, and at the same distance from the central body, are, to one another, as the respective quantities of matter in the two bodies.

2. The centrifugal forces of two equal bodies, which perform their revolutions round the central body in the same time, but at different distances from it, are to one another as their respective distances from the central body.

3. The centrifugal forces of two bodies which perform their revolutions in the same time, and whose quantities of matter are inversely as their distances from the center, are equal to one another.

4. The centrifugal forces of two equal bodies, moving at equal distances from the central body, but with different velocities, are to one another as the squares of their velocities.

5. The centrifugal forces of two unequal bodies, moving at equal distances from the center, with different velocities, are to one another in the compound ratio of their quantities of matter, and the squares of their velocities.

6. The centrifugal forces of two equal bodies, moving with equal velocities, at different distances from the center, are inversely as their distances from the center.

7. The centrifugal forces of two unequal bodies, moving with equal velocities, at different distances from the center, are to one another as their quantities of matter multiplied by their respective distances from the center.

8. The centrifugal forces of two unequal bodies, moving with unequal velocities, at different distances from the central body, are in the compound ratio of their quantities of matter, the squares of their velocities, and their distances from the center.

FRICTION, OR RESISTANCE TO MOTION,

In bodies Rolling or Rubbing on each other.

The greater part of all that is known, with precision, respecting the laws and properties which govern *friction* is founded upon experiments instituted on a large scale, and submitted to a great variety of trials, by some of the most eminent philosophers of the last century. M. Coloumb, member of the Academy of Science, at Paris, and Professor Vince, of the University of Cambridge, have written the most elaborately of any; both uniformly maintaining that *friction does not increase with the increase of rubbing surfaces; or, in other words, however the magnitude of the surface of contact may vary, the friction will remain the same, so long as the pressure is unchanged.*

Friction supposes one body moving, or tending to move upon the surface of another. There are *three* species of friction; or, to speak more properly, three ways, in which one surface can move upon another, in each of which friction acts differently.

1. When one body slides upon the plane surface of another body.

2. When one body, being cylindrical, rolls upon the surface of another body.

3. When a solid cylinder is inserted in a hollow cylinder of a greater diameter, and, being pressed in any direction, with a certain force, revolves within it.

Coloumb also satisfactorily established, by repeated experiments, all of which were confirmed by the experiments of *Ximenes*, that, under the same circumstances, *the friction of one surface moving upon another, is in exact proportion to the pressure with which the surfaces are urged together*. They found that when a block of wood, or any substance, has several faces of different magnitudes, the friction will be the same on whatever face it is placed, *except* in an *extreme case*, where they found a slight deviation from this law ; as when the *pressures* used were *extremely intense*, it was found that the friction did not increase in quite so fast a proportion as the pressure. The deviation from the law, however, was so very inconsiderable, and happened only in such extreme cases, that it might, for the most part, be neglected.

2. When one cylinder rolls upon the surface of another body, the friction is proportional to the pressure ; while, with cylinders of the same substance, having *different diameters*, but *equal pressures*, the friction is *inversely* as the diameters. Again, cylinders of the same substance, differing both in diameter and pressure, the friction is *directly* as the pressure, and *inversely* as the diameters, or in a ratio compounded of the direct ratio of the pressure, and the inverse ratio of the diameters.

3. When a solid cylinder is inserted in a hollow cylinder of a greater diameter, without rolling. If the hollow cylinder, or box, be supposed to revolve around the *axle*, as happens in the case of a carriage-wheel, every part of the surface of the box will successively be exposed to the effect of friction, while no part of the axle will suffer this effect, except the side which comes in contact with the box.

Let F represent the friction ;
W, " the force necessary to overcome friction ;
BI, " the radius of the cylinder which turns upon the axle ;
EI, " the radius of the wheel :

Then, by the established properties of the lever, we have

$$F : W :: EI : BI ;$$

$$\text{Or,} \quad F = W\frac{EI}{BI} ;$$

that is, the friction is equal to the weight or force that overcomes friction and produces motion, multiplied by the radius of the wheel, and divided by the radius of the hollow cylinder, which plays upon the axle. Thus it appears that the friction is greater than the preponderating weight in the proportion of the radius of the wheel to the radius of the cylinder.

In the years 1831, 1832, and 1833, a very extensive set of experiments were made at Metz, by M. Morin, under the sanction of the French government, to determine as near as possible the laws of friction; and by which the following were fully established:

1. When no unguent is interposed, the friction of any two surfaces (whether of quiescence or of motion), is directly proportional to the force with which they are pressed perpendicularly together; so that for any two given surfaces of contact, there is a constant ratio of the friction to the perpendicular pressure of the one surface upon the other. Whilst this ratio is thus the same for the same surfaces of contact, it is different for different surfaces of contact. The particular value of it in respect to any two given surfaces of contact, is called the co-efficient of friction in respect to those surfaces.

2. When no unguent is interposed, the amount of the friction is, in every case, wholly independent of the extent of the surfaces of contact; so that the force with which two surfaces are pressed together being the same, their friction is the same, whatever may be the extent of their surfaces of contact.

3. That the friction of motion is wholly independent of the velocity of the motion.

4. That where unguents are interposed, the co-efficient of friction depends upon the nature of the unguent, and upon the greater or less abundance of the supply. In respect to the supply of the unguent, there are two extreme cases, that in which the surfaces of contact are but slightly rubbed with the unctuous matter, as, for instance, with an oiled or greasy cloth, and that in which a continuous stratum of unguent remains continually interposed between the moving surfaces; and in this state the amount of friction is found to be dependent rather upon the nature of the unguent than upon that of the surfaces of contact. M. Morin found that with unguents, hog's lard and olive oil, interposed in a continuous stratum between surfaces of wood on metal, wood on wood, metal on wood, and metal on metal, when in motion, have all of them very near the same co-efficient of friction, being in all cases included between ·07 and ·08.

The co-efficient for the unguent tallow is the same, except in that of metals upon metals. This unguent appears to be less suited for metalic substances than the others, and gives for the mean value of its co-efficient, under the same circumstances, ·10. Hence, it is evident, that where the extent of the surface sustaining a given pressure is so great as to make the pressure less than that which corresponds to a state of perfect separation, this greater extent of surface tends to increase the friction by reason of that adhesiveness of the unguent, dependent upon its greater or less viscosity, whose effect is proportional to the extent of the surfaces between which it is interposed.

Mr. G. Rennie found, from a mean of experiments with different unguents on axles in motion, and under different pressures, that, with the unguent tallow, under a pressure of from 1 to 5 cwt., the friction did not exceed $\frac{1}{39}$th of the whole pressure; when soft soap was applied, it became $\frac{1}{34}$th; and, with the softer unguents applied,

such as oil, hog's lard, &c., the ratio of the friction to the pressure increased; but with the harder unguents, as soft soap, tallow, and anti-attrition composition, the friction considerably diminished; consequently, to render an unguent of proper efficiency, the nature of the unguent must be measured by the pressure or weight tending to force the surfaces together.

TABLE

Of the Results of Experiments on the Friction of Unctuous Surfaces

By M. Morin.

Surfaces of Contact.	Co-efficients of Friction.	
	Friction of Motion.	Friction of Quiescence.
Oak upon oak, the fibres being parallel to the motion - - - - - - - - - -	0·108	0·390
Ditto, the fibres of the moving body being perpendicular to the motion - - - -	0·143	0·314
Oak upon elm, fibres parallel - - - -	0·136	
Elm upon oak, do. - - - - - - - -	0·119	0·420
Beech upon oak, do. - - - - - - -	0·330	
Elm upon elm, do. - - - - - - - -	0·140	
Wrought iron upon elm, do. - - - -	0·138	
Ditto upon wrought iron, do. - - - -	0·177	
Ditto upon cast iron, do. - - - - - -	- -	0·118
Cast iron upon wrought iron, do. - - -	0·143	
Wrought iron upon brass, do. - - - -	0·160	
Brass upon wrought iron, do. - - - -	0·166	
Cast iron upon oak, do. - - - - - - -	0·107	0·100
Ditto upon elm do., the unguent being tallow	0·125	
Ditto, do., the unguent being hog's lard and black lead - - - - - - - - - -	0·137	
Elm upon cast iron - - - - - - - -	0·135	0·098
Cast iron upon cast iron - - - - - -	0·144	
Ditto upon brass - - - - - - - - -	0·132	
Brass upon cast iron - - - - - - - -	0·107	
Ditto upon brass - - - - - - - - -	0·134	0·164
Copper upon oak - - - - - - - - -	0·100	
Yellow copper upon cast iron - - - -	0·115	
Leather (ox-hide) well tanned upon cast iron, wetted - - - - - - - - - - -	0·229	0·267
Ditto upon brass, wetted - - - - - -	0·244	

In these experiments, the surfaces, after having been smeared with an unguent, were wiped, so that no interposing layer of the unguent prevented intimate contact.

TABLE

Of the Results of Experiments on Friction, with Unguents interposed.
By M. Morin.

Surfaces of Contact.	Co-efficients of Friction. Friction of Motion.	Friction of Quiescence.	Unguents.
Oak upon oak, fibres parallel	0·164	0·440	Dry soap.
Do. do. - - - -	0·075	0·164	Tallow.
Do. do. - - - -	0·067	- -	Hog's lard.
Do., fibres perpendicular - -	0·083	0·254	Tallow.
Do. do. - - - -	0·072	- -	Hog's lard.
Do. do. - - - -	0·250	- -	Water.
Do. upon elm, fibres parallel	0·136	- -	Dry soap.
Do. do. - - - -	0·073	0·178	Tallow.
Do. do. - - - -	0·066	- -	Hog's lard.
Do. upon cast iron - - - -	0·080	- -	Tallow.
Do. upon wrought iron - -	0·098	- -	Tallow.
Beech upon oak, fibres parallel	0·055	- -	Tallow.
Elm upon oak, do. - - - -	0·137	0·411	Dry soap.
Do. do. - - - -	0·070	0·142	Tallow.
Do. do. - - - -	0·070	- -	Hog's lard.
Elm upon elm, do. - - - -	0·139	0·217	Dry soap.
Do. upon cast iron - - - -	0·066	- -	Tallow.
Wrought iron upon oak, fibres parallel - - - - - - - -	0·256	0·649	Greased and saturated with water.
Do. do. - - - -	0·214	- -	Dry soap.
Do. do. - - - -	0·085	0·108	Tallow.
Do. upon elm, do. - - - -	0·078	- -	Tallow.
Do. do. - - - -	0·076	- -	Hog's lard.
Do. do. - - - -	0·055	- -	Olive oil.
Do. upon cast iron, do. - -	0·103	- -	Tallow.
Do. do. - - - -	0·076	- -	Hog's lard.
Do. do. - - - -	0·066	0·100	Olive oil.
Do. upon wrought iron, do. -	0·082	- -	Tallow.
Do. do. - - - -	0·081	- -	Hog's lard.
Do. do. - - - -	0·070	0·115	Olive oil.
Wrought iron upon brass, do.	0·103	- -	Tallow.
Do. do. - - - -	0·075	- -	Hog's lard.
Do. do. - - - -	0·078	- -	Olive oil.
Cast iron upon oak, do. - -	0·189	- -	Dry soap.
Do. do. - - - -	0·218	0·646	Greased and saturated with water.
Do. do. - - - -	0·078	0·100	Tallow.
Do. do. - - - -	0·075	- -	Hog's lard.
Do. do. - - - -	0·075	0·100	Olive oil.
Do. upon elm, do. - - - -	0·077	- -	Tallow.

Surfaces of Contact.	Co-efficients of Friction.		Unguents.
	Friction of Motion.	Friction of Quiescence.	
Cast iron upon elm, fibres parallel - - - - - - - -	0·061	- -	Olive oil.
Do. do. - - - -	0·091	- -	Hog's lard and plumbago.
Do. upon wrought iron - -	- -	0·100	Tallow.
Do. upon cast iron - - - -	0·314	- -	Water.
Do. do. - - - - -	0·197	- -	Soap.
Do. do. - - - - -	0·100	0·100	Tallow.
Do. do. - - - - -	0·070	0·100	Hog's lard.
Do. do. - - - - -	0·064	- -	Olive oil.
Do. do. - - - - -	0·055	- -	Hog's lard and plumbago.
Do. upon brass - - - - -	0·103	- -	Tallow.
Do. do. - - - - -	0·075	- -	Hog's lard.
Do. do. - - - - -	0·078	- -	Olive oil.
Copper upon oak, fibres parallel - - - - - - - - -	0·069	0·100	Tallow.
Yellow copper upon cast iron	0·072	0·103	Tallow.
Do. do. - - - - -	0·068	- -	Hog's lard.
Do. do. - - - - -	0·066	- -	Olive oil.
Brass upon cast iron - - -	0·086	0·106	Tallow.
Do. do. - - - - -	0·077	- -	Olive oil.
Do. upon wrought iron - -	0·081	- -	Tallow.
Do. do. - - - - -	0·089	- -	Lard and plumbago.
Do. do. - - - - -	0·072	- -	Olive oil.
Brass upon brass - - - -	0·058	- -	Olive oil.
Steel upon cast iron - - -	0·105	0·108	Tallow.
Do. do. - - - - -	0·081	- -	Hog's lard.
Do. do. - - - - -	0·079	- -	Olive oil.
Do. upon wrought iron - -	0·093	- -	Tallow.
Do. do. - - - - -	0·076	- -	Hog's lard.
Do. upon brass - - - - -	0·056	- -	Tallow.
Do. do. - - - - -	0·053	- -	Olive oil.
Do. do. - - - - -	0·067	- -	Lard and plumbago.
Tanned ox-hide upon cast iron - - - - - - - - -	0·365	- -	Greased and saturated with water.

The extent of the surfaces in these experiments bore such a relation to the pressure, as to cause them to be separated from one another throughout, by an interposed stratum of the unguent.

TABLE

Of the Results of Experiments on the Friction of Gudgeons, or Axle-ends, in motion upon their bearings. By M. Morin.

Surfaces in Contact.	State of the Surfaces.	Co-efficient of Friction.
Cast iron axles in cast iron bearings,	Coated with oil of olives, with hog's lard, tallow, and soft gome - - - -	0·07 to 0·08
	With the same and water	0·08
	Coated with asphaltum -	0·054
	Greasy - - - - - - - -	0·14
	Greasy and wetted - - -	0·14
Cast iron axles in cast iron bearings,	Coated with oil of olives, with hog's lard, tallow, and soft gome - - - -	0·07 to 0·08
	Greasy - - - - - - - -	0·16
	Greasy and damped - -	0·16
	Scarcely greasy - - - -	0·19
Wrought iron axles in cast iron bearings,	Coated with oil of olives, tallow, hog's lard, or soft gome - - - - - - - -	0·07 to 0·08
Wrought iron axles in brass bearings,	Coated with oil of olives, hog's lard, or tallow - -	0·07 to 0·08
	Coated with hard gome -	0·09
	Greasy and wetted - - -	0·19
	Scarcely greasy - - - -	0·25
Iron axles in lignum-vitæ bearings,	Coated with oil or hog's lard - - - - - - - -	0·11
	Greasy - - - - - - - -	0·19
Brass axles in brass bearings,	Coated with oil - - - -	0·10
	With hog's lard - - - -	0·09

From a paper lately read at the Institution of Civil Engineers, in London, on the Comparative Friction of Steam Engines of different Modifications, it appears that, as respects the friction caused by the strain, if the beam-engine be taken as the standard of comparison,—

The vibrating engine - - - - has a gain of 1·1 per cent.
The direct-action engine with slides " loss of 1·8 "
Ditto, with rollers - - - - - - - " gain of 0·8 "
Ditto, with a parallel motion - - - " gain of 1·3 "

It also states, as an opinion, that excessive allowance for friction has hitherto been made in calculating the effective power of engines in general; as it is found practically by experiments with the engines at the Blackwall railway, and also with other engines, that, where the pressure upon the piston is about 12 lbs. per square inch, the friction does not amount to more than 1½ lbs.; and also that, by experiments with an indicator on an engine of 50-horse power, at Trueman, Hanbury & Co.'s brewery, the whole amount of friction did not exceed 5-horse power, or one-tenth of the whole power of the engine.

The following is from Adcock:

THE FRICTION OF PIVOTS.

In order to examine the friction of pivots, Mr. Coulumb made several experiments equally valuable with the preceding. When the angle of the summit of the pivot which supported the planes was about 18 or 20 degrees, and the planes highly polished, the friction for garnet was $\frac{1}{1088}$ to $\frac{1}{1050}$, agate $\frac{1}{884}$, rock crystal $\frac{1}{784}$, glass, $\frac{1}{570}$ to $\frac{1}{589}$, and steel, plane tempered and polished, $\frac{1}{510}$. Hence it appears that garnet is far superior to any other substance for the caps of pivots. At an angle of 45 degrees, the friction for garnet was $\frac{1}{2500}$, agate $\frac{1}{2100}$, glass $\frac{1}{1400}$, and steel $\frac{1}{2000}$; and at an angle of 6 or 7 degrees, agate $\frac{1}{800}$, glass $\frac{1}{450}$, and steel $\frac{1}{230}$. Hence, for a plate of agate, the friction of the pivot, at an angle of 45°, is $\frac{1}{2100}$, at an angle of 15°, $\frac{1}{1200}$, and at an angle of 6°, $\frac{1}{800}$; consequently, it appears, that in garnet, agate, and glass, the friction increases as the pivots become more acute, and follows nearly the same ratio. The case, however, is different with steel. In agate and steel the frictions upon a pivot of 45° are nearly equal, being $\frac{1}{2100}$ for agate, and $\frac{1}{2000}$ for steel; whereas on a pivot of 6° or 7°, the friction for agate is $\frac{1}{800}$, and $\frac{1}{230}$ for steel.

FRICTION AND RIGIDITY OF ROPES.

By some this is supposed to vary as the diameters, as the curvature, and as the tension.

Wet ropes, if small, are a little more flexible than dry; if large, a little less flexible.

Tarred ropes are stiffer by about $\frac{1}{6}$th, and in cold weather somewhat more so The stiffness of ropes increases after a little rest.

THE FLY-WHEEL.

It is an object of great importance in machines to have means of accumulating power when the moving force is in excess, and of expending it when the moving force operates more feebly, or the resistance increases. This equalization of motion is obtained by what is called the *Fly-wheel*, which is generally made in the form of a heavy wheel. The fly-wheel being made to revolve about its axis, keeps up its force by its own inertia, and distributes it in all parts of its revolution. Fly-wheels are capable of accumulating power to a great extent, and when thus accumulated, they assist in bringing the crank past its centers; but much of their efficacy depends on the position assigned them in the machinery. If the *fly* is used as a regulator of force, it should be placed near the prime mover; but if, on the other hand, it be used as a magazine of power, it should be near to the working point. No general rules can be given for its exact position.

To find the Weight of the Rim or Ring of a Fly-wheel proper for a Steam Engine.

Rule.—Multiply the constant number, 1368, by the number of horses' power that the engine is equal to; divide the product by the diameter of the wheel, in feet, multiplied by the number of revolutions per minute, and the quotient is the weight of the ring in 100 pounds, nearly.

Example.—Required the weight of the rim of a fly-wheel proper for an engine of 30-horse power, the wheel to be 14 feet diameter, and making 40 revolutions per minute.

$$\frac{1368\times30}{14\times40}=\frac{41040}{560}=64{\cdot}1 \text{ cwt.}$$

NOTE. The fly-wheel of an engine for a flour mill ought to be of such a diameter that the velocity of the periphery of the wheel may exceed the velocity of the periphery of the stones, to prevent, as much as possible, any tendency to back lash, as it is termed.

The necessary weight and diameter of the wheel being found, *suppose a breadth of a rim.* Then,

To find the Thickness necessary to make the Weight in Cast Iron.

Rule.—Divide the required weight in pounds by the area of the ring in inches, multiplied by ·261, and the quotient is the thickness of the ring in inches.

Example.—What thickness must a ring of a fly-wheel be to equal 64·1 cwt., when the outer diameter is 14 feet, and the inner diameter 12 feet?

64·1 cwt. = 6410 lbs.

Diam. 14 feet = 168· in. and 12 = 144 in., and by Prob. xiii. in Men suration of the Circle, the area of the ring = 5881 sq. in.

$$\text{Then } \frac{6410}{5881\times{\cdot}261}=\frac{6410}{153{\cdot}50}=4{\cdot}17 \text{ in., nearly.}$$

NOTE. If the ring is to be of a cylindrical form, its diameter may be easily found by the following approximate

Rule.—Multiply the required weight in pounds by 1·62; divide the product by the diameter of the wheel in inches, and the square root of the quotient will be the diameter of the cross section of the ring in inches, nearly.

$$\text{Thus } \sqrt{\frac{6410\times162}{14+12}}=7{\cdot}7 \text{ inches.}$$

NOTE. The center of percussion in a fly-wheel, or wheels in general, is ¾ distant from the center of suspension, nearly.

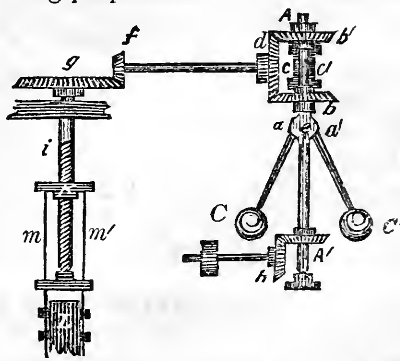
The centrifugal force is that power or tendency which all revolving bodies have to burst, or fly asunder in a direct line.

And the center of percussion in a revolving body is that point where the whole force or motion is collected, or that point which would strike any obstacle with the greatest effect. (*See Article on Percussion and Oscillation.*)

THE GOVERNOR.

The fly-wheel, already explained in the preceding article, is not the only regulator of force; and even in cases where it is used, we are sometimes obliged to have recourse also to other contrivances. In manufactories, it generally happens that there is one certain and determinate velocity with which the machinery should be moved, and which, if increased or diminished, would render the machine unfit to perform the work it was designed to execute. The application of a perfectly uniform power, aided by a fly-wheel, will not effect this.

Therefore, to maintain a uniform velocity with a varying resistance, one of the most beautiful and ingenious contrivances ever used is the *governor*; an instrument used in mill-works, but the appplication of which is most perspicuous in the steam engine, when that machine is applied to manufacturing purposes.



The principle on which the efficacy of this instrument depends, is easily explained. Let *AA'*, in the adjoining figure, be a vertical axis which is made to revolve by the bevel-wheel, *A'*, acted on or impelled by another bevel-wheel, *h*, so that it always revolves with a velocity proportioned to that of the fly-wheel.

Two heavy balls, *CC'*, are attached to metal rods, which work on pivots, at *aa'*, with shoulders that project into the upright axis, *e*, and become attached to a rod which works freely within the upright shaft, *AA'*. Now, by the revolution of the axis, *AA'*, the balls, *CC'*, acquire an obvious tendency to fly off from the axis, and this tendency is resisted by their weight; so that when the axis is revolving with a certain velocity, the balls will remain suspended. It follows that when the weights, *CC'*, diverge from the upright axis, the shoulders of the arms at *aa'*, will operate upon two clutches, *cc'*, by means of the fulcrums or shoulders of the arms, *aa'*, so as to raise or depress the clutches, *cc'*, by means of a sliding rod, *e*, attached to the shoulders of the metalic rods, *aC* *aC'*, so as to strike into the corresponding clutches in the bevel-wheels, *bb'*.

When the speed is such as to cause the arms to fall, the clutches *cc'*, become attached to the upper bevel-wheel, *b*, and this turns the horizontal bevel-wheel, *d*. This last-named wheel being connected to the bevel pinion, *f*, by means of a horizontal shaft, will turn another bevel-wheel, *g*, which is attached to a screw, *i*, causing the nut *K* to move upward. This will raise the sluice-gate, *L*, by means of metalic rods, *m m'*, extending from the nut, *K*, to the stem of the gate.

The impelling power is thus *increased* by an additional supply of water upon the wheel. Now, suppose an engine or wheel has been

working ten separate power presses, or runs of stone, and that five of them should be suddenly stopped. The engine or wheel will thus lose half their load, and would, if the same power of steam or water continued to be admitted, move with nearly twice their former velocity. But the moment the increased velocity is perceived in the machine, the balls, *CC′*, recede from the axis by their centrifugal force, and by the action of the shoulders of the arms, at *aa′*, upon the sliding rod, *e*, cause the clutches, *cc′*, to fall, and come in contact with the bevel-wheel, *b′*, which will obviously reverse the whole of the former operation, and check the supply of water or steam, by partially closing the gate or valve. The impelling power is thus diminished, and in exactly the same degree as the load, and the proper velocity will be thus restored. There are various other contrivances for regulating the motion of machinery, among which the pendulum of the clock may be mentioned, but none so universally used with success as the governor, above described.

(See Dr. Gregory's Mechanics.)

The revolutions per minute being given, to find the length of Pendulums or Arms of a Governor.

Rule.—Divide 375 by twice the number of revolutions per minute, and the square of the quotient will be the length required.

Example.—When the velocity of a governor is 38 revolutions per minute, what ought to be the length of the pendulums or arms?

$$375 \div 76 = 4{\cdot}93\text{; and } \overline{4{\cdot}93}^2 = 24{\cdot}3049 \text{ inches.}$$

EFFECTS OF HEAT.

TABLE,

Showing the Expansion of Water by Heat.

Temperature.	Expansion.	Temperature.	Expansion.	Temperature.	Expansion.
Deg. Fah.		Deg. Fah.		Deg. Fah.	
12	1·00236	82	1·00312	152	1·01934
22	1·00092	92	1·00477	162	1·02245
32	1·00022	102	1·00672	172	1·02575
42	1·00000	112	1·00880	182	1·02916
52	1·00021	122	1·01116	192	1·03265
62	1·00083	132	1·01367	202	1·03634
72	1·00180	142	1·01638	212	1·04012

TABLE,

Showing the Height of the Boiling Point, Fahrenheit's Thermometer, at different Heights of the Barometer.

Barometer.	Boiling Point.	Barometer.	Boiling Point.
Inches.	Degrees.	Inches.	Degrees.
31	213·57	28½	209·55
30½	212·79	28	208·69
30	212·00	27½	207·84
29½	211·20	27	206·96
29	210·38		

NOTE. In a vacuum, water boils at 98° to 100°, according as the vacuum is more or less perfect.

TABLE,

Showing the Progressive Dilatation of Metals by Heat, the Length at 62° Fahrenheit being 1000000.

Name of Metal.	At 212° Fahrenheit.	At 662° Fahrenheit.	Fusing Point.
Platinum - - -	1000735	1002995	1009926
Iron, wrought - -	1000984	1004483	1018378
Iron, cast - - -	1000893	1003943	1016389
Gold - - - - -	1001025	1004238	
Copper - - - -	1001430	1006347	1024376
Silver - - - -	1001626	1006886	1020640
Zink - - - - -	1002480	1008527	1012621
Lead - - - - -	1002323		1009072
Tin - - - - -	1001472		1003798

TABLE,

Showing the Power of Metals for conducting Heat.

Gold - - - - -	1000	Iron - - - - -	374·3
Silver - - - -	973	Zink - - - - -	363
Copper - - - -	898·2	Tin - - - - -	303·9
Platinum - - -	381	Lead - - - -	179·6

TABLE

Of Specific Heats.

Water - - -	1·0000	Carbonic acid -	0·2210
Hydrogen gas -	3·2936	Charcoal - - -	0·2631
Aqueous vapor	0·8470	Sulphur - - -	0·1850
Alcohol - - -	0·7000	Iron - - - -	0·1138
Ether - - -	0·6600	Zink - - - -	0·0955
Oil - - - -	0·5200	Mercury - - -	0·0332
Nitrogen gas -	0·2754	Platinum - - -	0·0324
Air - - - -	0·2669	Gold - - - -	0·0298
Oxygen - - -	0·2361		

PROGRESSIVE SPECIFIC HEAT.

	Between 32° and 212°.	Between 32° and 572°.
Of Mercury - -	0·0330	0·0350
Zink - - -	0·0927	0·1015
Antimony - -	0·0507	0·0547
Silver - - -	0·0557	0·0611
Copper - - -	0·0949	0·1013
Platinum - -	0·0335	0·0355
Glass - - -	0·1770	0·1900

SPECIFIC HEAT OF IRON.

From 32° to	212° - -	0·1098
—	392 - -	0·1150
—	572 - -	0·1218
—	662 - -	0·1255

TABLE

Of Latent Heats.

LIQUIDS.			
Of Water - -	140° F.	Of Bees' wax -	175° F.
Sulphur - -	143·7	Zink - - -	494
Spermaceti	145	Tin - - -	500
Lead - - -	162	Bismuth - -	550

VAPORS.	
Vapor of Water, at 212° F. -	1000
— Alcohol - - - -	457
— Ether - - - - -	312·9
— Oil of turpentine -	183·8
— Nitric acid - - -	550
— Ammonia - - - -	865·9
— Vinegar - - - -	903

NOTE. To find the latent heat of steam, or vapor of water, at any degree of temperature: subtract the sensible heat from the constant quantity, 1212, and the remainder is the latent heat.

One pound of steam will raise 3657 cubic feet of air 10°, and cause it to expand from 32° to 42°, about 3733 cubic feet.

The heat that would raise 1 pound of water 1°, would raise a pound of air 3°·7; 1 pound of air=about 11 cubic feet.

Expansion of Liquids in Volume, from 32° *to* 212° *Fahrenheit.*

1000 parts of	water - - -	become	1046
—	oil - - - - -	—	1080
—	mercury - -	—	1018
—	spirits of wine	—	1110
—	air - - - - -	—	1373

TABLE

Of Metals in their Order of Ductility.

	Wire-drawing Ductility.	Laminable Ductility.
1	Gold.	Gold.
2	Silver.	Silver.
3	Platinum.	Copper.
4	Iron, wrought.	Tin.
5	Copper.	Platinum.
6	Zink.	Lead.
7	Tin.	Zink.
8	Lead.	Iron, wrought.

The annexed table, by Mr. Gilbert, shows at one view those data which may be more implicitly relied upon.

TABLE,

Showing the Quantity of Heat producable from different kinds of Fuel.

Different kinds of Fuel experimented on.		Pounds of Water heated 1° by 1 pound of Fuel.	Pounds of Water at 52° converted into Steam at 220°.	Pounds of Fuel to transform a cubic foot of Water 52° into Steam 220°.
Newcastle or Swansea coal, according to Mr. Watt, ...	from	6950	5·93	10·5
	to	10400	8·9	7·0
	mean	8675	7·4	8·75
Newcastle, according to Dr. Black, .		9230	7·9	7·9
Ditto, Wall's-end coal, Tredgold, . .		10050	8·6	7·25
Wednesbury coal, according to Mr. Watt, ...	from	5200	4·45	14·0
	to	7800	6·68	9·34
	mean	6500	5·56	11·67
Pine wood, (dry,) Count Rumford, . .		3618	3·1	20·02
Oak wood, (dry,) ditto, ...		5662	4·85	12·9
Compact peat from Dartmore, Tredgold, ...		2400	2·05	30·5
Culm, Glasgow, ditto, ...		3330	2.85	22·0
Culm, Welch, ditto, ...		4175	3·56	17·5

According to Mr. Gilbert, who experimented largely on this subject in the mining districts of Cornwall, seven pounds of coal will convert one cubic foot of water into steam; or, what amounts to the same thing, one bushel of coals will convert fourteen cubic feet of water from the ordinary temperature into steam.

TABLE,

Showing the Results of Mr. Bull's Experiments.

Name of Wood.	Weight of a Cord.	Comparative Value per Cord.
	lbs.	lbs.
Red Oak, ...	3254	69
White Oak, ...	3821	81
Red-heart Hickory, ...	3705	81
Shell-bark Hickory, ...	4469	100
White Pine, ...	1868	42
Yellow Pine, ...	1904	43
Jersey Pine, ...	2137	54
Hard Maple, ...	2878	60

COMBUSTION.

Combustion denotes the combination of a body with any of the substances termed supporters of combustion. With reference to the generation of steam, we are restricted to but one of these combinations, and that is oxygen.

With equal weights of *fuel*, that which contains most hydrogen ought, in its combustion, to produce the greatest quantity of heat, where each kind is exposed under the most advantageous circumstances. For instance, pine wood is preferable to hard wood, and bituminous coal to anthracite coal.

Dr. Ure gives the subjoined numbers as representing, in degrees of Fahrenheit's thermometer, the insensible heats of the corresponding vapors.

Vapor of Water at 212, . . .	967°
Alcohol, ...	442
Ether, ...	302·379
Petroleum, ...	177·87
Turpentine, oil of, ...	177·87
Nitric acid, ...	531·99
Liquid ammonia, ...	837·28
Vinegar, ...	875

The following Table shows the Power of various Species of Fuel.

Species of Fuel.	Effect in lbs. of water heated one degree by one lb. of fuel.	Effect in lbs. of water converted into steam of 220°.	Quantity to convert a cubic foot of water into low pressure steam.	Quantity to convert a cubic foot of water into steam, allowing 10 per cent. for loss.
	lbs.	lbs.	lbs.	lbs.
Caking coal, . . .	9800	8·4	7·45	8·22
Coke, ...	9000	7·7	8·1	9·00
Splint coal, ...	7900	6·75	9·25	10·28
Oak wood, dry, . .	6000	5·13	12·2	13·6
Ordinary oak, . . .	3600	3·07	20·31	22·6
Peat compact, of ordinary dryness, .	3250	2·8	22·5	25·0

TABLE

Of the Elastic Force of Steam, and Corresponding Temperature of the Water with which it is in Contact.

Pressure on a Square Inch.*	Elastic Force in inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.	Pressure on a Square Inch.	Elastic Force in inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.
lbs.				lbs.			
14·7	30·00	212·0	1700	54	110·16	288·1	516
15	30·60	212·8	1669	55	112·20	289·3	508
16	32·64	216·3	1573	56	114·24	290·5	500
17	34·68	219·6	1488	57	116·28	291·7	492
18	36·72	222·7	1411	58	118·32	292·9	484
19	38·76	225·6	1343	59	120·36	294·2	477
20	40·80	228·5	1281	60	122·40	295·6	470
21	42·84	231·2	1225	61	124·44	296·9	463
22	44·88	233·8	1174	62	126·48	298·1	456
23	46·92	236·3	1127	63	128·52	299·2	449
24	48·96	238·7	1084	64	130·56	300·3	443
25	51·00	241·0	1044	65	132·60	301·3	437
26	53·04	243·3	1007	66	134·64	302·4	431
27	55·08	245·5	973	67	136·68	303·4	425
28	57·12	247·6	941	68	138·72	304·4	419
29	59·16	249·6	911	69	140·76	305·4	414
30	61·21	251·6	883	70	142·80	306·4	408
31	63·24	253·6	857	71	144·84	307·4	403
32	65·28	255·5	833	72	146·88	308·4	398
33	67·32	257·3	810	73	148·92	309·3	393
34	69·36	259·1	788	74	150·96	310·3	388
35	71·40	260·9	767	75	153·02	311·2	383
36	73·44	262·6	748	76	155·06	312 2	379
37	75·48	264·3	729	77	157·10	313·1	374
38	77·52	265·9	712	78	159·14	314·0	370
39	79·56	267·5	695	79	161·18	314·9	366
40	81·60	269·1	679	80	163·22	315·8	362
41	83·64	270·6	664	81	165·26	316·7	358
42	85·68	272·1	649	82	167·30	317·6	354
43	87·72	273·6	635	83	169·34	318·4	350
44	89·76	275·0	622	84	171·38	319·3	346
45	91·80	276·4	610	85	173·42	320·1	342
46	93·84	277·8	598	86	175·46	321·0	339
47	95·88	279·2	586	87	177·50	321·8	335
48	97·92	280·5	575	88	179·54	322·6	332
49	99·96	281·9	564	89	181·58	323·5	328
50	102·00	283·2	554	90	183·62	324·3	325
51	104·04	284·4	544	91	185·66	325·1	322
52	106·08	285·7	534	92	187·70	325·9	319
53	108·12	286·9	525	93	189·74	326·7	316

* This includes the pressure of the atmosphere.

Pressure on a Square Inch.	Elastic Force in inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.	Pressure on a Square Inch.	Elastic Force in inches of Mercury.	Temperature in degrees of Fahrenheit.	Volume of Steam compared with the Volume of Water.
lbs.				lbs.			
94	191·78	327·5	313	130	265·23	352·1	233
95	193·82	328·2	310	140	285·61	357·9	218
96	195·86	329·0	307	150	306·03	363·4	205
97	197·90	329·8	304	160	326·42	368·7	193
98	199·92	330·5	301	170	346·80	373·6	183
99	201·96	331·3	298	180	367·25	378·4	174
100	204·01	332·0	295	190	387·61	382·9	166
110	224·40	339·2	271	200	408·04	387·3	158
120	244·82	345·8	251				

TABLE

Of the Force and Temperature of Steam in Atmospheres.

Atmos.	Temp. Fah.	Atmos.	Temp. Fah.	Atmos.	Temp. Fah.
	Deg.		Deg.		Deg.
1	212·00	10	358·88	19	413·78
2	250·52	11	366·85	20	418·46
3	275·18	12	374·00	21	422·96
6	293·72	13	380·66	22	427·28
4	307·50	14	386·94	23	431·42
5	320·36	15	392·86	24	435·56
7	331·70	16	398·48	25	439·34
8	341·78	17	403·82		
9	350·78	18	408·92	50	510·60

TABLE

Of the Heating Power of various Combustible Substances, exhibiting the utmost Quantity of Water evaporated by the Given Weights, and the smallest Quantity of Air capable of producing Total Combustion.—Dr. Ure.

Species of Combustible.	Pounds of Water which a Pound can heat, from 0° to 212°.	Pounds of Boiling Water, evaporated by 1 Pound.	Weight of Atmospheric air at 32°, to burn 1 Pound.
Wood, in its ordinary state, -	26	4·72	4·47
Wood charcoal, - - - - - -	73	13·37	11·46
Pit coal, - - - - - - - -	60	10·90	9·26
Coke, - - - - - - - - -	65	11·81	11·46
Turf, - - - - - - - - -	30	5·45	4·60
Turf charcoal, - - - - - -	64	11·63	9·86
Carburetted hydrogen, - - -	76	13·81	14·58
Oil, Wax, Tallow, - - - - - - -	78	14·18	15·00
Alcohol of commerce, - - -	52	9·56	11·60

The best proportions of capacity and effective heating surface for steam engine boilers, according to Mr. Armstrong, are the following:

$$\left.\begin{array}{l}\text{1 cubic yard capacity,}\\ \text{1 square yard heating surface,}\\ \text{1 square foot of furnace bar,}\end{array}\right\}\text{ to each horse-power.}$$

He also gives the following formula for finding the horse-power, the area of fire grate, and the area of effective heating surface, so as to approximate the same in effect, when not of similar proportions:

$$\begin{array}{ll}\text{Horse power,} & =\sqrt{SF},\\ \text{Fire grate,} & =P^2\div S,\\ \text{Heating surface,} & =P^2\div F;\end{array}$$

where S represents the extent in square yards of effective heating surface; F, the area of fire grate, in square feet; and P, the horse-power that the boiler will be equal to.

Example.—Suppose a boiler to have 16 square yards of effective heating surface, and 25 square feet of furnace bar:

$$\sqrt{25\times 16}=20\text{-horse power;}$$
$$\text{or, } 20^2\div 25=16 \text{ square yards of heating surface;}$$
$$\text{and } 20^2\div 16=25 \text{ square feet of fire grate.}$$

For Cylindrical Boilers.

Multiply the number of square feet of heating surface to a horse-power, by the number of horses' power, and divide the product by 1·57 times the diameter; the quotient equals the length of the boiler in feet.

FORM AND DIRECTION OF STEAM-PIPES.

Enlargements in steam pipes, succeeded by contractions, tend more or less to retard the velocity of the steam, according to the nature of the contraction; and the like effect is produced by bends and angles; they should, therefore, be made as straight, and their internal surface as uniform and free from inequalities as may be practicable. Mr. Tredgold gives the following proportions of velocity for steam, passing by different passages or apertures:

The velocity of motion that would result from the direct, unretarded action of the column of fluid which produces it, being unity, - - - - - -	=1000, or 8·
The velocity through a conical tube, or mouth-piece, of the form of the contracted vein, - - - - -	=·983, or 7·9.
The velocity through a tube from two to three diameters in length, projecting outwards, - - - - -	=·813, or 6·5.
The velocity through a tube of the same length, projecting inwards, - - - - - - - - - - - - -	=·681, or 5·45.
The velocity through an aperture in a thin plate, by the same pressure, - - - - - - - - - - - -	=·625, or 5·

UNIT OF NOMINAL POWER FOR STEAM ENGINES.

The usual estimate of the dynamical effect per minute of a horse called by engineers a horse-power, is 33,000 lbs., at a velocity of 1 foot per minute; or, the effect of a load of 200 lbs., raised by a horse for 8 hours a day, at the rate of $2\frac{1}{2}$ miles per hour; or 150 lbs. at the rate of 220 feet per minute.

To determine the Diameter of a Cylinder for an Engine of a required Nominal Power.

Divide 5500 by the velocity of the piston, in feet, per minute, and the quotient equals the number of square inches to a horse-power which multiply by the required number of horses' power, and the product is the cylinder's area, against which, in the Table of Areas (pp. 91, &c.), is the diameter required.

Example.—Required the diameter of the cylinder for a 25-horse engine, with a velocity of 230 feet per minute:

$\frac{5500}{230}=23{\cdot}913\times25=597{\cdot}825$ inches area; or $27\frac{5}{8}$ inches diameter nearly.

Proportionate Velocities for the Pistons of Stationary Condensing Engines.

Length of Stroke, in Feet.	Velocity in Feet, per Minute.	Number of Revolutions, per Minute.
8	256	16
7	245	$17\frac{1}{2}$
6	240	20
$5\frac{1}{2}$	$236\frac{1}{2}$	$21\frac{1}{2}$
5	230	23
$4\frac{1}{2}$	$220\frac{1}{2}$	$24\frac{1}{2}$
4	214	$26\frac{3}{4}$
$3\frac{1}{2}$	203	29
3	192	32
$2\frac{1}{2}$	$177\frac{1}{2}$	$35\frac{1}{2}$
2	160	40

To estimate the Amount of Effective Power of an Engine by an Indicator.

Multiply the area of the piston in square inches, by the average force of the steam, in lbs., and by the velocity of the piston, in feet per minute; divide the product by 33,000, and $\frac{7}{10}$ths of the quotient equal the effective power.

Example.—Suppose an engine, with a cylinder of $37\frac{1}{2}$ inches diameter, a stroke of 7 feet, and making 17 revolutions per minute, or 238 velocity, and the average indicated pressure of the steam, 16·73 lbs. per square inch: required the effective power.

$$\text{Area}=\frac{1104{\cdot}4687\text{ inches}\times16{\cdot}73\text{ lbs.}\times238\text{ feet}}{33000}=\frac{133{\cdot}26\times7}{10}=$$

93·282 horses' power.

To find the Greatest Quantity of Water required for Steam.

Multiply the area of the cylinder, in feet, by the piston's velocity, in feet, per minute, add $\frac{1}{10}$th for cooling and waste, divide the sum by the volume of steam, compared to the volume of water (as per Table of Pressures), and the quotient equals the quantity, in cubic feet, per minute.

NOTE.—For single-acting engines only about half the quantity is required.

Example.—Required the quantity of water for steam to supply an engine, whose cylinder is 2 feet diameter, the piston's velocity 214 feet per minute, and the pressure of steam 18 lbs. per square inch, including the pressure of the atmosphere.

Area $= 3{\cdot}1416 \times 214 = 672{\cdot}3 + 67{\cdot}23 = 739{\cdot}53 \div 1411 = {\cdot}523$ cubic feet per minute.

To find the Quantity of Water required for Injection.

From 1212, subtract the temperature of the condensed water; divide the result by the temperature of the condensed water, minus the temperature of the cold water; multiply the quotient by the quantity of steam, in cubic feet, in a given time, and the product equals the quantity of water, in cubic inches.

Example.—Required the quantity of water, at 60°, to condense 500 cubic feet of steam to water, at 95° Fahrenheit.

$$\frac{1212-95}{95-60} = 31{\cdot}9 \times 500 = 15950 \text{ cubic inches;}$$
$$\text{or, } 15950 \times {\cdot}00058 = 9{\cdot}251 \text{ cubic feet.}$$

Proportions of Cylinder, Condenser, and Air-pump.

The length of the cylinder, and, consequently, the stroke of the piston, ought not to be less than twice the cylinder's diameter; as then the least surface is exposed, in proportion to the capacity; and the longer the stroke, the greater the effect, from the principle of expansive force.

The capacities of the condenser and air-pump, are each $\frac{1}{8}$th the capacity of the cylinder.

Advantages derived from the Expansive Properties of Steam.

If steam of a uniform elastic force be employed throughout the whole ascent or descent of the piston of a steam engine, the amount of effect is as the quantity expended. But suppose the steam to be shut off at any portion of the stroke, say, for instance, at one half, it expands, by degrees, until the termination of the stroke, and then exerts half its original force; hence an accumulation of effect in proportion to the quantity of steam.

To obtain or calculate the Amount of Effect.

Divide the length of the stroke by the length of the space into which the dense steam is admitted, and find the hyperbolic loga

rithm of the quotient, to which add 1·, and the sum is the ratio of the gain.

Example.—Suppose an engine, with a stroke of 6 feet, and the steam cut off when the piston has moved through 2: required the ratio of uniform, elastic force.

6÷2=3; hyperbolic logarithm of 3=1·0986+1=2·0986, ratio of effect: that is, supposing the whole effect of the steam to be 3·, the effect by the steam being cut off at $\frac{1}{3}$=2·0986.

TABLE

Of Hyperbolic Logarithms.

No.	Logarithm.	No.	Logarithm.	No.	Logarithm.	No.	Logarithm.
$1\frac{1}{4}$	·22314	$3\frac{1}{2}$	1·25276	$5\frac{3}{4}$	1·74919	8	2·07944
$1\frac{1}{2}$	·40546	$3\frac{3}{4}$	1·32175	6	1·79175	$8\frac{1}{2}$	2·14006
$1\frac{3}{4}$	·55961	4	1·38629	$6\frac{1}{4}$	1·83258	9	2·19722
2	·69314	$4\frac{1}{4}$	1·44691	$6\frac{1}{2}$	1·87180	$9\frac{1}{2}$	2·25129
$2\frac{1}{4}$	·81093	$4\frac{1}{2}$	1·50507	$6\frac{3}{4}$	1·90954	10	2·30258
$2\frac{1}{2}$	·91629	$4\frac{3}{4}$	1·55814	7	1·94591	12	2·48490
$2\frac{3}{4}$	1·01160	5	1·60943	$7\frac{1}{4}$	1·98100	14	2·63905
3	1·09861	$5\frac{1}{4}$	1·65822	$7\frac{1}{2}$	2·01490	16	2·77258
$3\frac{1}{4}$	1·17865	$5\frac{1}{2}$	1·70474	$7\frac{3}{4}$	2·04769	18	2·89037

LOCOMOTIVE ENGINES.

For the following useful Tables we are indebted to Mr. Lloyd, of the Paris and Versailles Railroad.

TABLE

Containing the Velocity of the Pistons, that of the Circumference of the Driving Wheels being taken as 1.

Stroke of the Pistons.	Diameters of Driving Wheels.								
in.	4ft. 0in.	4ft. 6in.	5ft. 0in.	5ft. 6in.	6ft. 0in.	6ft. 6in.	7ft. 0in.	7ft. 6in.	8ft. 0in.
20	0·2652	0·2393	0·2122	0·1929	0·1768	0·1632	0·1516	0·1414	0·1326
19	0·2519	0·2273	0·2016	0·1832	0·1679	0·1550	0·1440	0·1343	0·1259
18	0·2386	0·2153	0·1910	0·1736	0·1591	0·1468	0·1364	0·1272	0·1194
17	0·2254	0·2034	0·1803	0·1640	0·1503	0·1387	0·1288	0·1202	0·1127
16	0·2121	0·1914	0·1697	0·1543	0·1415	0·1305	0·1213	0·1131	0·1061
15	0·1989	0·1795	0·1591	0·1447	0·1326	0·1224	0·1137	0·1060	0·0994
14	0·1856	0·1675	0·1485	0·1350	0·1237	0·1141	0·1061	0·0990	0·0928
13	0·1724	0·1555	0·1379	0·1254	0·1149	0·1061	0·0985	0·0919	0·0862
12	0·1591	0·1436	0·1273	0·1157	0·1061	0·0979	0·0909	0·0848	0·0796

Application of this Table for finding the Tractive Power of Locomotive Engines.

Multiply the sum of the areas of the two pistons by the *effective pressure* of the steam in pounds, and further, that product by the co-effi-

cient in the Table, (belonging to its driving wheels and stroke of the pistons,) and this new product will be the traction of the engine in pounds.

Example.—A locomotive engine to have 5 feet 6 inch driving wheels, cylinders of 13 inches diameter by 18 inches stroke, and the effective pressure of the steam to be 40 lbs. on the square inch: what is its traction?

$$\frac{(2\times 132{\cdot}66)40\times 0{\cdot}1736}{1842{\cdot}39 \text{ lbs. of traction.}}$$

If it be required to know the number of tons the engine is able to draw on a level, divide its traction by the *friction* in pounds.

If the engine is to go up inclines, then add to that friction the *gravity* in pounds due to a ton on that incline, and use this sum as a divisor for the traction, the quotient will be the number of tons the engine is capable to rise up that incline with. In both cases is the weight of the engine and its tender in the quotient included.

Explanations.—By *effective pressure* is understood the pressure of the steam above the pressure of the atmosphere, less the number of pounds necessary to keep the engine by itself just in motion.

Friction, the power necessary to move a mass along, which is generally taken to be on railroads equal to 10 lbs. for every ton.

Gravity, the power to overcome the tendency of a mass or load to descend an incline, being always equal to the quotient of the product of the load, and height of the incline, divided by the length of the incline.

Therefore the above engine would draw

$$\frac{1842{\cdot}39}{10} = 184 \text{ tons on a level;}$$

and on an inclined plain, say as one 1 in 300

$$\text{Friction} = 10 \text{ lbs.}$$
$$\text{Gravity} = \frac{1\times 2240}{300} = \underline{\ \ 7{\cdot}466} \text{ lbs.}$$
$$17{\cdot}466 \text{ lbs.} \quad \text{Consequently,}$$

$$\frac{1842{\cdot}39}{17{\cdot}466} = 105{\cdot}5 \text{ tons up an incline of 1 in 300.}$$

If the weight of the engine with its tender be taken at 18 tons, it will draw a net gross load of

166 tons on a level, and
87·5 tons up an incline of 1 in 300.

TABLE

Showing the Circumferences of different Driving Wheels.

Diam. of Wheel. ft.	in.	Length of Circumference. ft.	Diam. of Wheel. ft.	in.	Length of Circumference. ft.
4	0	12·566	6	6	20·419
4	6	13·927	7	0	21·990
5	0	15·707	7	6	23·561
5	6	17·278	8	0	25·132
6	0	18·849			

TABLE

Of Dimensions of the Principal Parts of Locomotive Engines.

PORTIONS OF THE ENGINES.	GAUGE OF RAILWAY. 4 Ft. 8½ In.		5 Feet.		7 Feet.	
	Feet.	In.	Feet.	In.	Feet.	In.
Cylinders and steam passages.						
Diameters of cylinders - - - -	1	1	1	2	1	3
Distance from center to center of do.	2	5	2	5	3	0
Length of stroke - - - - - - -	1	6	1	6	1	8
steam and eduction ports	0	11	0	11	0	11½
Width of steam ports - - - -	0	1¼	0	1¼	0	1¼
eduction ports - - -	0	2½	0	2½	0	2⅝
Breadth of bridges between ports	0	0⅝	0	0⅝	0	0⅝
Cylindrical part of the boiler and tubes.						
Diameter of the boiler - - - -	3	4	3	7	4	0
Length " do. - - - -	8	0	8	6	8	6
" tubes - - - -	8	5	8	11	9	0
Diameter " do. - - - - - -	0	2	0	2	0	2
Thickness " do. by wire gauge	No. 14		No. 14		No. 14	
Number " do. - - - - - -	121		131		137	
Inside and outside fire boxes.						
Length of inside fire box - - -	3	0	3	4	3	8½
Breadth " do. - - -	3	7	3	5	3	11
Height above the fire bars - -	3	10	3	10	3	11
Area of fire grate - - - - - -	10	9	11	4	14	8
Length of outside fire box - - -	3	6	3	10	4	3
Breadth " do. - - -	4	1	4	0	4	6
Thickness of plates - - - - - -	0	0⅜	0	0⅜	0	0⅜
Ex. thickness where tubes inserted	0	0⅜	0	0⅜	0	0⅜
Smoke box, chimney, and blast pipe.						
Length of smoke box, inside - -	2	1	2	1	2	2½
Breadth " do. " - -	4	2	4	2¾	5	2½
Thickness of plates - - - - -	0	0¼	0	0¼	0	0¼
Diameter of chimney - - - -	1	1½	1	2¼	1	4
Height to top of do. from rails -	13	4	13	6	14	10
Diameter of blast pipe - - - -	0	3¼	0	3¼	0	3⅝
Wheels, springs, &c.						
Diameter of driving wheels - -	5	6	6	0	7	0
leading wheels - -	4	0	4	0	4	0
trailing wheels - -	3	6	4	0	4	0
Breadth of tires - - - - - - -	0	5½	0	5½	0	5½
Thickness, average - - - - - -	0	1¾	0	1¾	0	1¾
Diameter of axle bearings - - -	0	3½	0	3¾	0	4
Length of axle bearings - - -	0	6¾	0	6¾	0	7
driving-wheel springs -	2	9	2	9	2	6
Breadth of do. do. -	0	3½	0	3½	0	3½
Number of plates - - - - - - -	1 at ⅜ and 12 at 5$\frac{5}{16}$		1 at ⅜ and 14 at 5$\frac{5}{16}$		2 at ⅜ and 10 at $\frac{5}{16}$	

PORTIONS OF THE ENGINES.	GUAGE OF RAILWAY.					
	4 Ft. $8\frac{1}{2}$ In.		5 Feet.		7 Feet.	
	Feet.	In.	Feet.	In.	Feet.	In.
Wheels, Springs, &c.						
Length of leading wheel-springs -	2	5	2	5	2	3
Breadth of do. - - - - - - - - -	0	$3\frac{1}{2}$	0	$3\frac{1}{2}$	0	$3\frac{1}{2}$
Number of plates - - - - - - -	1 at $\frac{3}{8}$ and 14 at $\frac{5}{16}$		1 at $\frac{3}{8}$ and 14 at $\frac{5}{16}$		2 at $\frac{3}{8}$ and 10 at $\frac{5}{16}$	
Trailing-wheel springs - - - -	2	2	2	2	2	3
Breadth of do. - - - - - - - - -	0	$3\frac{1}{4}$	0	$3\frac{1}{4}$	0	$3\frac{1}{4}$
Number of plates - - - - - - -	1 at $\frac{3}{8}$ and 6 at $\frac{5}{16}$		1 at $\frac{3}{8}$ and 6 at $\frac{5}{16}$		2 at $\frac{3}{8}$ and 8 at $\frac{5}{16}$	
Diameter of safety valves - - - -	0	$3\frac{1}{2}$	0	$3\frac{1}{2}$	0	4
piston rods - - - -	0	2	0	$2\frac{1}{16}$	0	$2\frac{1}{4}$
valve spindles - - -	0	$1\frac{1}{8}$	0	$1\frac{1}{8}$	0	$1\frac{1}{4}$
crank pins - - - -	0	$5\frac{1}{2}$	0	$5\frac{3}{4}$	0	$6\frac{1}{2}$
pump rams - - - -	0	2	0	2	0	$2\frac{1}{4}$
Extreme breadth of outside frame -	6	5	6	$8\frac{1}{2}$	8	$9\frac{1}{4}$
length - - - - - - - -	18	$2\frac{1}{2}$	18	$2\frac{1}{2}$	20	4

Various modifications have lately been added to the list of improvements in locomotive engines, and among the most efficient is that of using steam more or less expansively, as required,— a system very successfully adopted on the South-Western Railway. The arrangement is such, that the quantity of steam, of uniform density, is immediately changed to suit either the load or inclinations of the line.

Another essential improvement on the same line of railway is that of insuring a uniform equilibrium of the engine, in case of fracture in the front axle; such an occurrence being the only real cause of fear in running a six-wheeled engine.

To calculate for the Travel of the Valves, Throw of Eccentrics, &c.

Let T represent the travel of the valve;
L, the length of lever, to which the eccentric rod is attached;
t, the throw of eccentrics; and
l, the lever immediately connected with the valve.

$$\text{Then, 1. } \frac{TL}{l}=t. \quad 2. \ \frac{TL}{t}=l. \quad 3. \ \frac{tl}{T}=L. \quad 4. \ \frac{tl}{L}=T.$$

Example.—Suppose, in a locomotive engine, the valves require a travel of $3\frac{3}{4}$ inches; the top lever, or lever immediately connected with the valve spindle, being $9\frac{1}{4}$ inches, and the bottom lever 8 inches: what must be the throw of eccentrics?

$$\frac{3\cdot75\times8}{9\cdot25}=3\cdot243 \text{ inches, or } 3\tfrac{1}{4}\text{, nearly.}$$

NOTE. The travel of a valve equal the width of the two steam openings, plus the lap of the valve over each opening, and the whole length of its movement, or face, upon the cylinder, equals twice the travel of the valve, plus the distance between the two steam openings.

To ascertain the Amount of Weight or Pressure on the Safety-valve of a Locomotive Engine.

Divide the length of the lever by the distance from its center of motion to center of valve, and multiply the indicated pressure on the spring balance by the quotient, to which add the action or pressure of the lever and spring balance; divide the sum by the area of the valve, and the quotient equals the pressure on each inch of the boiler.

Example.—Suppose an engine with valve levers of $22\frac{1}{2}$ inches in length, and the distance from the pin to center of valve $2\frac{1}{2}$ inches; the action of the lever and spring balance 5 lbs.; the indicated pressure 50 lbs., and the area of the valve 7 inches: required the pressure on each square inch.

$$22{\cdot}5 \div 2{\cdot}5 = 9; \text{ and } \frac{9 \times 50 + 5}{7} = 65 \text{ lbs. per square inch.}$$

To determine the Pressure of Steam equal to the Resistance of a given load, by a Locomotive Engine.

It is ascertained, by experiment, that 6 lbs. per ton of the engine's weight is expended in overcoming the friction of its parts, when unloaded; 9 lbs. per ton of gross load for horizontal traction, and additional friction, caused by the load; and 14·7 lbs. per square inch, the pressure of the atmosphere. Hence, to 9 times the gross load of the train, in tons, add the friction of the engine; multiply the sum by the diameter of the driving-wheels, in inches; divide the product by the cylinder's capacity, in inches, and the quotient, plus 14·7, equal the pressure of the steam, in lbs., per square inch, on the piston.

Example.—Required the pressure, per square inch, on the piston's area, to overcome a load of 120 tons, gross, on a level line, the engine being 13 tons, driving-wheels $5\frac{1}{2}$ feet, cylinders 13 inches diameter, and length 18:

$120 - 13 = 107 \times 9 = 1080$; and $13 \times 6 = 78$, the force of traction; $13^2 \times 18 = 3042$. Then $\dfrac{1080 + 78 \times 66}{3042} = 25{\cdot}12 + 14{\cdot}7 = 39{\cdot}82$ lbs. per square inch.

TABLE

Showing the Approximate of Useful Effect of an Ordinary Locomotive Engine, at different Velocities. By MR. N. WOODS.

Load in Tons.	Miles per hour.	Miles per hour.	Load in Tons.
25	30·90	10	250
50	25·15	$12\frac{1}{2}$	184
75	22·54	15	138
100	18·18	$17\frac{1}{2}$	106
125	15·98	20	83
150	14·29	$22\frac{1}{2}$	65
175	13·28	25	50
200	11·20	$27\frac{1}{2}$	38
225	10·77	30	28

The increased resistance to traction on ascending inclined planes is as the increased gravity of the load, caused by the inclination of the plane; or, as the length of the plane to the perpendicular height. Hence, divide 2240 (or the number of lbs. in a ton), by the inclination of the plane to 1, or unity, to the quotient of which add 9, and the sum equal the total resistance, in lbs., per ton, upon the plane.

Example.—Required the resistance to traction, per ton, on an inclined railway, rising 1 in 300.

$$\frac{2240}{300}=7{\cdot}47+9=16{\cdot}47 \text{ lbs. per ton.}$$

Useful effect of a single-acting pumping engine, with a consumption of 1 bushel of coals:

Diameter of cylinder,	5 feet	3 inches.	
Length of stroke,	7 "	9 "	
Stroke of pump,	6 "	9 "	

Number of strokes per minute, 6·1.
Effective pressure, per square inch, of piston, 13·4 lbs.
Water lifted 1 foot high, per minute, 38,063,288 lbs.

The following are the Principal Dimensions of the Great Western New Locomotive Passenger Engine:—

Outside fire-box, ...	6 ft. wide × 5 ft. 6 in. long.
Inside do.	5 ft. 3 in. wide × 4 ft. 10 in. long.
Depth of bars, ...	4 ft. 11 in.
Diameter of boiler, . . .	4 ft. 6 in.
Length, ...	10 ft. 6 in.
Number of tubes, . . .	280
Diameter, ...	2 in.
Diameter of cylinders, . .	18 in.
Length of stroke, . . .	2 ft.
Diameter of waste-pipe, . .	$5\frac{1}{2}$ in.
Diameter of driving-wheels,	8 ft.
Do. of small wheels,	4 ft. 6 in.
Weight of engine in working condition.	29 tons.
Weight on driving-wheels,	10 do.
Do. on leading-wheels,	$9\frac{1}{2}$ do.
Do. on trailing-wheels,	$9\frac{1}{2}$ do.

TABLE

Showing the Number of Revolutions of the Driving Wheels, or Strokes of the Piston, per Minute, while the Engine is performing a known Number of Miles per Hour.

Diam. of Wheel 4 ft.	Diam. of Wheel 4 ft. 6 in.	Diam. of Wheel 5 ft.	Diam. of Wheel 5 ft. 6 in.	Diam. of Wheel 6 ft.	Diam. of Wheel 6 ft. 6 in.	Diam. of Wheel 7 ft.	Diam. of Wheel 7 ft. 6 in.	Diam. of Wheel 8 ft.	Number of Miles performed per Hour.
No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	No. of revol. or strokes per min.	
35·03	31·59	28·03	25·47	23·34	21·55	20	18·67	17·5	5
70·06	63·17	56·06	50·95	46·68	43·10	40	37·34	35·0	10
105·09	94·76	84·09	76·42	70·02	64·65	60	56·01	52·5	15
140·12	126·36	112·12	101·70	93·36	86·20	80	74·68	75·0	20
175·15	157·95	140·20	127·37	116·70	107·75	100	93·35	87·5	25
210·18	189·54	168·18	152·85	140·04	129·30	120	112·02	105·0	30
245·21	221·13	196·21	177·29	163·38	150·85	140	130·69	122·5	35
280·24	252·72	224·24	203·76	186·72	172·40	160	149·36	140·0	40
315·27	284·30	252·27	229·23	210·06	193·95	180	168·03	157·5	45
350·30	315·90	280·30	254·75	233·40	215·50	200	186·70	175·0	50
385·33	347·49	308·33	280·17	256·74	237·05	220	205·37	192·5	55
420·36	379·08	336·36	305·64	280·08	258·60	240	224·04	210·0	60
455·39	416·67	364·39	331·11	303·42	280·15	260	242·71	227·5	65
490·42	442·19	392·42	356·58	326·76	301·70	280	261·38	245·0	70
525·45	473·85	420·50	372·05	350·10	323·25	300	280·05	262·5	75
560·48	505·50	448·48	407·60	373·44	344·80	320	298·72	280·0	80
595·51	536·89	477·01	432·99	396·78	366·35	340	317·39	297·5	85
630·54	568·60	504·51	458·55	420·12	387·90	360	336·06	315·0	90
665·57	600·19	532·57	483·93	443·46	409·45	380	354·73	332·5	95
700·60	631·78	560·63	509·50	466·80	431·00	400	373·40	350·0	100

NOTE.—To find the velocity the piston is traveling at in feet per minute, multiply the number of revolutions of its driving wheels, in the table, by twice the length of its stroke in feet.

Example.—What is the speed of the piston of an engine with 6 feet driving wheels and 15-inch stroke, when going at the rate of 50 miles an hour?

By means of the table:—

$$233{\cdot}4 \text{ revolutions} \times \left(\frac{2\times 5}{4}\right) = 583{\cdot}5 \text{ feet per minute.}$$

The number of revolutions of driving wheels are inversely as their diameters, and in direct proportion to the number of miles performed.

Example.—How many revolutions have the driving wheels of an engine to make when it is going at 95 miles an hour, their diameter being 9 feet 6 inches?

According to the table, a 4-feet wheel would have to make 665·57 revolutions, therefore

$$9{\cdot}5 : 4 :: 665{\cdot}57 : \frac{4\times 665{\cdot}57}{9{\cdot}5} = 280 \text{ revolutions.}$$

The driving wheels of an engine make 35·03 revolutions when

going at the rate of 5 miles an hour, how many will they make when going at 9 miles?

$$5 : 9 :: 35{\cdot}03 : \frac{9\times35{\cdot}03}{5}=63{\cdot}05 \text{ revolutions.}$$

TABLE

Of the Areas of Cylinders from 9 to 15 Inches Diameter.

Diameter of Cylinder.	Area of Cylinder.	Diameter of Cylinder.	Area of Cylinder.
Inches.	Square Inches.	Inches.	Square Inches.
9	63·58	12½	122·65
10	78·5	13	132·66
10½	86·56	13½	143·02
11	95·01	14	153·96
11½	103·84	14½	165·04
12	113·07	15	176·62

NOTE.—The areas of cylinders are as the squares of their diameters.

PRACTICAL RULES

For the Management of a Locomotive Engine in the Station, on the Road, and in cases of Accident. By CHARLES HUTTON GREGORY, *Civil Engineer.*

REMARKS.

THE substance of the following pages was written several months since, and subsequently sent to the Institution of Civil Engineers, where it was read in abstract on the 16th of February in the present session.

While our engineering literature contains several valuable treatises on the theory and construction of the locomotive engine, it has, as yet, produced no work illustrating its use. This circumstance, added to the recommendation of several competent authorities, has induced the writer to apply to the Council of the Institution of Civil Engineers for permission to lay before the public these "Practical Rules for the Management of a Locomotive Engine," drawn up from individual experience, in the hope that they may be acceptable, at a period when any subject connected with the efficacy and safety of railroad traveling is deservedly engaging attention.

At the end of the paper will be found some regulations for the first appointment of engine-men, adopted by the directors of the London and Croydon railroad, and framed by the writer in his official capacity as their resident engineer.

I. THE MANAGEMENT OF A LOCOMOTIVE ENGINE IN THE STATION.

The careful examination of a locomotive engine when in the station, and its judicious management while running, are essential to the

full performance of its duty, and to insure the safety of the passengers by the train.

While an engine is stopping at the station before a trip, the fire should be properly kept up; the throttle-valve should be closed and locked; the tender-break screwed down tight; the reversing-lever fixed in the middle position, so that the sliders may be out of gear; the cocks of the oil-vessels and feed-pipes turned off; and the steam blowing off from the safety-valve at a pressure of 70 lbs. per square inch; (for low-pressure engines 35 lbs.;) if blowing off in any excess, the waste steam may be turned into the tender-cistern to heat the water, and the door of the smoke-box may be opened to check the fire, but it should be fastened up again ten or fifteen minutes before the time of starting.

Before an engine starts with a train, the attention of the engine-man should first be directed to its being in complete working order; with this view he should go beneath the engine, and carefully examine the working gear in detail.

The connecting-rod is a very important part, and more liable perhaps than any other to fail for want of proper examination. The gibs and keys must be secure, and in case the brasses have too much play, they must be tightened up; observing, however, that brasses should never be set so hard as to cause friction. If there are set-screws at the side of the gibs and keys, they should be tight, and all gibs and keys should have a split-pin at the bottom for greater security. The gibs and keys which fasten the piston-rods to the cross-heads should be firm in their place, as well as the set-screws, keys, or other connections, by which the feed-pump pistons are secured to the piston-rod.

The brasses of the inner framing which carry the inside bearings of the cranked axle must be examined, and any considerable play prevented by screwing them up if necessary. The wheels ought to be accurately square and firm on their axles, and the keys driven up tight. All the pins, bolts, &c., by which the side-valve gear is connected, and the lifting-links, must be secure in their proper places; the eccentric-hooks ought to be fast on the lifting and weigh-bars, and the studs on the eccentrics of the weigh-bars should be particularly noticed, as, if loose, they may be shaken off on the road and cause the stoppage of the engine. A similar examination must be extended to the hand-gear, if there be any; and the bolts which fasten the plummer-blocks of the weigh-bars, &c., must be screwed up if they are loose.

The straps of the eccentrics should work with sufficient freedom, and the eccentrics must be firm in their right position on the axle, or the engine will beat unevenly: if any escape of steam has been observed in the stuffing-boxes of the piston-rod and side-valve spindle, or of water from the joints of the feed-pumps and suction-pipes, they must be screwed up; and any dirt that may have collected near any of the bearings or connections must be carefully wiped off with cotton waste.

The inspection beneath the engine being complete, the engine-man

should examine the ends of the tubes of the boiler, and if there should be leakage to any serious extent, it would be prudent to drive in a plug at each end of the defective tube. A small quantity of tallow should occasionally be introduced into the steam-chests and cylinders, to grease the slides and pistons. This is done either by cocks on the outside of the smoke-box or in the cylinder covers, or through holes. secured by plugs, in the steam-chest covers. The ashes should be emptied out of the smoke-box, and the small ash-door carefully secured.

Occasionally the gauge should be applied to the wheels, and the engine should never be allowed to run when they are found to be at all incorrect or out of the square.

If there are oil-vessels at the side of the engine with pipes to the pistons, bearings, &c., the engine-man must see that they are filled, and the cotton wicks in the top of the pipes, and hanging over into the oil; that the grease boxes of the axle-bearings are filled; and the pins, links, &c., of the springs right and sound. The draw-bar, connecting the engine and tender, must be secure, and the safety-chains attached.

The tender must be replenished with wood and water. An engine-man should never run with an engine without knowing what stock of both the tender will carry. It is impossible to lay down any general rule for the quantity of water evaporated and the wood consumed per mile with the same engine, as the amount depends entirely on the extent of duty performed. The stock of wood is usually about three times as much as that of water; the water which most tenders contain is ordinarily sufficient for running twenty miles with certainty; but when the gradients are steep, the load heavy, and stoppages frequent, additional water may be oftener required; and, on the other hand, with light duty, an engine may sometimes run further without any stoppage. The inconvenience attached to the necessity of frequent stoppages, and the expense of maintaining a large number of wood and water stations, have lately induced the manufacture of a larger class of tender on eight wheels, which, from superior capacity, will admit of a much longer run.

After a little practice, the examination described above occupies a very short time: it ought to be completed, and the engine in its position at the head of the train, at least five minutes before the hour of starting, when oil must be copiously supplied by the small oiling-can, to the oil-cups of the guides, connecting-rods, &c., and to all rubbing parts not fed by the oiling-pipes; the cocks of the large oil-vessels must be opened, and the safety-valve screwed down to the working pressure, say 85 lbs. per square inch.

Several articles should be constantly carried on the tender, as either being frequently required in the working of the engine, or occasionally in cases of derangement or accident. The following may be taken as a list:

One large can of oil, and one or two small oiling-cans and an oiling-tube, a box of tallow, a quantity of cotton waste, hemp, and gasken, a hand-brush, keys fitted to all the principal bolts, and one large and one small monkey-wrench.

A number of iron or wooden plugs, an iron plug-holder, and a 7-lb. maul, two cold chisels, a hammer and a file, spare washers, and duplicates of the principal bolts, nuts, pins, gibs and keys, &c., a quantity of thick and thin cord, and some tarred line, a fire-bucket, two long crow-bars, a spare coupling-chain, with shackle and hook complete, several wooden wedges about two feet long, four or five inches wide, and three inches thick; and, if running long journeys, two spare ball-clacks, and a screw-jack.

II. THE MANAGEMENT OF A LOCOMOTIVE ENGINE ON THE ROAD.

In the management of a locomotive engine, many unforeseen circumstances may occur, requiring the use of that discretion which experience alone can confer, and which it would be almost impossible to comprise in the particular instructions contained in the following pages, which, however, the writer believes to contain all the leading principles of engine-driving.

On receiving the signal to start, the engine-man should only slightly open the regulator, and let the train run for several yards before he opens it, by slow degrees, to the full extent. The object of thus giving a slight aperture to the regulator in starting, is to avoid any jerk to the carriages, by which passengers might be annoyed, or even the coupling-irons broken; to prevent the slipping of the driving-wheels, from their adhesion being unequal to the inertia of the train, when the full power of the engine is suddenly used; and because fully opening the regulator at starting generally causes the engine to *prime* considerably, from the quantity of water condensed in the cylinders and steam-passages while the engine was standing. When *priming* occurs at starting, the discharge-cocks of the cylinders should be open to remove the water. On leaving the station, and frequently on the road, the engine-man should watch the train behind him, to see that it is all right and its motion regular.

The engine-man should now be standing on the foot-board of the engine, which he ought never to leave unless the machinery is out of order, when he may leave the fireman in his place; he should, as much as possible, be in such a position as to command, without moving from his place, the reversing-lever, the whistle, and the regulator or throttle-valve, these being the parts which he is most frequently obliged to use at the shortest notice: his hand should be upon the regulator, which, when he has arrived at a good speed, he will gradually ease off, so as to economize steam without retarding the train: his eye should be constantly directed to the rails in front of him, that he may be immediately aware of any obstruction, and at the same time his full attention must be given to the maintaining a sufficiency of steam at an equable pressure; this is to be done by using the requisite care in the manner and time of supplying *water* and *fuel*.

Water is supplied by opening the cocks in the feed-pipes, which allow the pumps to act; and the height of water in the boiler is com-

monly shown by three gauge-cocks at the side, which should be opened from time to time, (especially when stopping,) as they afford a correct indication of the quantity of water and steam.

One pump, if constantly at work, would, in most engines, supply as much, or rather more water than is required by the engine as equivalent to the steam consumed; so that by turning on or off either or both pumps, the engine-man has the power of regulating the height of the water in the boiler at discretion.

It may be laid down as an invariable rule, that water alone should always blow off from the bottom cock (which is from 1 inch to 1½ inch above the top of the fire-box) in order that there may be enough water over the fire-box and tubes to prevent their burning; and few engines will carry their water much above the top cock without *priming*, so that the height of the water may be made to range between these two points, according as more or less steam is required.

The water is higher when the engine is running than when stopping: a good working height for it in most engines is when *water* blows off from the top cock while running, and *water* and *steam* in the middle when stopping: an engine-man is sometimes obliged to run the water rather lower, if he has heavy work; but it is always better to keep the level of the water as high as possible.

It is observed that when any variation takes place in the pressure of the steam, a corresponding change occurs in the level of the water,—that when the pressure of the steam rises or falls, the height of the water rises or falls simultaneously. Partly for this reason, and partly to allow the more rapid generation of steam, the feed-pumps are not generally allowed to act when the engine starts: a knowledge of this fact also shows the necessity of the water being above the ordinary level, before a decrease is allowed in the pressure of the steam.

When the engine is highest on an inclined plane, rather a greater height of water must be kept over the fire-box than on a level, in order that the chimney-ends of the tubes may be well covered.

The most favorable time for allowing the feed-pumps to act, is when the steam is blowing off with force from the safety-valve, and the fire strong; and the least favorable time, is when the steam and fire are low: indeed, the engine-man should manage that it may never be necessary in the latter case, as the addition of water rapidly lowers the steam.

In order to know the force of the steam, one hand may occasionally lift or depress for a moment the lever of the safety-valve, according as the steam is under or over the working pressure; and a little practice will soon enable a person to judge the extent of excess or deficiency.

Both feed-pumps should not commence working at the same time.

The water should never be allowed to run low before arriving at any part of the road where considerable power is required, as steam is produced more rapidly when both pumps are turned off,—a measure which is imprudent unless the water is high.

When "the feed" is turned on, the engine-man should try the pet-cock to see whether the pump is acting freely; the water thrown from it should be in forcible intermittent jets; warm water with a little steam will frequently escape from it at first; if this should continue, it may be concluded that the upper clack does not act; and if the water is a continuous stream without pulsation, the lower clack is out of order. In either case it will not be prudent to trust too much to the faulty pump, but the evil may frequently be remedied by working the pump a short time with the pet-cock open, or alternately turned on and off.

The supply of fuel should be regular, and so arranged that the fire may have burned up well by the time the steam is most required. As the addition of fuel causes a temporary reduction of the force of the fire, wood should not be supplied immediately before arriving at an inclined plane or any part of the road where much power is required; but when ascending an incline, wood should be gradually added when the engine begins to *beat heavily*,—the draught is then powerful, and a regular supply of fuel required to keep up the fire.

In other circumstances, provided the fire is low enough to require fuel, the best time to put in wood is when the water is sufficiently high to turn off the feed-pumps, the steam slightly blowing off, and the engine traveling at a good speed.

No definite instructions can be given for the frequency with which wood must be supplied to keep up the fire, as it varies according to the duty to be done, and the water consequently to be evaporated: in cases of heavy duty and bad gradients, it may at times be necessary even at as short an interval as 1 mile; under contrary circumstances an engine may sometimes run as far as 2 or 3 miles without adding a fresh supply of fuel.

The fire should be allowed to run rather low before arriving at the top of an inclined plane down which the steam will not be used; on beginning to descend the plane, fuel should be put on the fire, which will burn up by the time the train reaches the bottom of the plane.

If it is wished to keep up the steam, it is better not to supply water and fuel at the same time.

By observing the above rules for the supply of water and wood, an efficient pressure and quantity of steam will be produced, which it must be the study of the engine-man to economize. With this view the regulator should never be kept too far open;—as soon as the train has acquired the velocity wished, the aperture may be considerably reduced without diminishing the speed. As any diminution in the amount of steam used causes a corresponding diminution in the quantity of wood consumed, the skill of the engine-man should be unceasingly directed to the reduction of so heavy an item of railroad expenditure.

If there should be, at any time, an unnecessary quantity and force of steam, it is readily reduced by opening the fire-door, and by turning on the feed-pumps; if there should be too little, the engine-man must be content to run slowly for a short time, keep the regulator only partially open, and put on a gradual supply of wood.

When the water in the boiler is high, some engines begin to prime, especially after running for several days. When this occurs, the aperture of the regulator should be diminished, and the fire-door and the discharge cocks of the cylinders opened: if the height of the water will allow it, the blow-off cock of the boiler may be opened for a short time to carry off the sediment, which will be found advantageous.

The engine-man should frequently look to the working-gear, to see that it is in proper order, and to rectify any deficiency at the next station.

On nearing a station where it is intended to stop, the regulator should be gradually eased off at about five-eighths of a mile from the station, so that the train may be more under control, and when from a quarter to half a mile distant, according to the velocity and weight of the train, the steam should be completely shut off, and the train brought to rest by the breaks. In approaching terminal stations, the steam should be shut off at a greater distance than at the intermediate stations, to prevent the possibility of overrunning the mark from the failure of breaks. It must be borne in mind that the breaks act much less efficiently in wet or frosty weather, when it becomes necessary to shut off the steam further from the stations. The use of the reversing-lever ought, as much as possible, to be avoided: it may sometimes be placed in the middle position, (in which the valves do not act,) but it should never be completely reversed unless absolutely necessary for the stoppage of the train.

At the intermediate stations, the fireman should frequently oil all the bearings not supplied by the large oil-vessels, and fill the oil-cups of the connecting-rods, slides, &c., and if any of the bearings, brasses, &c., are hot, they should be more copiously oiled, and eased if necessary. He should also examine all the working gear cursorily to see if it is in a complete state; particular attention should be given to the axle-bearings, and especially those of the cranked axle, which sometimes becomes so hot by running as to require cooling by throwing on water.

In case of the driving wheels slipping much in starting from a station, the opening of the regulator should be reduced, and only gradually opened as the wheel bites; the fireman is sometimes obliged to scatter ashes, sand, &c., before the wheels: some engines are now furnished with hoppers in front, opened by a handle from the foot-board, by means of which sand may be dropped on the rails in front of the driving wheels.

If slipping is observed to an unusual extent, it may be inferred that there is not sufficient weight on the driving wheels, and the springs ought to be tightened by screwing up the nuts of the bearing bolts: or where the framing is hung to the springs by plain links, the spring pins must be lengthened the next time the engine is in the repairing shops. A deficiency of weight on the front or hind wheels is indicated by the pitching of the engine, and should be remedied in a similar manner.

The regulator should be gradually and completely closed, when

the engine or train pitches or rocks violently,—in passing a series of points and crossings,—in very sharp curves, especially if double,—in rough parts of the permanent way,—and in descending planes whose inclination is sufficient to carry the train down, without steam, at a velocity of 20 miles per hour. In descending such an inclined plane, if it should be found that the velocity is greater than 20 miles per hour, it should be reduced by gently applying the break.

On every railroad there is a prescribed limit to the pressure of the steam, and no circumstance should induce the engine-man to use steam at a higher pressure, or in any case to weight the lever, or hold it down for more than a moment. When there are two safety-valves, that which is out of reach may be set at the limit of pressure, and the valve next the foot-board some pounds lower. It is an advantage to have a stop placed below the lever of the safety-valve on the screw of the spring balance, to prevent its being inadvertently screwed down to more than the working pressure.

The steam whistle is obviously intended to give notice of danger: on this account its use is forbidden on some railroads, excepting on occasions of extreme emergency; but the variety of modulation of which it is susceptible has in others induced its adoption as a frequent warning. When the latter is the case, it has been found a safe measure to sound the whistle directly the steam has been shut off previously to stopping at a station, and to give two short whistles the moment before starting, to warn parties of the approach and departure of the train. When this system is practiced, the engine-man should not turn on the full power of the whistle, but reserve it exclusively for cases of danger.

When near the end of the trip very little fire is wanted, and both feed-pumps should be turned on for a short distance before arriving at the station, unless the engine is to start again immediately. If it is intended to remain at the station about an hour, the water should be considerably above the middle cock, (when the engine is standing,) which will be effected by keeping on both feed-pumps from a half to three-quarters of a mile. The safety-valve should, at the same time, be eased off to 70 lbs.

If the train is brought into the station by a tow-rope, great care must be taken to stretch the rope gradually by a gentle advance of the engine, which must be stopped at a signal from the tow-rope man.

It would be prudent to conduct the examination described at the commencement, directly the engine arrives at the station, in order to leave time for any repair which may be required.

When an engine is running the last trip for the day, no fuel need be put on for the last 2 miles; indeed, the fire may be nearly run out by the time the engine stops, if the gradients, &c., are favorable. For a considerable distance before stopping both pumps should be at work, so that the water in the boiler may be at or above the top cock when the engine stops, and the safety-valve should be eased off to 70 lbs. per square inch.

The practice of blowing off all the water from a boiler by the pressure of the steam should never be allowed, without an express order

from the superintendent of locomotives, when the boiler is unusually full of mud; as, if frequently practiced, it will seriously injure the fire-box and tubes.

III. THE MANAGEMENT OF A LOCOMOTIVE ENGINE IN CASES OF ACCIDENT.

An engine is liable to several accidents while running, and it is important that the engine-man should know how to act promptly under the circumstances. In the following list several cases are enumerated, with the particular steps to be taken in each.

1. *The bursting of a tube.*—The engine-man should stop the engine, and drive a plug into each end of the tube. It frequently happens that the water and steam blow out with so much force that it is impossible even to discover the defective tube: by running the engine for a short distance with both pumps acting, the pressure of the steam will perhaps be sufficiently reduced to enable the engine-man to work with safety; but if the escape of water and steam is still too great to do so, he must run his engine and train, if possible, off the main line into a siding, and draw the fire, to prevent its injuring the fire-box and tubes: when the water has run out down to the level of the defective tube, it may be easily plugged, and a fresh fire lighted. A tube will frequently leak to a considerable extent without absolutely requiring the stoppage of the train; but in this case great care is necessary not to use much steam, or urge the fire too far.

The bursting of a tube or other causes will sometimes lead to the lagging or casing of the boiler catching fire, which should be extinguished by throwing on water from the tender-cistern in a fire-bucket, or from the water-crane at a station.

2. *The failing of one of the feed-pumps.*—In this case the adequate supply of water may, with care, be maintained by one pump only. The supply of wood must be regular, and not in large quantities; and the steam must be economized, or the water may run low. The pump should be repaired as soon as possible; this may frequently be done in the interval between two trips.

3. *The breaking of a spring.*—This is an accident which does not necessarily involve the stoppage of the train; but as working the engine in such a state causes an unequal strain, it should run very gently over rough parts of the road; and if the derangement is considerable, and can not be repaired at the stations, the engine should cease running as soon as possible.

4. *The breaking of a connecting-rod, or its disconnection by the loss of the gibs and keys, fracture of the straps, &c.*—This accident, or any disconnection which allows the piston to be driven from end to end of the cylinder without restraint, causes expensive damage to the cylinders and covers; and the connecting-rod, if loose, will seriously injure the smaller gear, or may even throw the engine off the road. The engine should therefore be instantly stopped, and if possible the connection restored; if that cannot be done, the connecting-rod must be taken off, and if on a level or a descending gradient,

the train may sometimes be drawn by a single cylinder: to do so, the slide-valve spindle of the defective cylinder must be detached from the valve gear, by unscrewing the nuts, and setting the slide at the middle of its stroke so as to cover both ports.

If it should be found impracticable to move the train, the engine might run on alone for assistance; but in any case where the engine is obliged to remain stationary, the fire must be drawn directly the water is down to the bottom cock.

5. *The fracture or disconnection of the eccentrics, or any of the slide-valve gear.*—In engines without hand-gear, if the connection can not be restored, the attempt may be made, as in the previous instance, to work with one cylinder. When the slide-valve gear is disabled, engines with hand-gear possess an advantage which others want, in being able to be worked by hand, when a single cylinder would be unequal to the duty, from not being able to move the crank over the centers at starting.

6. *The fracture of the strap which holds the slide-valve*, renders unavailable the cylinder on that side where it occurs, without affecting the other side. The slide should be detached and placed in the middle of its stroke, and the attempt made to work with one cylinder.

7. *The disconnection of a piston*, by the fracture of either of the gibs, is sometimes caused by shutting off the steam too suddenly when the engine is traveling fast with a heavy load. In this case also the slide should be detached and set in the middle position, and the piston-rod uncoupled from the connecting-rod, which should be removed to prevent its damaging the small gear.

8. *The breaking of an axle*, in a four-wheeled engine, is an accident which is almost of necessity attended with the overturn of the engine. In a six-wheeled engine it requires the stoppage of the train until assistance arrives.

9. *The engine running off the rails.*—With an engine-man who drives carefully, watching well the position of the switches, and the signals given him, and stopping when he sees any danger attending his further course, this is an accident of very rare occurrence. If the engine should run off on hard ground and near the rails, it may sometimes be lifted on again at once, by screw-jacks, crow-bars, and long sways; but if on soft ground or far from the rails, the fire must be drawn, and instant attention given to prevent its sinking deep into the ground.

The engine should first be separated from the tender, which, being a lighter weight, may be pushed out of the way, and leave more room for operating on the engine; this, if it has fallen over on its side, should be lifted as quickly as may be into a vertical position; to do so, a purchase should be obtained under the framing on the lowest side, in two places if possible; two long and tough sways should be brought to bear on these points, and several men placed to weigh upon each; and as the engine is gradually lifted by the sways, every movement should be followed up and supported by screw-jacks bedded on timber blocking. When the engine has been lifted upright, it should be firmly supported by timbers placed as stanchions under

the framing; the earth may then be cautiously removed from under the wheels, and a length of rail introduced, taking care to bed it as securely as possible on the blockings previously laid down, without disturbing them; the same process should be repeated on the other side, and cross sleepers driven in under both rails to secure the foundation. As soon as the engine is in a vertical position and rails inserted under the wheels, a temporary railway may be laid down in continuation, and the engine again drawn on the main line. It will much facilitate the raising of the engine if the water is drawn away out of the boiler as soon as it is sufficiently cool.

In all cases of accident, involving the stoppage on the main line, it is of the highest importance that some person should immediately be sent back about three-quarters of a mile along the road, to give the proper signal of obstruction, and prevent any following train from running in unexpectedly.

The most essential personal qualifications of an engine-man are, sobriety and steadiness, activity, presence of mind, and unceasing watchfulness; and wherever these are combined with an accurate knowledge of the construction of a locomotive engine and the principles of its management, they tend in no small degree towards rendering railroads, what they properly are, the safest as well as the most agreeable mode of traveling.

Regulations for the first appointment of an Engine-man, adopted by the Directors of the London and Croydon Railway. 1840.

1. The candidate must not be under twenty-one years of age, and must produce a certificate of a sound constitution and steady habits.
2. He must be able to read and write, and understand the rudimental principles of mechanics.
3. It will be a great recommendation if he has served his time to any mechanical art, especially as a fitter of locomotive engines; and, if possible, he should produce testimonials stating his qualifications as such.
4. If the candidate has been a fitter or a stationary engine-man, he must, for several months at least, have been a fireman on a locomotive engine, under the direction of a steady and competent engine-man; and before his appointment, he should produce a testimonial from the superintendent of locomotives, or at least from the engine-man under whom he has served, stating full confidence in his acquaintance with the construction of an engine and the principles of its management.
5. If the candidate has not been a fitter or a stationary engine man, he must have served as a fireman for at least two years, and produce the testimonials named in the preceding rules.

6. If required by the board of directors, for greater security, the candidate must undergo an examination from their engineer, superintendent of locomotives, or other competent person, as to his knowledge of an engine and its management, and the general result of this examination must be committed to paper, signed by the examiner, and presented to the board.

7. The engineer or superintendent of locomotives of the railroad to which the candidate is desirous of being appointed, shall sign a certficate stating that he has conversed with him, has seen him drive, and has confidence in his steadiness and ability.

8. Before being allowed to take the entire charge of an engine and train, the candidate must drive for several days under the direction of an experienced engine-man, who must be on his engine, and certify to his ability.

9. All certificates and testimonials must be deposited with the secretary of the company, who will restore them to the owner on his leaving their service.

TABLE,

SHOWING THE AMOUNT OF WAGES

Per Week and Year, at a given sum per Hour, Day, &c.

NEW YORK CURRENCY.				NEW ENGLAND CURRENCY.			
Per Hour.	Per Day of 10 hours.	Per Week of 6 days.	Per Year of 52 weeks.	Per Hour.	Per Day of 10 hours.	Per Week of 6 days.	Per Year of 52 weeks.
cts. mills.	$ cts.	$ cts.	$ cts.	cts. mills.	cts. mills.	$ cts.	$ cts.
2·5	0·25	1·50	75·00	1·7	16·7	1·00	52·00
3·1	0·31¼	1·87½	97·50	2·5	25·0	1·50	78·00
3·7	0·37½	2·25	117·00	3·3	33·3	2·00	104·00
4·4	0·43¾	2·62½	136·50	4·2	41·6	2·50	130·00
5·0	0·50	3·00	156·00	5·0	50·0	3·00	156·00
5·6	0·56¼	3·37½	174·50	5·8	58·3	3·50	178·00
6·2	0·62½	3·75	195·00	6·7	66·7	4·00	208·00
6·9	0·68¾	4·12½	214·50	7·5	75·0	4·50	234·00
7·5	0·75	4·50	232·00	8·3	83·3	5·00	260·00
8·1	0·81¼	4·87½	253·50	10·0	100·0	6·00	312·00
8·7	0·87½	5·25	273·00	11·7	116·7	7·00	364·00
9·4	0·93¾	5·62½	292·50	12·5	125·0	7·50	390·00
10·0	1·00	6·00	312·00	13·3	133·3	8·00	416·00
11·2	1·12½	6·75	351·00	14·1	141·6	8·50	441·79
12·5	1·25	7·50	390·00	15·0	150·0	9·00	468·00
13·7	1·37½	8·25	429·00	16·7	166·7	10·00	520·00
15·0	1·50	9·00	468·00	17·5	183·0	10·00	536·00
17·5	1·75	10·50	546·00	20·0	200·0	12·00	624·00
20·0	2·00	12·00	624·00	22·5	225·0	13·50	702·00

THE following Table, made expressly for the use of cotton and woolen mills, will be found very convenient for computing hastily the superficial contents of various sized cylinders.

TABLE

Showing the number of square feet of filleting cord required to cover the convex surface of any cylinder, from 1 *inch diameter, varying by* ¼ *of an inch, to* 20 *inches diameter, and, in length, with variations from* 18 *inches to* 49 *inches.*

	REQUIRED AREA.							
Given Diameter.	18 Inches.	24 Inches.	30 Inches.	36 Inches.	40 Inches.	42 Inches.	48 Inches.	49 Inches.
1	0·39¼	0·52	0·65	0·795	0·87	0·92	1·05	1·07
¼	·49	·65	·82	·98	1·09	1·14	1·31	1·31
½	·589	·79	·98	1·18	1·31	1·37	1·57	1·60
¾	·687	·92	1·07	1·37	1·53	1·60	1·83	1·91
2	·785	1·05	1·31	1·57	1·74	1·83	2·09	2·12
¼	·883	1·18	1·37	1·77	1·83	2·06	2·35	2·45
½	·981	1·30	1·63	1·96	2·18	2·29	2·62	2·67
¾	1·08	1·44	1·79	2·16	2·40	2·52	2·88	2·94
3	1·178	1·57	1·96	2·36	2·61	2·75	3·14	3·20
¼	1·278	1·70	2·03	2·55	2·70	2·98	3·40	3·52
½	1·374	1·83	2·29	2·75	3·05	3·20	3·66	3·74
¾	1·472	1·96	2·45	2·94	3·27	3·43	3·92	4·
4	1·57	2·09	2·62	3·14	3·49	3·66	4·19	4·23
¼	1·679	2·22	2·68	3·34	3·57	3·89	4·45	5·57
½	1·767	2·35	2·75	3·53	3·93	4·12	4·70	4·9
¾	1·865	2·48	3·11	3·73	4·14	4·35	4·97	5·07
5	2·963	2·62	3·27	3·93	4·36	4·60	5·23	5·31
¼	2·061	2·75	3·44	4·12	4·59	4·81	5·50	5·64
½	2·159	2·88	3·60	4·32	4·81	5·04	5·76	5·88
¾	2·257	3.01	3·76	4·51	5·02	5·27	6·02	6·15
6	2·356	3·14	3·92	4·72	5·24	5·50	6·28	6·41
¼	2·454	3·28	4·00	4·91	5·46	5·73	6·55	6·68
½	2·552	3·40	4·25	5·10	5·67	5·95	6·81	7·07
¾	2·65	3·53	4·42	5·30	5·89	6·19	7·07	7·13
7	2·748	3·66	4·58	5·47	6·11	6·41	7·33	7·48
¼	2·847	3·80	4·74	5·69	6·33	6·64	7·59	7·55
½	2·947	3·93	4.91	5·89	6·55	6·88	7·86	8·02
¾	3·047	4·06	5·08	6·09	6·77	7·11	8·12	8·19
8	3·140	4·19	5·23	6·28	6·98	7·33	8·37	8·47
¼	3·239	4·32	5·40	6·48	7·20	7·56	8·64	8·82
½	3·338	4·45	5·56	6·68	7·42	7·79	8·90	9·09
¾	3·436	4·58	5·73	6·88	7·64	8·03	9·17	9·26
9	3·534	4·71	5·89	7·07	7·85	8·24	9·42	9·61
¼	3·632	4·84	6·05	7·26	8·07	8·47	9·68	9·89
½	3·730	4·97	6·22	7·46	8·29	8·71	9·94	10·16
¾	3·828	5·11	6·38	7·66	8·51	8·94	10·22	10·38
10	3·927	5·23	6·54	7·85	8·70	9·16	10·47	10·66
¼	4·024	5·36	6·70	8·05	8·93	9·39	10·73	10·94
½	4·122	5·49	6·87	8·24	9·18	9·61	10·99	11·28
¾	4·220	5·63	7·04	8·45	9·38	9·85	11·26	11·50
11	4·319	5·76	7·20	8·64	9·59	10.07	11·52	11·76
¼	4·417	5·88	7·35	8·84	9·80	10·30	11·77	12·02

	REQUIRED AREA.							
Given Diameter.	18 Inches.	24 Inches.	30 Inches.	36 Inches.	40 Inches.	42 Inches.	48 Inches.	49 Inches.
11½	4·515	6·02	7·52	9·04	10·04	10·53	12·04	12·30
¾	4·61	6·15	7·68	9·23	10·25	10·76	12·30	12·56
12	4·712	6.28	7·85	9·42	10·47	10·99	12·56	12·82
¼	4·810	6·41	8·01	9·62	10·69	11·22	12·82	13·09
½	4·91	6·55	8·19	9·81	10·92	11·45	13·11	13·36
¾	5·	6·67	8·32	10·	10·09	11·66	13·33	13·61
13	5·10	6·80	8·50	10·21	11·34	11·91	13·60	13·89
¼	5·20	6·94	8·67	10·40	11·56	12·13	13·88	14·16
½	5·30	7·06	8·83	10·60	11·77	12·38	14·12	14·51
¾	5·40	7·20	9·	10·79	11·99	12·60	14·40	14·69
14	5·49	7·33	9·16	10·99	12·21	12·82	14·65	14·96
¼	5·59	7·45	9·31	11·18	12·42	13·04	14·90	15·21
½	5·69	7·59	9·49	11·38	12·65	13·28	15·18	15·56
¾	5·78	7·71	9·65	11·56	12·84	13·48	15·43	15·74
15	5·88	7·85	9·82	11·79	13·	13·75	15·71	16·04
¼	5·98	7·97	9·97	11·96	13·29	13·95	15·94	16·27
½	6·08	8·10	10·12	12·16	13·51	14·18	16·20	16·54
¾	6·18	8·24	10·30	11·36	13·73	14·42	16·48	16·82
16	6·28	8·37	10·46	12·56	13·95	14·65	16·75	16·93
¼	6·38	8·50	10·62	12·76	14·17	14·88	17·	17·35
½	6·48	8·63	10·79	12·96	14·38	15·12	17·26	17·62
¾	6·57	8·76	10·95	13·14	14·60	15·36	47·52	17·88
17	6·67	8·90	11·12	13·34	14·81	15·58	17·80	18·17
¼	6·77	9·02	11·28	13·55	15·04	15·80	18·05	18·35
½	6·87	9·17	11·46	13·75	15·27	16·05	18·34	18·52
¾	6·97	9·29	11·61	13.94	15·84	16·26	18·58	18·96
18	7·07	9·42	11·77	14·13	15·70	16·48	18·84	19·22
¼	7·17	9·56	11·95	14·34	15·93	16·73	19·12	19·51
½	7·26	9·68	12·10	14·52	16·13	16·94	19·36	19·75
¾	7·36	9·80	12·25	14·72	16·33	17·17	19·61	20·01
19	7·46	9·94	12·42	14·92	16.57	17·40	19·88	20·31
¼	7·55	10·07	12·59	15·10	16·78	17·61	20·14	20·58
½	7·66	10·21	12·64	15·32	16·85	17·87	20·42	20·76
¾	7·76	10·34	12·92	15·52	17·22	18·10	20·68	21·15
20	7·85	10·46	13·08	15·70	17·40	18·32	20·92	21·32

EXPLANATION.—The length of any cylinder will be found at the head of each column, and the diameter in the left-hand column of the table.

Find the diameter of any cylinder, in inches, in the left-hand column of the table; then move to the right, on the *same line*, till you come under the length, in inches, and you will have the convex surface in square feet, and decimals of a foot.

Example.—What is the convex surface of a cylinder, 5¾ inches diameter, and 40 inches in length?

Against 5¾, and under 40, stands 5·02, which is the true answer sought in square feet, and decimals of a foot. If a cylinder should exceed, in length, any provision which has been made in this table, its convex surface would be double what is shown for half its length. A cylinder 60 inches long would contain double the convex surface as is shown in the table to be the surface of one 30 inches long.

IRON.

Cast iron expands $\frac{1}{162000}$ of its length for one degree of heat; the greatest change in the shade, in this climate, is $\frac{1}{1170}$ of its length; exposed to the sun's rays, $\frac{1}{1000}$; shrinks in cooling from $\frac{1}{85}$ to $\frac{1}{98}$ of its length; is crushed by a force of 93·000 lbs. upon a square inch; will bear, without permanent alteration, 15·300 lbs. upon a square inch, and an extension of $\frac{1}{1200}$ of its length.

Wrought iron expands $\frac{1}{143000}$ of its length for one degree of heat; will bear on a square inch, without permanent alteration, 17·800 lbs., and an extension in length of $\frac{1}{1400}$; cohesive force is diminished $\frac{1}{3000}$ by an increase of 1 degree of heat.

Weight of modulus of elasticity, for a base of one square inch, 24·920·000 lbs.; height of modulus of elasticity, 7 550·000 feet.

ANNEALING CAST IRON.

In the process of *annealing* cast iron, it is not desirable that the metal should be brought to any more than a *red* heat, as otherwise the smaller pieces and their bars might not only bend, but even melt. When the iron has become red hot, in a charcoal fire, it should be kept completely covered, and the fire allowed to go gradually out of its own accord. Or the iron, when red hot, may be buried in dry saw-dust, and in that state allowed to anneal. One great advantage of annealing cast iron is, that if it is afterwards subjected to a partial heating, it is less liable to warp than it would otherwise be. The character of cast iron is not in any way altered by annealing, except it is rendered more malleable. But the greatest benefit arising from this process will be seen from the following rule, which is exceedingly useful to iron founders, as by this means a great saving of expense may oftentimes be obtained in making new patterns.

To make a Casting of precisely the same Size of a broken Casting, without the original Pattern.

Rule.—Put the pieces of the broken casting together and mould them, and cast from this mould.

When the casting is drawn from the sand, place it in a charcoal fire and *anneal* it as above described; it will *expand* to the original size of the pattern, and there remain in that expanded state. This rule is of great importance to iron founders, as well as others, as by its observance much may be saved.

TABLE,

Exhibiting the Weight of a Lineal Foot of Square Rolled Iron, in lbs., from ¼ to 12 square inches.

Size in inches.	Weight in Pounds.	Size in inches.	Weight in Pounds.	Size in inches.	Weight in Pounds.	Size in inches.	Weight in Pounds.
¼	·211	2½	21·120	4¾	76·264	8	216·336
⅜	·475	⅝	23·292	⅞	80·333	¼	230·068
½	·845	¾	25·560	5	84·480	½	244·220
⅝	1·320	⅞	27·939	⅛	88·784	¾	258·800
¾	1·901	3	30·416	¼	93·168	9	273·792
⅞	2·588	⅛	33·010	⅜	97·657	¼	289·220
1	3·380	¼	35·704	½	102·240	½	305·056
⅛	4·278	⅜	38·503	⅝	106·953	¾	321·332
¼	5·280	½	41·408	¾	111·756	10	337·920
⅜	6·390	⅝	44·418	⅞	116·671	¼	355·136
½	7·604	¾	47·534	6	121·664	½	372·672
⅝	8·926	⅞	50·756	¼	132·040	¾	390·628
¾	10·352	4	54 084	½	142·816	11	408·960
⅞	11·883	⅛	57·517	¾	154·012	¼	427·812
2	13·520	¼	61·055	7	165·632	½	447·024
⅛	15·263	⅜	64·700	¼	177·672	¾	466·684
¼	17·112	½	68·448	½	190·136	12	486·656
⅜	19·066	⅝	72·305	¾	203·024		

Example.—What is the weight of a bar of rolled iron 1¾ inches square and 1 foot in length?

In column first, find 1¾, and opposite to it is 10·352 pounds, which is 10 lbs. and $\frac{352}{1000}$ of a lb.

If the lesser denomination of ounces is required, the result is obtained as follows: Multiply the remainder by 16, pointing off the decimals as in multiplication of decimals, and the figures remaining on the left of the point indicate the number of ounces.

$$\text{Thus, } \frac{352}{1000} \text{ of a lb.} = \begin{array}{r} 352 \\ 16 \\ \hline 2112 \\ 352 \\ \hline 5{\cdot}632 \text{ oz} \end{array}$$

The weight, then, is 10 lbs. $5\cdot\frac{632}{1000}$ ounces.

If the weight of a piece of iron less than a foot be required, first find the weight for a foot, and then take an aliquot part for the answer.

Thus, the weight of a bar, $2\frac{3}{4}$ inches square and 8 inches long, is obtained as follows:

$2\frac{3}{4}$ and 12 inches long, = 25·560 lbs.

And, 8 inches = $\frac{2}{3}$ of 12 inches; therefore, 25·560 × $\frac{2}{3}$ = 17·040 lbs.

TABLE,

Exhibiting the Weight of a Lineal Foot of Round Rolled Iron, from $\frac{1}{4}$ to 12 inches diameter.

Diam. in inches.	Weight in pounds.	Diam. in inches.	Weight in pounds.	Diam. in inches.	Weight in pounds.	Diam. in inches.	Weight in pounds.
$\frac{1}{4}$	·165	$2\frac{1}{2}$	16·688	$4\frac{3}{4}$	59·900	8	169·856
$\frac{3}{8}$	·373	$\frac{5}{8}$	18·293	$\frac{7}{8}$	63·094	$\frac{1}{4}$	180·696
$\frac{1}{2}$	.663	$\frac{3}{4}$	20·076	5	66·752	$\frac{1}{2}$	191·808
$\frac{5}{8}$	1·043	$\frac{7}{8}$	21·944	$\frac{1}{8}$	69·731	$\frac{3}{4}$	203·260
$\frac{3}{4}$	1·493	3	23·888	$\frac{1}{4}$	73·172	9	215·040
$\frac{7}{8}$	2·032	$\frac{1}{8}$	25·926	$\frac{3}{8}$	76·700	$\frac{1}{4}$	227·152
1	2·654	$\frac{1}{4}$	28·040	$\frac{1}{2}$	80·304	$\frac{1}{2}$	239·600
$\frac{1}{8}$	3·360	$\frac{3}{8}$	30·240	$\frac{5}{8}$	84·001	$\frac{3}{4}$	252·376
$\frac{1}{4}$	4·172	$\frac{1}{2}$	32·512	$\frac{3}{4}$	87·776	10	266·288
$\frac{3}{8}$	5·019	$\frac{5}{8}$	34·886	$\frac{7}{8}$	91·634	$\frac{1}{4}$	278·924
$\frac{1}{2}$	5·972	$\frac{3}{4}$	37·332	6	95·552	$\frac{1}{2}$	292·688
$\frac{5}{8}$	7·010	$\frac{7}{8}$	39·864	$\frac{1}{4}$	103·704	$\frac{3}{4}$	306·800
$\frac{3}{4}$	8·128	4	42·464	$\frac{1}{2}$	112·160	11	321·216
$\frac{7}{8}$	9·333	$\frac{1}{8}$	45·174	$\frac{3}{4}$	120·960	$\frac{1}{4}$	336·004
2	10·616	$\frac{1}{4}$	47·952	7	130·048	$\frac{1}{2}$	351·104
$\frac{1}{8}$	11·988	$\frac{3}{8}$	50·815	$\frac{1}{4}$	139·544	$\frac{3}{4}$	366·536
$\frac{1}{4}$	13·440	$\frac{1}{2}$	53·760	$\frac{1}{2}$	149·328	12	382·208
$\frac{3}{8}$	14·975	$\frac{5}{8}$	56·788	$\frac{3}{4}$	159·456		

NOTE. The application of this table is the same as the preceding one.

The weight of bar iron being 1;

The weight	of	cast iron	=	·95
"	"	steel	=	1·02
"	"	copper	=	1·16
"	"	brass	=	1·09
"	"	lead	=	1·48

TABLE,

Exhibiting the Weight of a Lineal Foot of Flat Bar Iron, in lbs., from ½ to 5½ inches in Breadth, and ⅛ to 5 inches in Thickness.

B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds
½	⅛	0·211	1⅜	1½	2·325	1⅞	2⅜	2·376
	¼	0·422		⅝	2·904		½	3·168
	⅜	0·634		¾	3·484		⅝	3·960
⅝	⅛	0·264		⅞	4·065		¾	4·752
	¼	0·528		1	4·646		⅞	5·544
	⅜	0·792		1⅛	5·227		1	6·336
	½	1·056		1¼	5·808		1⅛	7·129
¾	⅛	0·316	1½	⅛	0·633		1¼	7·921
	¼	0·633		¼	1·266		1⅜	8·713
	⅜	0·950		⅜	1·900		1½	9·505
	½	1·265		½	2·535		1⅝	10·297
	⅝	1·584		⅝	3·168		1¾	11·089
⅞	⅛	0·369		¾	3·802	2	⅛	0·845
	¼	0.738		⅞	4·435		¼	1·689
	⅜	1·108		1	5·069		⅜	2·534
	½	1·477		1⅛	5·703		½	3·379
	⅝	1·846		1¼	6·337		⅝	4·224
	¾	2·217		1⅜	6·970		¾	5·069
1	⅛	0·422	1⅝	⅛	0·686		⅞	5·914
	¼	0·845		¼	1·372		1	6·758
	⅜	1·267		⅜	2·059		1⅛	7·604
	½	1·690		½	2·746		1¼	8·448
	⅝	2·112		⅝	3·432		1⅜	9·294
	¾	2·534		¾	4·119		1½	10·138
	⅞	2·956		⅞	4·805		1⅝	10·983
1⅛	⅛	0·475		1	5·492		1¾	11·828
	¼	0·950		1⅛	6·178		1⅞	12·673
	⅜	1·425		1¼	6·864	2⅛	⅛	0·898
	½	1·901		1⅜	7·551		¼	1·795
	⅝	2·375		1½	8·237		⅜	2·693
	¾	2·850	1¾	⅛	0·739		½	3·591
	⅞	3·326		¼	1·479		⅝	4·488
	1	3·802		⅜	2·218		¾	5·386
1¼	⅛	0·528		½	2·957		⅞	6·283
	¼	1·056		⅝	3·696		1	7·181
	⅜	1·584		¾	4·435		1⅛	8·079
	½	2·112		⅞	5·178		1¼	8·977
	⅝	2·640		1	5·914		1⅜	9·874
	¾	3·168		1⅛	6·653		1½	10·772
	⅞	3·696		1¼	7·393		1⅝	11·670
	1	4·224		1⅜	8·132		1¾	12·567
	1⅛	4·752		1½	8·871		1⅞	13·465
1⅜	⅛	0·580		1⅝	9·610		2	14·362
	¼	1·161	1⅞	⅛	0·792	2¼	⅛	·950
	⅜	1·742		¼	1·584		¼	1·900

B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.
2¼	⅜	2·851	2½	2¼	19·008	2⅞	1	9·716
	½	3·802		2⅜	20·064		1⅛	10·931
	⅝	4·752	2⅝	⅛	1·109		1¼	12·145
	¾	5·703		¼	2·218		1⅜	13·360
	⅞	6·653		⅜	3·327		1½	14·574
	1	7·604		½	4·436		1⅝	15·789
	1⅛	8·554		⅝	5·545		1¾	17·003
	1¼	9·505		¾	6·654		1⅞	18·218
	1⅜	10·455		⅞	7·763		2	19·432
	1½	11·406		1	8·872		2⅛	20·647
	1⅝	12·356		1⅛	9·981		2¼	21·861
	1¾	13·307		1¼	11·090		2⅜	23·076
	1⅞	14·257		1⅜	12·199		2½	24·290
	2	15·208		1½	13·308		2⅝	25·505
	2⅛	16·158		1⅝	14·417		2¾	26·719
2⅜	⅛	1·003		1¾	15·526	3	⅛	1·267
	¼	2·006		1⅞	16·635		¼	2·535
	⅜	3·009		2	17·744		⅜	3·802
	½	4·013		2⅛	18·853		½	5·069
	⅝	5·016		2¼	19·962		⅝	6·337
	¾	6·019		2⅜	21·071		¾	7·604
	⅞	7·022		2½	22·180		⅞	8·871
	1	8·025	2¾	⅛	1·162		1	10·138
	1⅛	9·028		¼	2·323		1⅛	11·406
	1¼	10·032		⅜	3·485		1¼	12·673
	1⅜	11·035		½	4.647		1⅜	13·940
	1½	12·038		⅝	5·808		1½	15·208
	1⅝	13·042		¾	6·970		1⅝	16·475
	1¾	14·045		⅞	8·132		1¾	17·742
	1⅞	15·048		1	9·294		1⅞	19·010
	2	16·051		1⅛	10·455		2	20·277
	2⅛	17·054		1¼	11·617		2¼	22·811
	2¼	18·057		1⅜	12·779		2½	25·346
2½	⅛	1·056		1½	13·940		2¾	27·881
	¼	2·112		1⅝	15·102	3¼	⅛	1·373
	⅜	3·168		1¾	16·264		¼	2·746
	½	4·224		1⅞	17·425		⅜	4·119
	⅝	5·280		2	18·587		½	5·492
	¾	6·336		2⅛	19·749		⅝	6·865
	⅞	7·392		2¼	20·910		¾	8·237
	1	8·448		2⅜	22·072		⅞	9·610
	1⅛	9·504		2½	23·234		1	10·983
	1¼	10·560		2⅝	24·395		1⅛	12·356
	1⅜	11·616	2⅞	⅛	1·215		1¼	13·730
	1½	12·672		¼	2·429		1⅜	15·102
	1⅝	13·728		⅜	3·644		1½	16·475
	1¾	14·784		½	4·858		1⅝	17·848
	1⅞	15·840		⅝	6·072		1¾	19·221
	2	16·896		¾	7·287		1⅞	20·594
	2⅛	17·952		⅞	8·502		2	21·967

B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.
3¼	2¼	24·712	4	¾	10·138	4¾	1	16·052
	2½	27·458		1	13·518		1¼	20·066
	2¾	30·204		1¼	16·897		1½	24·079
	3	32·950		1½	20·277		1¾	28·092
3½	⅛	1·479		1¾	23·656		2	32·105
	¼	2·957		2	27·036		2¼	36·118
	⅜	4·436		2¼	30·415		2½	40·131
	½	5·914		2½	33·795		2¾	44·144
	⅝	7·393		2¾	37·174		3	48·157
	¾	8·871		3	40·554		3¼	52·170
	⅞	10·350		3¼	43·933		3½	56·184
	1	11·828		3½	47·313		3¾	60·197
	1⅛	13·307		3¾	50·692		4	64·210
	1¼	14·785	4¼	⅛	1·795		4¼	68·223
	1⅜	16·264		¼	3·591		4½	72·235
	1½	17·742		½	7·181	5	¼	4·224
	1⅝	19·221		¾	10·772		½	8·449
	1¾	20·699		1	14·364		¾	12·673
	1⅞	22·178		1¼	17·953		1	16·897
	2	23·656		1½	21·544		1¼	21·122
	2¼	26·613		1¾	25·135		1½	25·346
	2½	29·570		2	28·725		1¾	29·570
	2¾	32·527		2¼	32·316		2	33·795
	3	35·485		2½	35·907		2¼	38·019
	3¼	38·441		2¾	39·497		2½	42·243
3¾	⅛	1·584		3	43·088		2¾	46·468
	¼	3·168		3¼	46·679		3	50·692
	⅜	4·752		3½	50·269		3¼	54·916
	½	6.336		3¾	53·860		3½	59·140
	⅝	7·921		4	57·450		3¾	63·365
	¾	9·505	4½	¼	3·802		4	67·589
	⅞	11·089		½	7·604		4¼	71·813
	1	12·673		¾	11·406		4½	76·038
	1⅛	14·257		1	15·208		4¾	80·262
	1¼	15·841		1¼	19·010	5¼	¼	4·436
	1⅜	17·425		1½	22·812		½	8·871
	1½	19·009		1¾	26·614		¾	13·307
	1⅝	20·594		2	30 415		1	17·742
	1¾	22·178		2¼	34·217		1¼	22·178
	1⅞	23·762		2½	38·019		1½	26·613
	2	25 346		2¾	41·820		1¾	31·049
	2¼	28·514		3	45·623		2	35·484
	2½	31·682		3¼	49·425		2¼	39·920
	2¾	34·851		3½	53·226		2½	44·355
	3	38·019		3¾	57·028		2¾	48·791
	3¼	41·187		4	60·830		3	53·226
	3½	44·355		4¼	64·632		3¼	57·662
4	⅛	1·690	4¾	¼	4·013		3½	62·097
	¼	3·380		½	8·026		3¾	66·533
	½	6·759		¾	12·039		4	70·968

B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.	B'dth Inch.	Thickn'ss Inches.	Weight in Pounds.
$5\frac{1}{4}$	$4\frac{1}{4}$	75·404	$5\frac{1}{2}$	$1\frac{1}{4}$	23·234	$5\frac{1}{2}$	$3\frac{1}{4}$	60·408
	$4\frac{1}{2}$	79·839		$1\frac{1}{2}$	27·881		$3\frac{1}{2}$	65·055
	$4\frac{3}{4}$	84·275		$1\frac{3}{4}$	32·527		$3\frac{3}{4}$	69·701
	5	88·710		2	37·174		4	74·348
$5\frac{1}{2}$	$\frac{1}{4}$	4·647		$2\frac{1}{4}$	41·821		$4\frac{1}{4}$	78·995
	$\frac{1}{2}$	9·294		$2\frac{1}{2}$	46·468		$4\frac{1}{2}$	83·642
	$\frac{3}{4}$	13·940		$2\frac{3}{4}$	51·114		$4\frac{3}{4}$	88·288
	1	18·587		3	55·761		5	92·935

Example.—What is the weight of a bar of iron $4\frac{3}{4}$ inches in breadth by $1\frac{3}{4}$ inches thick ?

Find $4\frac{3}{4}$ inches in the column of *Breadths*, and below it in the column of *Thickness* find $1\frac{3}{4}$; and opposite to that is 28·092, which is 28 lbs. and $\frac{092}{1000}$ of a lb. Ans.

NOTE.—For parts of a foot operate precisely according to the rule laid down for table, page 183.

TABLE

Of the weight of one square foot of different Metals in avoirdupois pounds, from $\frac{1}{16}$ to 1 inch in thickness, advancing by $\frac{1}{16}$.

THICKNESS	WEIGHT.				
	Malleable Iron.	Cast Iron.	Copper.	Brass.	Lead.
INCHES.	LBS.	LBS.	LBS.	LBS.	LBS.
$\frac{1}{16}$	2·535	2·345	2·860	2·738	3·693
$\frac{1}{8}$	5·070	4·690	5·720	5·476	7·386
$\frac{3}{16}$	7·605	7·035	8·580	8·214	11·079
$\frac{1}{4}$	10·140	9·380	11·440	10·952	14·772
$\frac{5}{16}$	12·675	11·725	14·300	13·690	18·465
$\frac{3}{8}$	15·216	14·670	17·160	16·428	22·158
$\frac{7}{16}$	17·851	16·415	20·020	19·166	25·851
$\frac{1}{2}$	20·280	18·760	22·880	21·904	29·544
$\frac{9}{16}$	22·815	21·105	25·740	24·642	33·237
$\frac{5}{8}$	25·350	23·450	28·600	27·380	36·930
$\frac{11}{16}$	27·885	25·795	31·640	30·118	40·623
$\frac{3}{4}$	30·410	28·140	34·320	32·856	44·316
$\frac{13}{16}$	32·945	30·485	37·180	35·594	48.009
$\frac{7}{8}$	35·480	32·880	40·040	38·332	51·702
$\frac{15}{16}$	38·015	35·225	42·900	41·170	55·405
1	40·550	37·570	45·760	43·908	59·098

NOTE. The weight of cast iron being 1 ;

The weight of bar iron = 1·07
" " steel = 1·08
" " brass = 1·16
" " copper = 1·21
" " lead = 1·56

TABLE,

Exhibiting the Weight of Malleable Iron, Copper and Lead Pipes, 12 inches long, of various thicknesses, and any diameter required.

Thickness.	Malleable Iron.	Copper.	Lead.
IN.			
$\frac{1}{32}$	·104	·121	·1539
$\frac{1}{16}$	·208	·2419	·3078
$\frac{3}{32}$	·3108	·3628	·4616
$\frac{1}{8}$	·414	·4838	·6155
$\frac{1}{8}$ and $\frac{1}{32}$	·518	·6047	·7694
$\frac{1}{8}$ and $\frac{1}{16}$	·621	·7258	·9282
$\frac{1}{8}$ and $\frac{1}{32}$	·725	·8466	1·0771
$\frac{1}{4}$	·828	·9678	1·231

Rule.—Multiply the circumference of the pipe in inches by the numbers opposite the thickness required, and by the length, in feet; the product will be the weight in avoirdupois pounds, nearly.

Example.—Required the weight of a copper pipe 12 feet long, 15 inches in circumference, $\frac{1}{8}$ and $\frac{1}{16}$ of an inch in thickness.

$$\cdot7258 \times 15 = 10\cdot817 \times 12 = 130\cdot644 \text{ lbs., nearly.}$$

TABLE,

Showing the Weight of Copper Pipes 12 inches in length, and from $\frac{1}{2}$ inch to 3 inches in diameter, and $\frac{1}{8}$ of an inch in thickness.

Diam. of Bore.	Weight in lbs.	Diam of Bore.	Weight in lbs.	Diam. of Bore.	Weight in lbs.	Diam. of Bore.	Weight in lbs.
IN.		IN.		IN.		IN.	
$\frac{1}{2}$	·94	$1\frac{1}{4}$	2·08	$1\frac{7}{8}$	3·03	$2\frac{1}{2}$	3·97
$\frac{3}{4}$	1·33	$1\frac{3}{8}$	2·23	2	3·21	$2\frac{5}{8}$	4·12
$\frac{7}{8}$	1·51	$1\frac{1}{2}$	2·42	$2\frac{1}{8}$	3·39	$2\frac{3}{4}$	4·34
1	1·69	$1\frac{5}{8}$	2·67	$2\frac{1}{4}$	3·58	$2\frac{7}{8}$	4·56
$1\frac{1}{8}$	1·89	$1\frac{3}{4}$	2·87	$2\frac{3}{8}$	3·78	3	4·78

TABLE

Of the Weight of a Square Foot of Sheet Iron in lbs. Avoirdupois, the thickness being the number on the Birmingham Wire-guage.

No. 1 is $\frac{5}{16}$ of an inch; No. 4, $\frac{1}{4}$; No. 11, $\frac{1}{8}$, &c.

No. on wire-guage,	1	2	3	4	5	6	7	8	9	10	11
Pounds avoir.	12·5	12	11	10·	9	8	7·5	7	6	5·68	5
No. on wire-guage,	12	13	14	15	16	17	18	19	20	21	22
Pounds avoir.	4·62	4·31	4	3·95	3	2·5	2·18	1·93	1·62	1·5	1·37

TABLE

Of the Weight of Copper Bolts, from ¼ *to* 2⅛ *in. Diameter, and* 12 *inches long.*

Diameter.	Pounds.	Diameter.	Pounds.	Diameter.	Pounds.
$\frac{1}{4}$	·189	$\frac{13}{16}$	1·998	1 and $\frac{3}{8}$	5·723
$\frac{5}{6}$	·296	$\frac{7}{8}$	2·318	1 and $\frac{7}{16}$	6·255
$\frac{3}{8}$	·425	$\frac{15}{16}$	2·661	1 and $\frac{1}{2}$	6·811
$\frac{7}{16}$	·579	1	3·016	1 and $\frac{9}{16}$	7·390
$\frac{1}{2}$	·757	1 and $\frac{1}{16}$	3·417	1 and $\frac{5}{8}$	7·933
$\frac{9}{16}$	·958	1 and $\frac{1}{8}$	3·831	1 and $\frac{3}{4}$	9·270
$\frac{5}{8}$	1·182	1 and $\frac{3}{16}$	4·269	1 and $\frac{7}{8}$	10·642
$\frac{11}{16}$	1·431	1 and $\frac{1}{4}$	4·730	2	12·061
$\frac{3}{4}$	1·703	1 and $\frac{5}{16}$	5·214	2 and $\frac{1}{8}$	13·668

TABLE

Of the Weight of a Lineal Foot of Cast Iron Pipes, in lbs., from 1 *inch to* 30 *inches Bore.*

Bore.	Thickness.	Weight.	Bore.	Thickness.	Weight.	Bore.	Thickness.	Weight.
Inches.	Inches.	Pounds.	Inches.	Inches.	Pounds.	Inches.	Inches.	Pounds.
1	¼	3·06	3¾	½	20·90		⅞	63·18
	⅜	5·05		⅝	26·83	7	½	36·66
1¼	¼	3·67		¾	33·07		⅝	46·80
	⅜	6·	4	½	22·05		¾	56·96
1½	⅜	6·89		⅝	28·28		⅞	67·60
	½	9·80		¾	34·94		1	78·39
1¾	⅜	7·80	4¼	½	23·35	7½	½	39·22
	½	11·04		⅝	29·85		⅝	49·92
2	⅜	8·74		¾	36·73		¾	60·48
	½	12·23	4½	½	24·49		⅞	71·76
2¼	⅜	9·65		⅝	31·40		1	83·28
	½	13·48		¾	38·58	8	½	41·64
2½	⅜	10·57	4¾	½	25·70		⅝	52·68
	½	14·66		⅝	32·91		¾	64·27
	⅝	19·05		¾	40·43		⅞	76·12
2¾	⅜	11·54	5	½	26·94		1	88·20
	½	15·91		⅝	34·34	8½	½	44·11
	⅝	20·59		¾	42·28		⅝	56·16
3	⅜	12·28	5½	½	29·40		¾	68·
	½	17·15		⅝	37·44		⅞	80·50
	⅝	22·15		¾	45·94		1	93·28
	¾	27·56	6	½	31·82	9	½	46·50
3¼	½	18·40		⅝	40·56		⅝	58·92
	⅝	23·72		¾	49·60		¾	71·70
	¾	29·64		⅞	58·96		⅞	84·70
3½	½	19·66	6½	½	34·32		1	97·98
	⅝	25·27		⅝	43·68	9½	½	48·98
	¾	31·20		¾	53·30		⅝	62·02

Bore.	Thickness.	Weight.	Bore.	Thickness.	Weight.	Bore.	Thickness.	Weight.
Inches.	Inches.	Pounds.	Inches.	Inches.	Pounds.	Inches.	Inches.	Pounds.
9½	¾	75·32	14	⅝	89·61	19	¾	145·20
	⅞	88·98		¾	108·46		⅞	170·47
	1	102·90		⅞	127·60		1	195·92
10	½	51·46		1	147·03	20	⅝	126·33
	⅝	65·08	14½	½	73·72		¾	152·53
	¾	78 99		⅝	92·66		⅞	179·02
	⅞	93·24		¾	112·10		1	205·80
	1	108·84		⅞	131·86	21	⅝	132·50
10½	½	53·88		1	151·92		¾	159·84
	⅝	68·14	15	½	75·96		⅞	187·60
	¾	82·68		⅝	95·72		1	215·52
	⅞	97·44		¾	115·78	22	⅝	138·60
	1	112·68		⅞	136·15		¾	167·24
11	½	56·34		1	156·82		⅞	196·46
	⅝	71·19	15½	½	78·40		1	225·38
	¾	86·40		⅝	98·78	23	⅝	144·77
	⅞	101·83		¾	119·48		¾	174·62
	1	117·60		⅞	140·40		⅞	204·78
11½	½	58·82		1	161·82		1	235·28
	⅝	74·28	16	½	80·87	24	⅝	150·85
	¾	90·06		⅝	101·82		¾	181·92
	⅞	106·14		¾	123·14		⅞	213·28
	1	122·62		⅞	144·76		1	245·08
12	½	61·26		1	166·60	25	⅝	156·97
	⅝	77·36	16½	½	83·30		¾	189·28
	¾	93·70		⅝	104·82		⅞	221·94
	⅞	110·48		¾	126·79		1	254·86
	1	127·42		⅞	149·02	26	¾	196·62
12½	½	63·70		1	171·60		⅞	230·56
	⅝	80·40	17	½	85·73		1	264·66
	¾	97·40		⅝	107·96	27	¾	204·04
	⅞	114·72		¾	130·48		⅞	239·08
	1	132·35		⅞	153·30		1	274·56
13	½	66·14		1	176·58	28	¾	211·32
	⅝	83·46	17½	½	88·23		⅞	247·62
	¾	101·08		⅝	111·06		1	284·28
	⅞	118·97		¾	134·16	29	¾	218·70
	1	137·28		⅞	157·59		⅞	256·20
13½	½	68·64		1	181·33		1	294·02
	⅝	86·55	18	⅝	114·10	30	¾	226·20
	¾	104·76		¾	137·84		⅞	264·79
	⅞	123·30		⅞	161·90		1	303·86
	1	142·16		1	186·24		1⅛	343·20
14	½	71·07	19	⅝	120·24			

NOTE. These weights do not include any allowance for spigot and faucet ends.

The above table is found to be of great use in making out correct estimates of cast iron pipes. For instance, suppose it is required to know the weight of a range of pipes, 324 feet long, 8½ inches diam-

eter of bore, and metal $\frac{5}{8}$ of an inch thick. The table shows the weight of 1 foot of such pipe to be 56·16 lbs:

Then, $56{\cdot}16 \times 324 = 18195{\cdot}84$ lbs., or $9\frac{1}{10}$ tons, very nearly.

TABLE,

Showing the Weight of Solid Cylinders of Cast Iron, 12 inches long, in Avoirdupois Pounds.

Diameter in Inches.	Weight in lbs.	Diameter in Inches.	Weight in lbs.	Diameter in Inches.	Weight in lbs.	Diameter in Inches.	Weight in lbs.
$\frac{3}{4}$	1·394	$2\frac{1}{2}$	15·492	$4\frac{1}{2}$	50·193	8	158·638
$\frac{7}{8}$	1·897	$2\frac{5}{8}$	17·080	$4\frac{3}{4}$	55·926	$8\frac{1}{2}$	179·087
1 in.	2·478	$2\frac{3}{4}$	18·745	5	61·968	9	200·774
$1\frac{1}{8}$	3·137	$2\frac{7}{8}$	20·488	$5\frac{1}{4}$	68·319	$9\frac{1}{2}$	223·704
$1\frac{1}{4}$	3·873	3	22·308	$5\frac{1}{2}$	74·981	10	247·872
$1\frac{3}{8}$	4·686	$3\frac{1}{8}$	24·206	$5\frac{3}{4}$	81·952	$10\frac{1}{2}$	273·278
$1\frac{1}{2}$	5·577	$3\frac{1}{4}$	26·181	6	89·234	11	299·925
$1\frac{5}{8}$	6·545	$3\frac{3}{8}$	28·234	$6\frac{1}{4}$	96·825	$11\frac{1}{2}$	327·811
$1\frac{3}{4}$	7·591	$3\frac{1}{2}$	30·364	$6\frac{1}{2}$	104·726	12	356·935
$1\frac{7}{8}$	8·714	$3\frac{5}{8}$	32·572	$6\frac{3}{4}$	112·936	13	418·903
2	9·915	$3\frac{3}{4}$	34·857	7	121·457	14	485·830
$2\frac{1}{8}$	11·193	$3\frac{7}{8}$	37·219	$7\frac{1}{4}$	130·287	15	557·712
$2\frac{1}{4}$	12·548	4	39·660	$7\frac{1}{2}$	139·428	16	634·552
$2\frac{3}{8}$	13·981	$4\frac{1}{4}$	44·771	$7\frac{3}{4}$	148·878		

NOTE. Cubic inches of cast iron × ·263 = lbs. avoirdupois.
Circular inches of cast iron × ·2065 = lbs. avoirdupois.

TABLE,

Showing the Capacity and Weight of Cast Iron and Lead Balls, from 1 inch to $8\frac{1}{2}$ Diameter.

Diam. inches.	Capacity in Cubic Inches.	Cast Iron Pounds.	Lead Pounds.	Diam. inches.	Capacity in Cubic Inches.	Cast Iron Pounds	Lead Pounds.
1	·523	·136	·215	5	65·450	17·063	26·843
$1\frac{1}{2}$	1·767	·461	·725	$5\frac{1}{2}$	87·114	22·721	35·729
2	1·189	1·092	1·718	6	113·097	29·484	46·385
$2\frac{1}{2}$	8·181	2·133	3·355	$6\frac{1}{2}$	143·793	37·453	58·976
3	14·137	3·685	5·798	7	179·594	46·820	73·659
$3\frac{1}{2}$	22·449	5·852	9·207	$7\frac{1}{2}$	220·893	57·587	90·598
4	33·510	8·736	13·744	8	268·082	69·889	109·552
$4\frac{1}{2}$	47·713	12·439	19·569	$8\frac{1}{2}$	321·555	83·840	131·883

TABLE,

Showing the Number of Nails and Spikes to the Pound, of Various Sizes, as manufactured at the Troy Iron and Nail Factory, N. Y.

Size of Nails.	No. to the lb.	Boat Spikes.	Diameter of Rod.	No. Spikes to the lb.	Ship Spikes.	Diameter of Rod.	No. Spikes to the lb.
3 penny	600	No. 4	$\frac{1}{4}$	13	No. 4	$\frac{5}{16}$	8
4 "	360	" 5	$\frac{5}{16}$	8	" 5	$\frac{3}{8}$	6
6 "	200	" 6	$\frac{5}{8}$	5	" 6	$\frac{3}{8}$	5
8 "	110	" 7	$\frac{3}{8}$	4	" 7	$\frac{3}{8}$	$3\frac{1}{2}$
10 "	88				" 8	$\frac{3}{8}$	3
12 "	68				" 9	$\frac{9}{16}$	2
20 "	40				" 10	$\frac{9}{16}$	$1\frac{1}{2}$

TABLE

Of the Weight of Lead Pipe per Yard, from ¼ to 4½ inches Diameter.

Diameter.	Weight in lbs.	and oz.	Diameter.	Weight in lbs.	and oz.
¼ inch medium	3	—	1½ inch extra light	9	—
“ strong	4	—	“ light	13	—
½ inch light	3	—	“ medium	15	8
“ medium	4	—	“ strong	19	—
“ strong	5	—	1¾ inch medium	16	—
“ extra strong	6	6	“ strong	20	—
⅝ inch light	5	—	2 inch light	16	12
“ medium	6	8	“ medium	20	—
“ strong	7	8	“ strong	23	—
“ extra strong	8	4	2½ inch light	25	—
¾ inch extra light	5	—	“ medium	30	—
“ light	6	4	“ strong	35	—
“ medium	8	—	3 inch light	30	—
“ strong	9	12	“ medium	35	—
“ extra strong	10	8	“ strong	44	—
1 inch extra light	6	14	3½ inch medium	45	—
“ light	8	5	“ strong	54	—
“ medium	10	5	“ extra strong	70	—
“ strong	12	4	4 inch waste, light		
1¼ inch extra light	8	5	“ “ medium	21	—
“ light	9	12	“ “ strong	26	—
“ medium	11	—	4½ inch “ light	—	—
“ strong	12	8	“ “ medium	24	—
“ extra strong	14	10	“ “ strong	29	—

VERY LIGHT PIPE.

Diameter.	Weight in lbs.	and oz.	Diameter.	Weight in lbs.	and oz.
¼ inch	1	—	¾ inch	3	6
⅜ “	1½	—	1 “	5	10
½ “	2	—	1¼ “	6	14
⅝ “	2½	—			

TABLE

Of the Weight of Tin Plate.

KIND OF TIN.	Size of sheet.	MEAN THICKNESS. No. on wire guage.	MEAN THICKNESS. Thickness of sheet.	Mean weight of one sheet.	REMARKS.
	IN.		IN.	LBS.	
Single Tin	10×14	31	0·0125 or 80 to 1 in.	0·5	There are usually 225 sheets in a box.
Double X	10×14	27	0·018 or 55 to 1 in.	0·75	

TABLE

OF

THE WEIGHT, IN LBS.

Of a foot in length of Cast Iron.

Side of the square or diameter.	Square.	Hex'on.	Oct'gon.	Circle.	Side of the square or diameter.	Square.	Hex'on.	Oct'gon.	Circle.
inches.					inches.				
½	·781	·675	·650	·612	6½	132·031	114·271	109·948	103·696
¾	1·756	1·528	1·471	1·387	6¾	142·381	123·231	118·534	111·825
1	3·125	2·703	2·603	2·454	7	153·125	132·528	127·478	120·372
1¼	4·881	4·225	4·065	3·854	7¼	161·256	142·162	136·743	128·986
1½	7·031	6·085	5·856	5·521	7½	175·781	152·037	146·337	138·056
1¾	9·568	8·281	7·971	7·515	7¾	187·693	162·449	156·259	147·415
2	12·520	10·815	10·412	9·815	8	200·000	173·099	166·503	157·078
2¼	15·818	13·990	13·168	12·425	8¼	212·693	184·087	177·071	167·049
2½	19·531	16·900	16·256	15·337	8½	225·781	195·412	187·365	177·328
2¾	23·631	20·450	19·671	18·559	8¾	239·256	207·078	199·127	187·912
3	28·125	24·340	23·412	22·087	9	253·125	219·078	210·721	199·203
3¼	33·009	28·565	27·475	25·921	9¼	266·781	231·418	222·600	210·800
3½	38·281	33·131	31·818	30·065	9½	282·031	244·100	234·793	221·506
3¾	43·943	38·031	36·581	31·512	9¾	296·968	257·105	247·315	233·318
4	50·000	43·271	41·621	39·268	10	312·500	270·471	260·163	245·437
4¼	56·443	48·353	46·990	44·331	10¼	328·318	284·159	273·341	257·859
4½	63·281	54·768	52·681	49·700	10½	344·531	298·193	286·828	270·593
4¾	70·506	61·021	58·696	55·375	10¾	351·131	312·559	300·646	283·633
5	78·125	67·515	65·040	61·359	11	378·125	327·268	314·796	296·978
5¼	86·131	74·549	71·701	67·709	11¼	393·216	342·315	329·268	310·631
5½	94·531	81·815	78·696	74·243	11½	410·281	357·693	344·062	324·587
5¾	103·318	89·421	86·015	81·126	11¾	429·023	373·325	359·187	338·856
6	112·500	97·368	93·656	88·354	12	450·000	389·475	374·631	353·428
6¼	122·058	105·640	101·621	95·871					

TABLE

OF

THE RELATIVE WEIGHT AND STRENGTH OF ROPES AND CHAINS.

Circumference of Rope in . .	Weight per fathom in	Diameter of Chain in	Weight per fathom in	Proof of strength in .		Circumference of Rope in . .	Weight per fathom in	Diameter of Chain in	Weight per fathom in	Proof of strength in	
inches.	lbs.	inches.	lbs.	tons.	cwts.	inches.	lbs.	inches.	lbs.	tons.	cwts.
3½	2¾	$\frac{5}{16}$	5½	1	5½	10	23	⅞	43	10	0
4¼	4¾	⅜	8	1	16¾	10¾	28	$\frac{15}{16}$	49	11	11
5	5¾	$\frac{7}{16}$	10½	2	10	11½	30½	1	56	13	8
5¾	7	½	14	3	5½	12¼	36	$1\frac{1}{16}$	63	14	18
6½	9¾	$\frac{9}{16}$	18	4	3½	13	39	1⅛	71	16	14
7	11¼	⅝	22	5	2	13¾	45	$1\frac{3}{16}$	79	18	11
8	15	$\frac{11}{16}$	27	6	4½	14½	48½	1¼	87	20	8
8¾	19	¾	32	7	7	15¼	56	$1\frac{5}{16}$	96	22	13
9½	21	$\frac{13}{16}$	37	8	13½	16	60	1⅜	106	24	18

TABLE

Of Hollow Cylindrical Shafts, for Large Mills, with Cores, according to the Sizes they are usually made.

Diameter of Shaft at small End.	Diameter of a Cross Section in the Middle.	Diameter of the Core at small End.	Diameter of the Core at the Middle.	Length of the Shaft in Feet.	No. of Cubic Inches in the Shaft.	Total Weight of Shaft in lbs.
inches.	inches.	inches.	inches.			
7	11	2	$3\frac{3}{4}$	12	8944	2236
8	12	2	4	12	10400	2600
9	14	3	$7\frac{1}{2}$	14	13849	3462
10	16	4	10	16	33744	8438

NOTE.—It has been fully tested that cast iron shafts of a large dimension with a core is stronger than without it. This no doubt arises from the fact that the case affords a sensible spring in case of a sudden strain, and therefore less liable to fracture.

Should any shaft vary from the above dimensions, as will frequently be the case, the weight and cubic contents will be found by the following

Rule.—To the areas of the *end* and middle diameters add the *square root of their product;* this sum, multiplied by $\frac{1}{3}$ of whole length of the shaft in *inches*, equals the solidity of the shaft. By the same rule the solidity of the core is found, which, when subtracted from the whole solidity of the shaft, leaves the true solidity. This last sum divided by 4 equals the weight in avoirdupois pounds.

That is,

$$A+a+\sqrt{A+a}\times\frac{l}{3}-\left(A'+a'+\sqrt{A'+a'}\right)\times\frac{l}{3}\div 4=W.$$

Where A equals the area of the middle diameter,
a " " " end "
l " the whole length in inches,
A′ " the area of the core at center,
a' " " " " at end,
l " length of core in inches, and
W " the whole weight.

NOTE.—Shafts are distinguished into long and short and square, solid cylindrical and hollow cylindrical. The cylindrical shaft is the best form; for when it is not cylindrical, the flexure will vary in different parts of the revolution, and cause a want of uniformity in the motion. Feathered shafts are preferable to square ones; cylindrical shafts to both; and, for large shafts, hollow cylindrical to solid.

Let W, be the weight or stress upon the shaft in cwt;
l, the length of the gudgeon in inches; and d, the diameter of the gudgeon;

$$\text{Then, } d=\sqrt[3]{(W\times l)}\times 0.42.$$

Where the gudgeon is not exposed to much wear, but where it is as in water-wheels, the number 0·6 should be used, instead of 0·42.

TABLE,

EXHIBITING THE EXPERIMENTAL STRENGTH

Of various species of Timber opposed to a transverse strain.

Kinds of Wood.	Specific Gravity.	Length in feet.	Breadth in inches.	Depth in inches.	Deflection at the time of fracture.	Breaking wt. in pounds.	Value of constant strength.	Authorities.
Oak, English, young tree	·863	2·	1	1	1·87	482	2892	Tredgold.
Do. old ship timber . .	·872	2·5	1	1	1·5	264	1980	do.
Do. from old tree . .	·625	2·	1	1	1·38	218	1308	do.
Do. medium quality . .	·748	2·5	1	1	‘	284	2130	Ebbels.
Do. green	·763	2·5	1	1	‘	219	1741	do.
Do. do.	1·063	11·75	8.5	8·5	3·2	24812	1785	Buffon.
Beech, medium quality	·690	2.5	1	1	,	271	2031	Ebbels.
Alder	·555	2·5	1	1	‘	212	1590	do.
Plane tree	·648	2·5	1	1	‘	243	1821	do.
Sycamore	·590	2·5	1	1	‘	214	1605	do.
Chestnut tree	·875	2·5	1	1	‘	180	1350	do.
Ash, from young tree .	·811	2·5	1	1	2·5	324	2430	Tredgold.
Do. medium quality .	·690	2·5	1	1	‘	254	1905	Ebbels.
Ash	·753	2·5	1	1	2·38	314	2355	Tredgold.
Elm, common	·544	2·5	1	1	,	216	1620	Ebbels.
Do. wycb, green . . .	·763	2·5	1	1	;	192	1440	do.
Acacia, green	·820	2·5	1	1	;	249	1866	do.
Mahogany, Spanish, dry	·852	2·5	1	1	‘	170	1275	Tredgold.
Do. Honduras, seasoned	·256	2·5	1	1	’	255	1911	do.
Walnut, green	·925	2·5	1	1	‘	195	1461	Ebbels.
Teak	·744	7·	2	2	4·00	820	2151	Barlow.
Willow	·405	2·5	1	1	3·	146	1095	Tredgold.
Birch	·720	2·5	1	1	‘	207	1551	Ebbels.
Cedar of Libanus, dry .	·586	2·5	1	1	2·75	165	1236	Tredgold.
Riga fir	·480	2·5	1	1	1·3	212	1590	do.
Memel fir	·553	2·5	1	1	1·15	218	1635	do.
Norway fir fm. Longsound	·639	2·	1	1	1·125	396	2376	do.
Mar forest fir	·715	7·	2	2	5·5	360	945	Barlow.
Scotch fir, Eng. growth	·529	2·5	1	1	1·75	233	1746	Tredgold.
Do. do.	·460	2·5	1	1	‘	157	1176	Ebbels.
Christiana white deal .	·512	2·	1	1	·937	343	2058	Tredgold.
American white spruce .	·465	2·	1	1	1·362	285	1710	do.
Spruce fir, British growth	·555	2·5	1	1	‘	186	1395	Ebbels.
American pine, Weymouth	·460	2·0	1	1	1·125	329	1974	Tredgold.
Larch, choice specimen	·640	2·5	1	1	3·0	253	1896	do.
Do. medium quality . .	·622	2·5	1	1	‘	223	1671	do.
Do. very young wood .	·396	2·5	1	1	1·78	129	966	do.
English oak	·934	7·	2	2	8·1	637	1672	Barlow.
Canadian, do.	·872	7·	2	2	6·0	673	1766	do.
Dantzic, do.	·756	7·	2	2	4·86	560	1457	do.
Adriatic, do.	·993	7·	2	2	5·73	526	1383	do.
Ash	·760	7·	2	2	8·92	772	2026	do.
Beech	·696	7·	2	2	5·73	593	1556	do.
Pitch pine	·660	7·	2	2	6·00	622	1632	do.
Red pine	·657	7·	2	2	5·83	511	1341	do.
New England fir . . .	·553	7·	2	2	4·66	420	1102	do.

TABLE,

Exhibiting the number of Threads to an inch in V.-thread Screws.

Diam. in inches.	No. of threads.	Diam. in in.	No. of threads.	Diam. in in.	No. of threads.
$\frac{1}{4}$	20	$1\frac{3}{8}$	6	$3\frac{1}{2}$	$3\frac{1}{4}$
$\frac{5}{16}$	18	$1\frac{1}{2}$	6	$3\frac{3}{4}$	3
$\frac{3}{8}$	16	$1\frac{5}{8}$	5	4	3
$\frac{7}{16}$	14	$1\frac{3}{4}$	5	$4\frac{1}{4}$	$2\frac{7}{8}$
$\frac{1}{2}$	12	$1\frac{7}{8}$	$4\frac{1}{2}$	$4\frac{1}{2}$	$2\frac{7}{8}$
$\frac{5}{8}$	11	2	$4\frac{1}{2}$	$4\frac{3}{4}$	$2\frac{3}{4}$
$\frac{3}{4}$	10	$2\frac{1}{4}$	4	5	$2\frac{3}{4}$
$\frac{7}{8}$	9	$2\frac{1}{2}$	4	$5\frac{1}{4}$	$2\frac{5}{8}$
1	8	$2\frac{3}{4}$	$3\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{5}{8}$
$1\frac{1}{8}$	7	3	$3\frac{1}{2}$	$5\frac{3}{4}$	$2\frac{1}{2}$
$1\frac{1}{4}$	7	$3\frac{1}{4}$	$3\frac{1}{4}$	6	$2\frac{1}{2}$

The depth of the threads should not be less than half their pitch.

The diameter of a screw, to work in the teeth of a wheel, should be such that the angle of the threads does not exceed 10°.

Friction constitutes the whole efficacy of the screw.

TABLE

Of the sizes of Nuts, equal in strength to their Bolts.

Diameter of bolt in inches.	Short diameter of nut in inches.	Diam. of bolt in in.	Short diameter of nut in inches.	Diam. of bolt in in.	Short diameter of nut in inches.
$\frac{1}{4}$	$\frac{3}{8}$	$1\frac{3}{8}$	$2\frac{7}{16}$	$2\frac{1}{2}$	$4\frac{7}{16}$
$\frac{3}{8}$	$\frac{5}{8}$	$1\frac{1}{2}$	$2\frac{11}{16}$	$2\frac{5}{8}$	$4\frac{3}{4}$
$\frac{1}{2}$	$\frac{7}{8}$	$1\frac{5}{8}$	$2\frac{7}{8}$	$2\frac{3}{4}$	$4\frac{15}{16}$
$\frac{5}{8}$	$1\frac{1}{16}$	$1\frac{3}{4}$	$3\frac{1}{8}$	$2\frac{7}{8}$	$5\frac{1}{8}$
$\frac{3}{4}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$3\frac{3}{8}$	3	$5\frac{3}{8}$
$\frac{7}{8}$	$1\frac{9}{16}$	2	$3\frac{9}{16}$	$3\frac{1}{4}$	$5\frac{7}{8}$
1	$1\frac{3}{4}$	$2\frac{1}{8}$	$3\frac{3}{4}$	$3\frac{1}{2}$	$6\frac{5}{16}$
$1\frac{1}{8}$	2	$2\frac{1}{4}$	4	$3\frac{3}{4}$	$6\frac{3}{4}$
$1\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$4\frac{1}{4}$	4	$7\frac{1}{8}$

Note. The depth of the head should equal the diameter of the bolt; the depth of the nut should exceed it, in the proportion of 9 or 10 to 8.

TABLE

Of Change Wheels for Screw Cutting, the leading screw being ½ inch pitch, or containing 2 threads in an inch.

Number of Threads in Inch of Screw.	Number of Teeth in: Lathe Spindle-wheel.	Number of Teeth in: Leading Screw-wheel.	Number of Threads in Inch of Screw.	Number of Teeth in: Lathe Spindle-wheel.	Number of Teeth in: Wheel in contact with Spindle-wheel.	Number of Teeth in: Pinion in contact with Screw-wheel.	Number of Teeth in: Leading Screw-wheel.	Number of Threads in Inch of Screw.	Number of Teeth in: Lathe Spindle-wheel.	Number of Teeth in: Wheel in contact with Spindle-wheel.	Number of Teeth in: Pinion in contact with Screw-wheel.	Number of Teeth in: Leading Screw-wheel.
1	80	40	8¼	40	55	20	60	19	50	95	20	100
1¼	80	50	8½	90	85	20	90	19½	80	120	20	130
1½	80	60	8¾	60	70	20	75	20	60	100	20	120
1¾	80	70	9½	90	90	20	95	20¼	40	90	20	90
2	80	90	3¾	40	60	20	65	21	80	120	20	140
2¼	80	90	10	60	75	20	80	22	60	110	20	120
2½	80	100	10½	50	70	20	75	22½	80	120	20	150
2¾	80	110	11	60	55	20	120	22¾	80	130	20	140
3	80	120	12	90	90	20	120	23¾	40	95	20	100
3¼	80	130	12¾	60	85	20	90	24	65	120	20	130
3½	80	140	13	90	90	20	130	25	60	100	20	150
3¾	80	150	13½	60	90	20	90	25½	30	85	20	90
4	40	80	13¾	80	100	20	110	26	70	130	20	140
4¼	40	85	14	90	90	20	140	27	40	90	20	120
4½	40	90	14¼	60	90	20	95	27½	40	100	20	110
4¾	40	95	15	90	90	20	150	28	75	140	20	150
5	40	100	16	60	80	20	120	28½	30	90	20	95
5½	40	110	16¼	80	100	20	130	30	70	140	20	150
6	40	120	16½	80	110	20	120	32	30	80	20	120
6½	40	130	17	45	85	20	90	33	40	110	20	120
7	40	140	17½	80	100	20	140	34	30	85	20	120
7½	40	150	18	40	60	20	120	35	60	140	20	150
8	30	120	18¾	80	100	20	150	36	30	90	20	120

TABLE,

OF

THE CURVATURE OF THE EARTH.

Dist. in miles.	Height.	Dist. in miles.	Height.	Dist. in miles.	Height.	Dist. in miles.	Height.
	inches.		ft. in.		feet.		feet.
¼	½	7	32 5	15	149	40	1064
½	2	8	42 5	16	170	45	1346
1	8	8	53 8	17	192	50	1662
	ft. in.	10	66 4	18	215	60	2394
2	2 6	11	80 2	19	240	70	3258
3	6	12	95 4	20	266	80	4255
4	10 6	13	112	25	415	90	5386
5	16 6	14	130	30	599	100	6649
6	23 9			35	814		

COPPER.

To find the Weight of Cast Copper.

Rule.—Ascertain the number of cubic inches in the piece, and multiply them by ·317 (for copper plate multiply by ·321), and the product will be the weight in avoirdupois pounds.

Example.—What is the weight of a piece of cast copper 1 inch thick and 20 inches square; and what of copper plate $\frac{1}{2}$ inch thick and 10 inches square?

$$20\times20\times1=400 \text{ and } 400\times{\cdot}317=126{\cdot}80 \text{ lbs.}$$
$$\text{And } 10\times10\times\tfrac{1}{2}=\ 50\ \text{“}\quad 50\times{\cdot}321=160{\cdot}50 \text{ lbs.}$$

BRASS.

To find the Weight of Brass Castings.

Rule.—Find the number of cubic inches in the piece, and multiply them by ·311, the product will be the weight in avoirdupois pounds.

Example.—Required the weight of a piece of casting, containing 117 cubic inches.

$$117\times{\cdot}311=36{\cdot}378 \text{ lbs.}$$

36·387

LEAD

To find the Weight of Lead.

Rule.—Ascertain by calculation, the number of cubic inches it contains, and multiply them by ·41, and the product will be the weight in avoirdupois pounds.

Example.—Required the weight of a leaden pipe 18 feet long, 3 inches bore in diameter, and $\frac{1}{2}$ inch thick.

By Areas of Circles and Rules in Mensuration, &c.

Area of 4 inch = 12·566
“ “ 3 inch = 7·068

18 feet = 216 inches. $5{\cdot}498\times216=1187{\cdot}568$ area of ring

$$\begin{array}{r} 1187{\cdot}568 \\ {\cdot}41 \\ \hline 486{\cdot}90288 \text{ lbs} \end{array}$$

TABLE,

Showing the Size of Cables and Anchors, proportioned to the Tonnage of Vessels.

Tonnage of Vessel.	Cables. Circumf. in Inches.	Chain Cables. Diameter in Inches.	Proof in Tons.	Weight of Anchor in Pounds.	Weight of a Fathom of Chain.	Weight of a Fathom of Cable.
5	3	$\frac{5}{16}$	$\frac{3}{4}$	56	$5\frac{1}{2}$	2·1
8	4	$\frac{3}{8}$	$1\frac{3}{4}$	84	8	4
10	$4\frac{1}{2}$	$\frac{7}{16}$	$2\frac{1}{2}$	112	11	4·6
15	5	$\frac{1}{2}$	4	168	14	6·5
25	6	$\frac{9}{16}$	5	224	17	8·4
40	$6\frac{1}{2}$	$\frac{5}{8}$	6	336	24	9·8
60	7	$\frac{11}{16}$	7	392	27	11·4
75	$7\frac{1}{2}$	$\frac{3}{4}$	9	532	30	13·
100	8	$\frac{13}{16}$	10	616	36	15·
130	9	$\frac{7}{8}$	12	700	42	18·9
150	$9\frac{1}{2}$	$\frac{15}{16}$	14	840	50	21·
180	$10\frac{1}{2}$	1	16	952	56	25·7
200	11	$1\frac{1}{16}$	18	1176	60	28·2

The proof in the U. S. Naval Service, is about $12\frac{1}{2}$ per cent. less than the above.

TABLE,

Showing what Weight a Good Hemp Cable will bear with Safety.

Circumference.	Pounds.	Circumference.	Pounds.	Circumference.	Pounds.
6·	4320·	9·50	10830·	13·	20280·
6·50	5070·	10·	12000·	13·50	21870·
7·	5880·	10·50	13230·	14·	23520·
7·50	6750·	11·	14520·	14·50	25230·
8·	7680·	11·50	15870·	15·	27000·
8·50	8670·	12·	17280·	15·50	28830·
9·	9720·	12·50	18750·		

To ascertain the Strength of Cables.

Rule.—Multiply the square of the circumference, in inches, by 120, and the product is the weight the cable will bear, in pounds, with safety.

TABLE,

Showing what Weight a Hemp Rope will bear with Safety.

Circumference.	Pounds.	Circumference.	Pounds.	Circumference.	Pounds.
1 in.	200	$3\frac{1}{2}$	2450	$5\frac{1}{2}$	6050
$1\frac{1}{4}$	312	$3\frac{3}{4}$	2812	$5\frac{3}{4}$	6612
$1\frac{3}{4}$	612	4	3200	6	7200
2	800	$4\frac{1}{4}$	4512	$6\frac{1}{4}$	7812
$2\frac{1}{4}$	1012	$4\frac{1}{2}$	4050	$6\frac{1}{2}$	8450
$2\frac{1}{2}$	1250	$4\frac{3}{4}$	4512	$6\frac{3}{4}$	9112
3	1800	5	5000	7	9800
$3\frac{1}{4}$	2112	$5\frac{1}{4}$	5512	8	12800

SIZES, STRENGTH, &c.

The *size of a rope* is designated by the circumference, measured with a thread; thus, a 3-inch rope measures 3 inches round.

The *utmost strength* of a good hemp rope is 6·400 lbs. to the square inch; the weight it will bear before breaking is expressed in *tons by $\frac{1}{5}$ of the square of the girth, in inches.* In practice, a rope should not be subjected to more than half this strain; it stretches from $\frac{1}{5}$ to $\frac{1}{7}$, and its diameter is diminished from $\frac{1}{4}$ to $\frac{1}{7}$, before breaking.

A difference in the quality of hemp may produce a difference of $\frac{1}{4}$ in the strength of ropes of the same size.

The strength of Manilla is about $\frac{1}{2}$ that of hemp.

White ropes are $\frac{1}{3}$ more durable.

The best *quality* of hemp is of a *pearl gray*; the next, *greenish*; then the *yellow*. A *brown* color indicates decay; the odor should be strong, and free from mustiness.

To find the Strength of Ropes.

Rule.—Multiply the square of the circumference by 200, the product will be the weight, in pounds, the rope will bear with safety.

PAINTS, LACKERS, &c.*

COMPOSITION AND PREPARATION.

The proportions are given for 100 parts, by weight of prepared colors, &c., when not otherwise designated.

NOTE. A gallon of linseed oil, New York measure, weighs 7·45 lbs.; of spirits turpentine, 7 lbs.

BOILED OIL.

Raw linseed oil - - - - - - -	103
Copperas - - - - - - - - -	3·15
Litharge - - - - - - - - - -	6·3

* Ordnance Manual, 1841.

Put the copperas and litharge in a cloth bag, and suspend it in the middle of the kettle. Boil the oil $4\frac{1}{2}$ hours, with a slow, even fire, so that it may not be burnt; then let it stand, and deposit the sediment.

DRYINGS.

Mixture of copperas and litharge, taken from the boiled oil	60
Spirits turpentine - - - - - - - - - - - - - - - - - -	56
Boiled oil - - - - - - - - - - - - - - - - - - -	2

The mixture taken from the boiled oil to be ground, and mixed with the turpentine and oil.

PUTTY,

For Filling Cracks in Wood.

Spanish whiting, pulverized - -	81·6
Boiled oil - - - - - - - - - -	20·4

Made into a stiff paste. If not intended for immediate use, raw oil should be used, as the putty made with boiled oil hardens quickly.

WHITE PAINT.

	For Inside Work.	For Outside Work.
White lead, ground in oil	80	80
Boiled oil - - - - - -	14·5	9
Raw oil - - - - - -	—	9
Spirits turpentine - -	8	4

Grind the white lead in the oil, and add the spirits of turpentine. New wood-work requires about 1 lb. to the square yard, for 3 coats.

LEAD COLOR.

White lead, ground in oil - - -	75·
Lampblack - - - - - - - - - -	1·
Boiled linseed oil - - - - - - -	23·
Litharge - - - - - - - - - - -	0·5
Japan varnish - - - - - - - -	0·5
Spirits turpentine - - - - - - -	2·5

The lampblack and the litharge are ground separately upon the stone, in oil, then stirred into the white lead and oil; the turpentine and varnish are added as the paint is required for use, or when it is packed in kegs for transportation.

GRAY, OR STONE COLOR, (FOR BUILDINGS.)

White lead, in oil - - - - - -	78·
Boiled oil - - - - - - - - -	9·5
Raw oil - - - - - - - - - -	9·5
Spirits turpentine - - - - - -	3·
Turkey umber - - - - - - -	0·5
Lampblack - - - - - - - - -	0·25

Mixed like the lead color.

1 square yard of new brick-work requires, for 2 coats, 1·1 lb.; for 3 coats, 1·5 lb.

CREAM COLOR, (FOR BUILDINGS.)

	1st Coat.	2d Coat.
White lead, in oil	66·66	70·
French yellow	3·33	3·33
Japan varnish	1·33	1·33
Raw oil	28·	24·5
Spirits turpentine	2·25	2·25

1 square yard of new brick-work requires for 1st coat, 0·75; for 2d coat, 0·3 lbs.

BLACK PAINT.

Lampblack	28·
Litharge	1·
Japan varnish	1·
Linseed oil, boiled	73·
Spirits turpentine	1·

Grind the lampblack in oil; mix it with the oil, then grind the litharge in oil, and add it, stirring it well into the mixture. The varnish and turpentine are added last. This paint is used for the iron work of carriages.

LIQUID OLIVE COLOR.

Olive paste	61·5
Boiled oil	29·5
Spirits turpentine	5·5
Dryings	3·5
Japan varnish	2·

Stirred together in a paint pot.

PAINT FOR TARPAULINS.

1. *Olive.*—Liquid olive color	100
Beeswax	6
Spirits turpentine	6

Dissolve the beeswax in the spirits of turpentine, with a gentle heat, and mix the paint warm.

2. Add 12 oz. of beeswax to 1 gallon of linseed oil, and boil it two hours; prime the cloth with this mixture, and use the same, in place of *boiled oil*, for mixing the paint. Give two coats of paint.

LACKER FOR IRON ORDNANCE.

1. Black lead, pulverized	12
Red lead	12
Litharge, Extract	5
Lampblack	5
Linseed oil	66

Boil it gently about twenty minutes, during which time it must be constantly stirred.

LACKER FOR WATER-PROOF PAPER.

Beeswax - - - - - - - - - - -	18
Spirits turpentine - - - - - - -	80
Boiled linseed oil - - - - - - -	3·5

All the ingredients should be pure, and of the best quality. Heat them together in a copper or earthen vessel, over a gentle fire, in a water bath, until they are well mixed.

LACKER FOR BRIGHT IRON WORK.

Linseed oil, boiled - - - - -	80·5
Litharge - - - - - - - - - - -	5·5
White lead, ground in oil - -	11·25
Rosin, pulverized - - - - - -	2·75

Add the litharge to the oil; let it simmer over a slow fire 3 hours; strain it, and add the rosin and white lead; keep it gently warmed, and stir it until the rosin is dissolved. Apply it with a paint brush.

PENETRATION OF SHOT AND SHELLS.*

EXPERIMENTS AT FORT MONROE ARSENAL, IN 1839.

Penetration in Masonry.

CALIBRE.	Charge.	Elevation.	Distance.	MEAN PENETRATION.		
				Dressed granite.	Potomac free stone.	Hard brick.
SHOT.	Lbs.		Yds.	In.	In.	In.
32-Pounder (gun)	8	1°	880	3·5	12·	15·25
SHELL.						
8-inch Seacoast Howitzer	6	1° 35′	880	1·	4·5	8·5

The solid shot broke against the granite, but not against the free-stone or brick.

The shells broke into small fragments against each of the three materials.

* Ordnance Manual.

EXPERIMENT IN NEW-YORK HARBOR, IN 1814.

Penetration in a Target of White Oak Timber, 5 feet thick.

CALIBRE	Charge.	Distance.	Penetrat'n.	REMARKS.
	Lbs.	Yds.	In.	
32-Pounder	11	100	60	Shot wrapped with leather, so as to destroy the windage.
	11	150	54	

PENETRATION OF LEADEN BALLS.

EXPERIMENTS MADE AT WEST POINT, IN 1837.

Penetration in Seasoned White Oak

ARM.	Charge.	DISTANCES IN YARDS.							REMARKS.
		3½	9	50	100	150	200	300	
	Grains.	In.	In.	In.	In.	In.	In.	In.	
Musket	134	2·00	1·60	1·43	1·	0·66	0·55	0·00*	* 1 Ball in 10 imbedded.
	125	1·60	–	–	–	–	–	–	
	90	1·60	–	–	–	–	–	–	
Common Rifle	92	2·10	1·80	1·43	0·94	0·65	0·29	0·00†	† Indentation 0·2 inches
Hall's Rifle	70	1·12	1·70	0·63	0·53	0·40	0·00‡	–	‡ 2 Balls in 10 imbedded.

The musket fired at 9 yards distance, with a charge of 134 grains, 1 ball and 3 buckshot, gave for the ball a penetration of 1·15 inches; buckshot, 0·41 inches.

EXPERIMENTS AT WASHINGTON ARSENAL, IN 1839.

Penetration in Seasoned White Oak.

ARM.	Charge.	Distance.	Penetration.	REMARKS.
	Grains.	Yds.	In.	
Musket - - -	144	5	3·00	Arm loaded with new musket powder.
Common Rifle -	100	5	2·05	
Hall's Rifle - -	100	5	2·00	
Hall's Carbine, musket calibre	70	5	0·60	
	80	5	0·80	
	90	5	1·10	Charges too great for service.
	100	5	1·20	
Pistol - - - -	51	5	0·725	

LIGHTNING RODS.

Extracted from a Report of the French Academy of Sciences.

In a lightning rod there may be distinguished two principal parts the pointed *stem* and the *conductor* with its roots.

The stem or upper part of the rod is a bar of square or round iron, or copper, drawn out into a pyramidal or conical form, 13 to 20 feet high (according to the height of the building and of the surrounding objects), and from 1·25 inches to 2 inches thick at the base, in order to give it sufficient stiffness in proportion to its height. The upper end is made of a conical brass rod, 20 inches long, gilt at the top, or pointed with platina, which is united to the brass with silver solder, the joint strengthened by a small brass collar. The iron and brass rods are joined together by an iron dowel, screwed into both, and secured by iron pins. The brass point may be used without being gilt or armed with platina.

The *stem* should be made, if possible, in one piece; if it becomes necessary to divide it, the joint should be at about one-third of the height from the bottom; the two parts connected together by a tapering tenon in the upper piece, and a corresponding mortice below, secured by an iron pin; or, the two parts may be screwed together. At the bottom of the stem, above the roof, is a shoulder or flanch to throw off the rain water, which might otherwise follow down the rod into the roof; just above this flanch the stem is rounded about 2 inches in height, to receive a hinged collar with two *ears*, between which the end of the conductor is secured by a bolt; or the stem and the conductor may be connected by a *square collar* or *strap*, and a cross bar, which is fastened over the conductor by two nuts screwed on the ends of two branches of the strap which pass through holes in the cross bar; the back of this strap has a *branch* projecting upwards, which is secured by a bolt to a *fork* formed in the end of the conductor.

The stem is fixed to the building in various ways, according to circumstances. The best method is to attach it to a chimney or gable, where it may be secured by straps passing round the chimney, or through the wall, keyed inside against an iron bar. On an arch, the stem terminates in three or four branches or *feet*, which are leaded into the masonry.

The *conductor* is made of bars of square or round iron or copper, from 0·5 inch to 1 inch thick, connected together by a bevel joint in the form of a *Z*, fastened by 2 pins; or the bars may be screwed together. The conductor runs parallel to the roof, from 4 inches to 6 inches above it, and is supported, at intervals of about 10 feet, by *forks*, the lower ends of which are not pointed, but flattened and bent at a right angle, and nailed to a rafter, to prevent the filtration of water into the wood; the rod is held in each fork by a rivet through the branches above the rod.

The conductor follows the cornice of the building without touching it, and descends in like manner along the wall, to which it is fastened by *cramps* or *staples*. About 18 inches below the surface of the

ground, it is bent perpendicularly to the wall, and extended in that direction about 15 feet, when it passes into a well, or if water is not met with, into a pit 15 feet deep, which may be filled with loose paving stones, if it is not convenient to let it remain as a well.

To facilitate the transmission of the electric fluid, and to prevent the rod from becoming rusty, it is enclosed in a trough filled with well-burnt charcoal, which should be packed about 1·5 inches thick all round the rod; this trough may be made of bricks, or of stone, tiles, wood, &c. The conductor passes out of this trough through the sides of the well; if the latter is within the building, the conductor should pass through the wall under ground. The communication should not be made with a well or cistern which is resorted to for use.

The conductor terminates generally in two or three *prongs* or roots, which should be so placed, if possible, as to be always immersed in water not less than 2 feet; in a dry well, the roots should be covered with charcoal well rammed, and the rod within the well should be enclosed in a pipe or trough filled with charcoal. In rock or dry ground, if the conductor cannot be led into moist earth, increase the number of branches. Lead the rain water, if practicable, over or through the trough which contains the rod, and into the well.

Chains or *ropes of metallic wire* may be used for conductors, and are convenient in some situations, but rods are preferable. Copper wire is better than iron for this purpose.

The greatest care must be taken, both in fixing the rod and in its subsequent preservation, to keep the communication perfect through every part of it; otherwise it will be *dangerous* instead of useful.

It is considered that a lightning rod can protect a circular space the radius of which is double the height of the rod above the roof of the building; but when it is attached to an elevated part of the building, such as a tower or steeple, it is safer to rely on its protection to the extent only of its height above the body of the building, and to erect others for protecting the more distant parts.

A building is better protected by two rods of 15 to 20 feet high, placed at a distance equal to the sum of their radii of action, than by one rod of double the height.

All the large pieces of metal about a building, such as metallic coverings of the roof, ridges, fastenings, gutters, and long bars, should communicate with the conductor through bars or wires about 0·25 inch thick; it is better, in this respect, not to use such materials in building when they are not indispensable; nothing is to be apprehended from the common iron work of buildings, as hinges, locks, &c.

The conductor should be carried to the ground in the shortest line; one conductor may sometimes be made to communicate with two stems, without increasing its diameter; there should be not less than two conductors for *three* stems or points; the feet of several conductors may be made to communicate with each other.

Conductors should be generally placed on the side toward the prevalent storms; the walls of that side being oftenest wet by the rain, might otherwise cause accidents by acting as imperfect conductors.

Lightning rods for powder magazines are attached to *masts* or poles planted from 6 to 10 feet from the walls of the building: the *stem* of the rod need not be thicker than the conductor or more than 6 feet high, but the mast should be of such a height that the point of the stem may be about 15 feet above the building. If a lightning rod is placed *on* a magazine, it will be prudent to have two conductors to it; one on the side of the prevailing storms, the other on the opposite side.

POWER OF CONDUCTING ELECTRICITY.

Copper	10,000
Gold	9,360
Silver	7,360
Zink	2,850
Platina	1,880
Iron	1,580
Tin	1,550
Lead	830
Mercury	345
Potassium	133

The conducting power of rods of the same metal, of equal diameter, is inversely as their lengths; of rods of equal lengths, it is proportional to the mass, and not to the surface.

The conducting power is increased by lowering the temperature, and diminished, and finally destroyed, by raising the temperature.

The metals are infinitely better conductors than any other substances. Charcoal, which has been exposed to a strong heat, is one of the best conductors, though greatly inferior in this respect to iron and platina.

PIT COAL.

Bituminous Coal.—There are two principal varieties.

Open-burning Coal kindles quickly, and burns well, but produces much flame and smoke, and is soon consumed; it lies open in the fire, and does not cake. Of this kind is the English cannel coal.

Close-burning Coal melts and swells in the fire, and runs together, forming what blacksmiths call a *hollow fire*, or a dome over the nozzle of the bellows, under which the iron is heated equally, and covered from the air. This kind of coal forms a very hot fire, and leaves little residuum; it is, therefore, the most suitable for smiths' use. The Newcastle coal, and the Virginia and Pennsylvania bituminous coals are of this kind.

Anthracite Coal is now extensively used for the forge; it ignites with difficulty, and does not cake or melt in the smallest degree, but produces a very hot, open fire.

Coal is not injured, but on the contrary, rather improved by exposure to air and moisture.

COKE.

Coke is produced by charring bituminous coal, in order to expel the bitumen and sulphur; this is usually done in close furnaces or ovens. Good coke has a dull fracture, is very porous, and cellular; it gives very little ashes when burnt; it is injured, like wood charcoal, by absorbing water.

Coal furnishes 60 to 70 per cent. of coke by weight; the volume being increased 5 to 15 per cent.

ARTIFICIAL LIGHT.

Artificial light, for domestic purposes, is commonly obtained from gas, oil, wax, or tallow.

It has been determined by Count Rumford, that, to obtain light of equal intensity, there must be burnt of wax, 101 parts; tallow, 100; oil in an argand lamp, 110; in a common lamp, 129; and of an ill-snuffed tallow candle, 229. And by several experiments made on coal gas, it has been found, that above 20 cubic feet are required to produce light equal in duration and in illuminating powers, to a pound of tallow candles, six to the pound, if set up and burnt out one after the other.

In the destructive distillation of coal, to obtain gas, the first products are carbonic acid, olefiant, and sulphuretted hydrogen gases; toward the end of the process, hydrogen gas and carbonic oxyd.

Coal produces the greatest quantity of gas when the retorts are maintained at bright red heat, and the process of distillation is then the most rapidly effected. When the retorts are heated either below or beyond a bright red heat, the production of gas is considerably less. From some experiments made at the Soho, it appears, that in distilling 56 lbs. weight of coal, the quantity of gas produced, in cubic feet, when the distillation was effected in 3 hours, was 41·3; in 7 hours, 37·5; in 2 hours, 33·5; and in 25 hours, 31·7.

A decided advantage is also obtained, by charging, when the retorts are at a bright red heat; it diminishes the time of distillation, produces gas in larger quantities, and of much better quality.

To detect sulphuretted hydrogen in gas, force a quantity of it through a solution of acetate of lead, formed by dissolving four grains in a 2-oz. phial of water; if the gas contain sulphuretted hydrogen, the solution will immediately assume a dark, cloudy appearance.

Water impregnated with sulphuretted hydrogen, will assume a black appearance on the addition of a drop or two of nitrate of silver.

A current of gas, containing sulphuretted hydrogen, directed against a paper which has been painted over with white lead, ground up in water, will discolor it.

MISCELLANEOUS NOTES.

Wood is from 7 to 20 times stronger transversely than longitudinally. It becomes stronger both ways when dry.

The *hardness of metals* is as follows: Iron, Platina, Copper, Silver, Gold, Tin, Lead.

A pipe of *cast iron*, 15 inches in diameter and ·75 inches thick, will sustain a head of water of 600 feet. One of *oak*, 2 inches thick, and of the same diameter, will sustain a head of 180 feet.
When the cohesion is the same, the thickness varies, as the height multiplied by the diameter.

Tides.—The difference in time between high water, averages about 49 minutes each day.

Silica is the base of the mineral world; *Carbon*, of the organized.

When one *beam* is let in, at an inclination to the depth of another, so as to bear in the direction of the fibres of the beam that is cut, the depth of the cut *at right angles to the fibres* should not be more than $\frac{1}{5}$ of the length of the piece, the fibres of which, by their cohesion, resist the pressure.

In *sandy soil*, the greatest force of a pile-driver will not drive a *pile* over 15 feet.

A *fall* of $\frac{1}{10}$ of an inch in a mile will produce a *current* in rivers.

Melted snow produces about $\frac{1}{8}$ of its bulk of water.

Sound passes in water at a velocity of 4708 feet per second.

In Buffon's experiments, b, d, and l, being the breadth, depth, and length of a beam of oak in inches, the weight which broke it in pounds, was $b\,d^2\left(\frac{54{\cdot}25}{l}+10\right)$.

Count Rumford found the cohesive strength of a cylinder of iron, an inch in diameter, 63320 pounds. This is only 1-20th more than Emerson.

SOLDERS.

For Lead.—Melt one part of block tin, and when in a state of fusion, add two parts of lead. If a small quantity of this, when melted, is poured out upon the table, there will, if it be good, arise little bright stars upon it. Resin should be used with this solder.

For Tin.—Take four parts of pewter, one of tin, and one of bismuth; melt them together, and run them into thin slips. Resin is also used with this solder.

For Iron.—Good tough brass, with a little borax.

CEMENTS.

A very strong glue is made by adding some powdered chalk to common glue when melted; and a glue which will resist the action of water may be formed by boiling one pound of common glue in two quarts (English measure) of skimmed milk.

Soft Cements.—For steam boilers, steam pipes, &c., red or white lead in oil, 4 parts; iron borings, 2 to 3 parts.

Hard Cements.—Iron borings and salt water, and a small quantity of sal-ammoniac with fresh water.

TABLE OF COMPOSITIONS.

Brass, &c.

Copper.	Tin.	Zink.	Materials.
2	0	1	For Yellow Brass.
3	0	1	" Spelter.
4	1	$\frac{1}{4}$	" Lathe bushes.
6	1	0	" Shaft bearings.
7	1	0	" " "
1	9		" " " *
5	1	$\frac{1}{2}$	" " " (hard.)
8	1	0	" Wheels, boxes, cocks, &c.
9	1	0	" Gun metal.
3	0	1	" Brass.
10	1	0	" Valves.
78	22	–	" Bells and Gongs.
80	10	5·6, and lead 4·3	

* Some use this composition of metals for shaft bearings with great satisfaction.

PHYSICAL DATA.

The average quantity of water which falls in rain and snow at Philadelphia, is 36 inches in a year.

Limits of Vegetation, in the Temperate Zone.

The vine ceases to grow at the height of about 2,300 feet above the level of the sea; Indian corn, 2,800; oak, 3,350; walnut, 3,600; ash, 4,800; yellow pine, 6,200; fir, 6,700.

Perpetual Snow.

Under the equator, at 15,800 feet above the level of the sea; in latitude 45°, at 8,400; in latitude 65°, 5,000.

Variation of the Magnetic Needle.

At West Point—April 16th, 1827,	- -	6°, 44′, 30″ west.
" " November 16th, 1839,	-	7°, 58′, 27″ west.
Dip of the Needle: September 5th, 1839,	-	73°, 26′, 30″

CONGELATION AND LIQUEFACTION.

Freezing water gives out 140°. Water may be cooled to 20°. All solids absorb heat when becoming a fluid. The particular quantity of heat which renders a substance a fluid, is called its caloric of fluidity, or latent heat.

The heat absorbed in liquefaction is given out again in freezing. Fluids boil in vacuo with 124° less of heat than when under the pressure of the atmosphere. On Mount Blanc, water boils at 187°.

MELTING POINTS OF SOLIDS.

Platinum, palladium, rhodium, lime, silex, fine porcelain, can be melted in small quantities by means of strong lenses, or of the hydro-oxygen blowpipe. *Manganese, iron, cobalt, nickel, plaster of Paris, common pottery,* at 150° to 180° of Wedgewood's pyrometer; say 20,000° to 24,000° Fahr. *Iron, red hot* (in day light), 1207° Fahr. = 1° Wedgewood.

		Fahr.		Wedgewood.	
	Gold	5,237°	=	32°	
	Silver	4,717°	=	28°	
	Copper	4,587°	=	27°	
	Brass	3,809°	=	21°	
	Flint glass	2,377°	=	10°	
	Antimony	809°			
	Zink	680°			
	Saltpetre	660°			
	Lead	612°			Experiments of committee of the Franklin Institute.
	Bismuth	506°			
	Tin	442°			
Alloys.	Lead 2, tin 1 (common solder)	475°			
Alloys.	Lead 1, tin 1	393°			
Alloys.	Lead 1, tin 2 (soft solder)	360°			
Alloys.	Lead 1, tin 1, bismuth 1	272°			
Alloys.	Lead 2, tin 3, bismuth 5	212°			
	Sulphur	220°			
	Beeswax, bleached	155°			
	Do. common	149°			
	Tallow	127°			

FREEZING POINTS OF LIQUIDS.

	Fahr.		Fahr.
Olive oil	36°	Sulphuric acid	1°
Water	32°	Brandy	— 7°
Milk	30°	Mercury	—39°
Vinegar	28°	Nitric acid	—55°
Spirits of turpentine	16°		

BOILING POINTS OF LIQUIDS.

Sulphuric ether	98°	Phosphorus	554°
Ammonia	140°	Spirits of turpentine	560°
Alcohol	176°	Sulphur	570°
Water, and essential oils	212°	Sulphuric acid	590°
Water, saturated with common salt	224°	Linseed oil	600°
Nitric acid	248°	Mercury	660°

FREEZING MIXTURES.

Nitrate of ammonia 1, water 1;	therm'r falls from 50° to	4°
Sulph. soda 8, muriatic acid 5	50° to	0°
Common salt 1, snow or ice 2	32° to—	4°
Cryst. chloride of lime 3, snow 2	32° to—	50°

STAINING WOOD AND IVORY.

Yellow.—Diluted nitric acid will often produce a fine yellow on wood; but sometimes it produces a brown, and if used strong, it will seem nearly black.

Red.—A good red may be made by an infusion of Brazil wood in stale urine, in the proportion of a pound to a gallon. This stain is to be laid on the wood boiling hot; and before it dries it should be laid over with alum water. For the same purpose, a solution of dragon's blood in spirits of wine may also be used.

Mahogany color may be produced by a mixture of madder, Brazil wood, and logwood, dissolved in water, and put on hot. The proportions must be varied by the artist according to the tint required.

Black.—Brush the wood several times over with a hot decoction of logwood, and then with iron lacquer; or, if this cannot be had, a strong solution of nutgalls.

Blue.—Ivory may be stained blue thus:—Soak the ivory in a solution of verdigris in nitric acid, which will make it green; then dip it into a solution of pearlash boiling hot, and it will turn blue.

To stain ivory black, the same process as for wood may be employed.

Purple may be produced by soaking the ivory in a solution of salammoniac into four times its weight of nitrous acid.

MISCELLANEOUS ALLOYS AND COMPOSITIONS.

Chinese white copper.—40·4 parts copper, 31·6 nickel, 25·4 zink, and 2·6 iron.

German silver.—1 part nickel, 1 zink, and 2 copper; when intended for rolling into plates, 25 nickel, 20 zink, and 60 copper, to which may be added 3 of lead.

Tombac, or red brass.—8 parts copper, 1 part zink.

Manheim gold.—3 parts copper, 1 zink, and a small quantity of tin.

Alloy of the standard measures used by government.—576 parts copper, 59 tin, and 48 brass.

Bath metal.—32 parts brass, 9 parts zink.

Speculem metal.—6 parts copper, 2 parts tin, and 1 of arsenic; or, 7 of copper, 3 of zink, and 4 of tin.

Hard solder.—2 parts copper and 1 part zink.

Blanched copper.—8 parts of copper and $\frac{1}{2}$ part arsenic.

Britannia metal.—4 parts of brass and 4 of tin; when fused, add 4 of bismuth and 4 of antimony. This composition is added at discretion to melted tin.

Plumber's solder.—Equal parts of lead and tin.

Tinman's solder.—2 parts of lead and 1 of tin.

Pewterer's solder.—2 parts of tin and 1 of lead.

Common Pewter.—4 parts of tin and 1 of lead.

Best Pewter.—100 parts of tin and 17 of antimony.

A metal that expands in cooling.—9 parts of lead, 2 of antimony, and 1 of bismuth. This metal is very useful in filling small defects in iron castings, &c.

Queen's metal.—9 parts of tin, 1 of antimony, 1 of bismuth, and 1 of lead.

Mock platinum.—8 parts of brass and 5 of zink.

Silver coin of Britain.—$11\frac{1}{10}$th pure silver and $9\frac{9}{10}$ths copper.

Gold coin of Britain.—11 parts pure gold and 1 copper. Previous to 1826, silver formed part of the alloy of gold coin; hence the different color of English gold money.

Ring gold.—6 dwts. 12 grains pure copper, 3 dwts. 16 grains fine silver, and 1 ounce 5 dwts. pure gold.

Mock gold.—Fuse together 16 parts of copper, 7 of platinum, and 1 of zink. When steel is alloyed with $\frac{1}{500}$th part of platinum, or with $\frac{1}{500}$th part of silver, it is rendered much harder, more malleable, and better adapted for every kind of cutting instrument.

NOTE. In making alloys, care must be taken to have the more infusible metals melted first, and afterward add the others.

Composition used in welding cast steel.—Take of borax 10 parts, sal-ammoniac 1 part; grind or pound them roughly together: then fuse them in a metal pot over a clear fire, taking care to continue the heat until all spume has disappeared from the surface. When the liquid appears clear, the composition is ready to be poured out to cool and concrete; afterward, being ground to a fine powder, it is ready for use

To use this composition, the steel to be welded is raised to a heat which may be expressed by "bright yellow;" it is then dipped among the welding powder, and again placed in the fire until it attains the same degree of heat as before; it is then ready to be placed under the hammer.

Cast iron cement.—Take of clean borings or turnings of cast iron 16 parts, of sal-ammoniac 2 parts, and flour of sulphur 1 part; mix them well together in a mortar, and keep them dry. When required

for use, take 1 part of the mixture and 20 parts of clean borings, mix thoroughly, and add a sufficient quantity of water.

NOTE. A little grindstone dust added improves the cement.

Booth's patent grease for railway axles.—Water 1 gallon, clean tallow 3 lbs., palm oil 6 lbs., common soda $\frac{1}{2}$lb., or tallow 8 lbs., and palm oil 10 lbs. The mixture to be heated to about 210°F., and well stirred till it cools down to about 70°, when it is ready for use.

Cement for steam-pipe joints, &c. with faced flanges.—To 2 parts of white lead mixed, add 1 part red lead dry, grind or otherwise mix them to a consistence of thin putty, apply interposed layers with one or two thicknesses of canvas or gauze wire, as the necessity of the case may be.

MATERIALS USED FOR MECHANICAL PURPOSES.

TIMBER.

WHITE OAK, (*Quercus alba*).—The bark is white, the leaf long, narrow, and deeply indented; the wood is of a straw color, with a somewhat reddish tinge, tough and pliable; it is the principal timber used for ordnance purposes, being employed for all kinds of artillery carriages.

WHITE BEECH—RED BEECH, (*Fagus sylvestris—Fagus ferruginea*), are the most suitable for fuses, mallets, and fuse setters; also, for plane stocks and various other tools.

WHITE ASH, (*Fraxinus Americana*), is straight grained, tough and elastic, and is therefore suitable for light carriage shafts; in artillery, it is used chiefly for sponge and rammer staves; sometimes for handspikes, and for sabots, and tool handles.

ELM, (*Ulmus Americana*), is well suited for fellies and for small naves.

HICKORY, (*Juglans tomentosa*), is very tough and flexible; the most suitable wood for handspikes and tool handles, and for wooden axletrees.

BLACK WALNUT, (*Juglans nigra*), is hard and fine grained; it is preferred for naves, and the plank is used for ammunition boxes; it is used exclusively for the stocks of small arms.

WHITE POPLAR, or TULIP-TREE, (*Liriodendron tulipifera*), is a soft, light, fine-grained wood, which grows to a great size; it is used for sabots, cartridge blocks, &c., and for the lining of ammunition boxes

WHITE PINE, (*Pinus strobus*), is used for all kinds of building; in artillery, it is used for arm chests, and packing boxes generally.

CYPRESS, (*Cupressus disticha*), is a soft, light, straight-grained wood, which grows to a very large size. On account of the difficulty

of procuring oak of suitable kind in the Southern States, cypress has been used for sea-coast and garrison carriages. It resists better than oak the alternate action of heat and moisture to which sea-coast carriages are particularly exposed in casemates; but being of inferior strength, a larger scantling of cypress than of oak is required for the same purpose; and on account of its softness, it does not resist sufficiently the friction and shocks to which such carriages are liable.

Bass-wood, or American Lime, (*Tilia Americana*), is very light, not easily split, and is excellent for sabots, cartridge blocks, and for large fuses.

Dog-wood, (*Cornus florida*) is hard and fine grained, suitable for mallets, drifts, &c.

SELECTION OF STANDING TREES.

The principal circumstances which affect the quality of growing trees, are *soil*, *climate*, and *aspect*.

In a moist soil, the wood is less firm, and decays sooner than in a dry, sandy soil; but in the latter, the timber is seldom fine; the best is that which grows in a dark soil, mixed with stones and gravel. This remark does not apply to the poplar, willow, cypress, and other light woods, which grow best in wet situations.

In the United States, the climate of the Northern and Middle States is most favorable to the growth of timber used for ordnance purposes, except the cypress.

Trees growing in the center of a forest, or on a plain, are generally straighter and more free from limbs than those growing on the edge of the forest, in open ground, or on the sides of hills; but the former are at the same time less hard. The aspect most sheltered from the prevalent winds is generally most favorable to the growth of timber. The vicinity of salt water is favorable to the strength and hardness of white oak.

The selection of timber trees should be made before the fall of the leaf. A healthy tree is indicated by the top branches being vigorous, and well covered with leaves; the bark is clear, smooth, and of a uniform color. If the top has a regular, rounded form—if the bark is dull, scabby, and covered with white and red spots, caused by running water or sap—the tree is unsound. The decay of the uppermost branches, and the separation of the bark from the wood, are infallible signs of the decline of the tree.

FELLING TIMBER.

The most suitable season for felling timber, is that in which vegetation is at rest, which is the case in midwinter and in midsummer; recent opinions, derived from facts, incline to give preference to the latter season, say the month of July; but the usual practice is to fell

trees for timber between the first of December and the middle of March. Some experiments are in progress with a view to determine the question with regard to oak timber for ordnance purposes.

The tree should be allowed to obtain its full maturity before being felled; this period in oak timber is generally at the age of from 75 to 100 years, or upwards, according to circumstances. The age of hard wood is determined by the number of rings which may be counted in a section of the tree.

The tree should be cut as near the ground as possible, the lower part being the best timber. The quality of the wood is in some degree indicated by the color, which should be nearly uniform in the heart wood, a little deeper toward the center, and without sudden transitions.

Felled timber should be immediately stripped of its bark, and raised from the ground.

DEFECTS OF TIMBER TREES, (ESPECIALLY OF OAK).

Sap, the white wood next to the bark, which very soon rots, and should never be used, except that of hickory. There are sometimes found rings of light-colored wood surrounded by good hard wood; this may be called the *second sap;* it should cause the rejection of the tree in which it occurs.

Brash-wood, is a defect generally consequent on the decline of the tree from age; the pores of the wood are open, the wood is reddish colored, it breaks short, without splinters, and the chips crumble to pieces. This wood is entirely unfit for mechanical purposes or artillery carriages.

Wood which has died before being felled, should in general be rejected; so should *knotty trees*, and those which are covered with tubercles or excrescences.

Twisted wood, the grain of which ascends in a spiral form, is unfit for use in large scantling; but if the defect is not very decided, the wood may be used for naves, and for some light pieces.

Splits, *checks* and *cracks*, extending toward the center, if deep and strongly marked, make the wood unfit for use, unless it is intended to be split.

Wind-shakes, are cracks separating the concentric layers of wood from each other; if the shake extends through the entire circle, it is a ruinous defect.

All the above-mentioned defects are to be guarded against in procuring timber for use in artillery constructions; the *center heart* is also to be rejected in nearly all cases.

SEASONING AND PRESERVING TIMBER.

As soon as practicable after the tree is felled, the sap-wood should be taken off, and the timber reduced, either by sawing or splitting, nearly to the dimensions required for use. Pieces of large scantling

or of peculiar form, such as those for the bodies of gun carriages and for chassis, are got out with the saw ; those of smaller dimensions—such as spokes, fellies, handspikes, &c.—are split with wedges. Naves should be cut to the proper length, and bored through the axis with a $1\frac{1}{2}$-inch auger, to facilitate their seasoning, and to prevent cracking as much as possible. They should be cut from the butt of the tree.

Timber of large dimensions is improved by *immersion in water* for some weeks, according to its size ; after which, it is less subject to warp and crack in seasoning.

For the purpose of seasoning, timber should be piled under shelter, where it may be kept dry, but not exposed to a strong current of air ; at the same time, there should be a free circulation of air about the timber, with which view slats or blocks of wood should be placed between the pieces that lie over each other, near enough together to prevent the timber from bending. In the sheds, the pieces of timber should be piled in this way, or in square piles, and classed according to age and kind. Each pile should be distinctly marked with the number and kind of pieces, and their age, or the date of receiving them. The piles should be taken down, and made over again at intervals, varying with the length of time which the timber has been cut. The seasoning of timber requires from two to four years, according to its size.

Gradual drying and seasoning in this manner is considered the most favorable to the durability and strength of timber, but various methods have been proposed for hastening the process. For this purpose, *steaming* or *boiling* timber has been applied with success ; and the results of experiments with Mr. Kyan's process of saturating timber with a solution of corrosive sublimate, have been highly satisfactory ; this is said to harden and season the wood, at the same time that it secures it from the dry rot, and from the attacks of worms. This process is now under trial in the Ordnance Department, and also that of Mr. Earle, which consists in saturating the wood with a solution of copperas and blue vitriol, mixed together *Kiln-drying* is serviceable only for boards and pieces of small dimensions, and is apt to cause cracks, and to impair the strength of wood, unless performed very slowly. *Charring* or *painting* is highly injurious to any but seasoned timber, as it effectually prevents the drying of the inner part of the wood, in which consequently fermentation and decay soon take place.

Oak timber loses about *one-fifth of its weight* in seasoning, and about *one-third of its weight* in becoming perfectly dry.

MEASURING TIMBER.

Sawed or hewn timber is measured by the cubic foot, or more commonly by *board measure*, the unit of which is a superficial foot of a board 1 inch thick. Small pieces, especially those which are got out by splitting—such as handspikes, spokes, &c.—and *shapes*, or

pieces roughed out to a particular pattern—such as stocks for small arms, fellies, &c.—are purchased by the piece.

Usual Rule for Measuring Round Timber.

Multiply the length by the square of $\frac{1}{4}$ the mean girth, for the solid contents; or, $\frac{LC^2}{16}$; L being the length of the log, and C half the sum of the circumferences of the two ends. But when round timber is procured for use in the Ordnance Department, it should be measured according to the square of good timber which can be obtained from the log.

IRON.

Iron is obtained from ores, in which it generally exists in the state of an oxide, combined with earthy or stony matters, and sometimes with *sulphur*, *arsenic*, *magnesia*, *manganese*, &c. Iron ores are classed and named according to their different combinations, as *magnetic*, *specular*, *micaceous*, *red hematite*, *brown hematite;* the last named is the ore from which the Salisbury and the Juniata irons are extracted.

MAKING PIG IRON.

Roasting.—To obtain pig iron, the ore is first roasted, to separate such of the foreign substances as can be consumed or volatilized by a moderate heat; for this purpose, the ore is distributed in layers, alternately with refuse coal or charcoal, and burnt in the open air, or in a kiln similar to that used for burning lime; when sufficiently calcined, the ore is easily broken into pieces of the proper size for smelting.

Smelting separates the iron from the refractory substances with which it is combined in the ore; it is effected in the blast furnace, by exposing the ore to a great heat, in conjunction with a suitable flux of limestone or clay, which, combining with the earthy matter, runs off over the dam, in a cinder, leaving the iron to settle at the bottom of the furnace, where it is protected from the blast by the cinder, and kept hot and fluid, until it is drawn off into open channels or molds, in the form of *pigs*, which is usually done every twelve hours.

The kinds of fuel used for smelting, are charcoal, bituminous coal, coke, and, very recently, anthracite coal. For many purposes—such as sheets for tinning, bars for converting into steel—*charcoal iron* is exclusively used; and for bar iron, it is superior to that made with bituminous coal; but for castings, the latter may be often used with advantage.

Pig iron, according to the proportion of carbon which it contains, is divided into *foundry iron* and *forge iron*, the latter being adapted

only to converting into malleable iron, while the former, containing the largest proportion of carbon, can be used either for casting or for making bar iron.

MALLEABLE IRON.

Malleable iron may be made directly from the pigs, by means of the bloomery, or puddling furnace; but when it is desired to obtain iron homogeneous and of the best quality, the pig iron should invariably be *refined;* otherwise, the bar iron will be full of black specks and cinder holes.

Refining.—This important operation deprives the iron of a considerable portion of its carbon; it is effected in a *blast furnace*, where the iron is melted by means of charcoal or coke, and exposed for some time to the action of a great heat; the metal is then run into a cast iron mold, by which it is formed into a large, broad plate, about 2·5 inches thick. As soon as the surface of the plate is chilled, cold water is poured on plentifully, to render it brittle.

The next process is to convert the metal into malleable iron, by depriving it of its remaining carbon and oxygen, which, in the United States, is usually effected in the *bloomery fire;* in England, by means of a *puddling furnace.*

Bloomery.—The bloomery resembles a large forge fire, where charcoal and a strong blast are used, and the refined metal or the pig iron, after being broken into pieces of the proper size, is placed before the blast, directly in contact with the charcoal; as the metal fuses, it falls into a cavity left for that purpose, below the blast, where the bloomer works it into the shape of a *ball*, which he places again before the blast, surrounded with fresh charcoal; this operation is generally again repeated, when the ball is ready for the *shingler.*

Puddling is effected in a reverberatory furnace, where the flame of bituminous coal is made to act directly on the metal, which has been previously broken into small pieces. When melted, it is thoroughly worked by the puddler, who separates his charge, on the cast iron bottom or hearth of the furnace, into five or six *puddlers' balls*, weighing from 80 to 100 pounds each. These balls are next passed to the shingler.

Shingling is best performed under the *tilt hammer*, weighing from two to four tons, although it is sometimes done under a *squeezer;* it has a double object, to form the ball into a shape to be received by the puddle rolls, and to express the liquid cinder which may remain in the ball; this is effected by from fifteen to twenty blows with the hammer; the ball, now called a *bloom*, is ready for being rolled or hammered.

Puddle Rolls.—By passing through the different grooves in these rolls, the bloom is reduced to a *rough bar*, from three to four

feet in length, its name conveying an idea of its condition, which is rough and imperfect.

The bloom may be *broken down* under the hammer, instead of rollers; for this purpose, the shingler works the bloom as long as the heat will permit, when it is re-heated and hammered, until it is reduced to one or more *anconies*, according to the size of bar which it is intended to make; these are again heated, and reduced to the required size and shape.

PILING.—To prepare rough bars for this operation, they are cut, either hot or cold, by means of a strong pair of *shears*, into such lengths as are best adapted to the size of the finished bar required; the sheared bars are piled, one over the other, to the number of from two to six or more pieces, according to the size required, when the pile is ready for

BALLING.—This operation is performed in the balling furnace, which is similar to the puddling furnace, except that its bottom or hearth is made up, from time to time, with sand instead of cast iron; it is used to give a welding heat to the piles, to prepare them for rolling.

FINISHING ROLLS.—The *balls* are passed successively between rollers of various forms and sizes, according to the shape of the finished bar required.

These bars are straightened on a cast iron bed, with heavy wooden beetles.

PROPERTIES OF IRON.

All iron contains more or less carbon, the hardest containing the least, and the proportion varies from $\frac{1}{2}$ to 2 per cent. It melts at 160° Wedgewood; it expands $\frac{1}{812}$th from 0° to 212° Fahr., and $\frac{1}{140}$th part, from 0° to a red heat. Specific gravity 7,788.

To test the Quality of Bar Iron.—The most convenient test is by the fracture; but this is not always sufficient, as the same iron will present different appearances, according to the manner in which it has been forged, and the degree of heat to which it has been subjected. In testing by the fracture, the sample should be 1 inch square, or if flat bar, $\frac{1}{2}$ inch thick; cut a notch on one side with a cold chisel, and bend the bar down over the edge of an anvil, or give it a heavy blow, when laying flat on the ground, with a sledge hammer; if the fracture exhibits long silky fibres, of a leaden-gray color, cohering together, and twisting or pulling apart before breaking, it denotes a tough, soft iron, which is easy to work and hard to break, suitable for sheet iron, wire, &c.; but it may weld badly. A medium, even grain, mixed with fibres as above, but without bright specks or dark spots, is also a favorable indication. In general, a short, blackish fibre indicates iron badly refined, and mixed with carbon, plumbago, or oxyd; if worked very hot, it may be improved, but there will be a great waste. *A very fine, close grain*, denotes a hard, steely iron, which is apt to be *cold-short*, hard to work with the hammer or file. *A coarse grain*, with a brilliant, crystalized fracture, or yellow or

brown spots, denotes a brittle iron, inclined to be *cold-short*, but working easily when heated, and making a good weld. Numerous cracks on the edges of the bar generally indicate a *hot-short* iron, which cracks or breaks when punched or worked at a red heat, and will not weld; it is strong when cold, and may be useful in that state; but, if worked, care should be taken not to subject it to strains at a red heat. *Blisters*, *flaws*, and *cinder holes*, are caused by imperfect welding at too low a heat, or by the iron not being properly worked, and do not always indicate an inferior quality. The above-mentioned characters are not often found separate in iron, and its quality must be determined by their combination, and by the predominance of one or the other of them. In general, good iron is readily heated, is soft under the hammer, and throws out but few sparks when taken from the fire.

The best test for iron, is to have a piece forged into the shape in which it is intended to be used. *Another test for iron, when cold*, is to cut a screw thread on a square bar, and bend it by striking the end with a hammer; also, by punching or drilling pieces which are to have holes in them; in the case of the square bar, it should be bent in different directions, at sharp angles; and if the bar is heavy, place the end on the corner of the anvil, and strike it with a heavy sledge until the piece is forced off. Examine the welding of pieces which are *jumped on*, or *upset*.

To test Iron when hot.—Draw out the iron, bend and twist it; split it, and turn back the two parts, to see if the split extends up; punch a long hole in the direction of the fibre, and another at right angles to it; punch holes of different forms; weld the iron to iron and steel; make chains from small rods; observe if cracks or flaws weld easily; finally, forge some of the most difficult pieces for which the iron is intended.

NOTE ON FORGING.—Good iron is often injured by being unskillfully worked: care should be taken that the iron, while heating, is not exposed to the air, which would assist in forming scales of oxyd on its surface. It is to prevent this, that the workman, from time to time, throws sand or clay on his iron to protect it. When iron is at a white heat, immediate contact with coal tends to carbonize it, and make it *steely*. Iron heated for any purpose, and especially for welding, should be heated as rapidly as possible, in order to expose it the least possible time to the action of the air and coal; for this purpose, the strongest fuel, with an abundant, steady blast, is necessary. Defects in iron, caused by unskillful working, may be remedied in part: if, for example, iron has been *burned*, give it a smart heat, protected as much as possible from the air; if the iron has been injured by *cold hammering*, a moderate annealing heat will restore it; if the iron has become hard and steely, give it one or more smart heats, to extract the carbon.

CAST IRON.

Iron castings for ordnance purposes are made of pig metal obtained from the smelting furnace. There are many varieties of cast iron, differing from each other by almost insensible shades; the two principal divisions are *gray* and *white*, so called from the color of the fracture when recent.

Gray Iron is softer and less brittle than white iron; it is in a slight degree malleable and flexible, and is not sonorous; it can be easily drilled and turned in the lathe, and does not resist the file. It has a brilliant fracture, of a gray, or sometimes a bluish-gray color; the color is lighter as the grain becomes closer, and its hardness increases at the same time. A coarse grain, without much luster, indicates a good quality of gray metal; a small, white, shining grain, the contrary. Its mean specific gravity is 7,200.

Gray iron melts at a higher heat than white iron, becomes more fluid, and preserves its fluidity longer; it runs smoothly; the color of the fluid metal is red, and it is deeper in proportion as the heat is lower; it does not stick to the ladle; it fills the molds well, contracts less, and contains fewer cavities than the white iron; the edges of a casting are sharp, and the surface smooth, convex, and covered with carburet of iron. Gray iron is the only kind suitable for making castings which require great strength, such as cannon. Its tenacity is increased by annealing at a red heat; but at a white heat it becomes brittle, and acquires a permanent increase of bulk.

White Iron is very brittle and sonorous; it resists the file and the chisel, and is susceptible of a high polish; the surface of a casting is concave; the fracture presents a silvery appearance, generally fine grained and compact, sometimes radiating or lamellar. Its mean specific gravity is 7,500.

When melted, it is white, and throws off a great number of sparks; its qualities are the reverse of those of gray iron; it is therefore unsuitable for ordnance purposes.

Mottled Iron is a mixture of white and gray; it has a spotted appearance; it flows well, and with few sparks; the casting has a plane surface, with edges slightly rounded. It is suitable for making shot and shells.

Besides these general divisions, the manufacturers distinguish more particularly the different varieties of pig metal by numbers, according to their relative hardness.

No. 1 is the softest iron, possessing in the highest degree the qualities described as belonging to gray iron; it has not much strength; but on account of its fluidity when melted, and of its mixing advantageously with old or scrap iron, and with the harder kinds of cast iron, it is of great use to the founder, and commands the highest price.

No. 2 is harder, closer grained, and stronger than No. 1; it has a gray color, and considerable luster. It is the kind of iron most suitable in general for making shot and shells.

No. 3 is still harder than No. 2; its color is gray, but inclining to white; it has considerable strength, but it is principally used by the founder for mixing with other kinds of iron.

No. 4 is *bright* iron.

No. 5, *mottled*.

No. 6, *white*, which is unfit for general use by itself.

The qualities of these various kinds of iron seem to depend on the proportion of carbon, and on the state in which it is found in the metal. In the darker kinds of iron, where the proportion is sometimes seven per cent. of carbon, it exists partly in the state of graphite or plumbago, which makes the iron soft. In white iron, the carbon is thoroughly combined with the metal, as in steel.

Cast iron frequently retains a portion of foreign ingredients from the ore, such as earths, or oxyds of other metals, and sometimes sulphur and phosphorus, which are all injurious to its quality. Sulphur hardens the iron, and, unless in a very small proportion, destroys its tenacity.

These foreign substances, and also a portion of the carbon, are separated by melting the iron in contact with air, and soft iron is thus rendered harder and stronger. The effect of remelting varies with the nature of the iron and the kind of ore from which it has been extracted: that from the hard ore, such as the magnetic oxyds, undergoes less alteration than that from the hematites; the latter being sometimes changed from *No.* 1 to *white* by a single remelting in the air furnace. The kind of iron most suitable for any special purpose, such as the casting of cannon, should be ascertained by trial for that purpose in the furnace in which it is to be used.

All cast iron expands forcibly at the moment of becoming solid, and again contracts in cooling; gray iron, as before remarked, expands more and contracts less than other iron.

The color and texture of cast iron depend greatly on the size of the casting and the rapidity of cooling; a small casting, which cools quickly, is almost always *white*, and the surface of large castings partakes more of the qualities of white metal than the interior.

STEEL.

Steel is a compound of iron and carbon, in which the proportion of the latter is from five to one per cent., and even less, in some kinds. Steel may be distinguished from iron by its fine grain; its susceptibility of hardening by immersion, when hot, into cold water; and with certainty, by the action of dilute nitric acid, which leaves a black spot on steel, and on iron a spot which is lighter colored in proportion as the iron contains less carbon.

There are many varieties of steel, the principal of which are:

Natural steel, which is obtained by reducing the rich and pure kinds of iron ore with charcoal, and refining the cast iron, so as to deprive it of a sufficient portion of carbon to bring it to a malleable state. It is made principally in Germany, and is used for making files and other tools.

The India steel called *Wootz* is said to be a natural steel, containing a small portion of other metals.

Blistered Steel, or *steel of cementation*, is prepared by the direct combination of iron and carbon. For this purpose, the iron in bars is put in layers alternating with powdered charcoal, in a close furnace, and exposed for seven or eight days to a heat of about 70° Wedgewood, and then suffered to cool for as many days more. The bars, on being taken out, are covered with blisters, have acquired a brittle quality, and exhibit in the fracture a uniform crystalline appearance. The degree of carbonization is varied according to the purposes for which the steel is intended, and the best qualities of iron (Russian and Swedish) are used for the finest kinds of steel.

Tilted Steel is made from blistered steel, moderately heated, and subjected to the action of a tilt hammer, by which means its tenacity and density are increased, and it is thus adapted to use.

Shear Steel is made from blistered or natural steel, refined by piling thin bars into faggots, which are brought to a welding heat in a reverberatory furnace, and hammered or rolled again into bars: this operation is repeated several times to produce the finest kinds of shear steel, which are distinguished by the names of *half-shear*, *single-shear*, and *double-shear*, or steel of 1 *mark*, of 2 *marks*, of 3 *marks*, &c., according to the number of times it has been piled.

Cast Steel, is made by breaking blistered steel into small pieces, and melting it in close crucibles, from which it is poured into iron moulds; the *ingot* is then reduced to a bar by hammering or rolling, as described under the head of malleable iron, these operations being performed with great care. Cast steel is the finest kind of steel, and best adapted for most purposes; it is known by a very fine, even, and close grain, and a silvery homogeneous fracture; it is very brittle, and acquires extreme hardness, but is difficult to weld without the use of a flux. The other kinds of steel have a similar appearance to cast steel, but the grain is coarser and less homogeneous; they are softer and less brittle, and weld more readily. A fibrous or lamellar appearance in the fracture indicates an imperfect steel. A material of great toughness and elasticity, as well as hardness, is made by forging together steel and iron, forming the celebrated *damask steel*, which is used for sword blades, springs, &c.; the damasked appearance is produced by the action of a diluted acid, which gives a black tint to the steel parts, whilst the iron remains white. Various *fancy steels*, or alloys of steel with *silver*, *platina*, *rhodium*, *aluminium*, have been made with a view to imitating the Damascus steel, wootz, &c., and improving the fabrication of some of the finer kinds of surgical and other instruments.

PROPERTIES OF STEEL.

The best steel possesses the following characters: heated to redness, and plunged into cold water, it becomes hard enough to scratch glass, and to resist the best files; the hardness is uniform throughout the piece: after being tempered, it is not easily broken; it welds readily; it does not crack or split; it bears a very high heat, and

preserves the capability of hardening after repeated working; the grain is fine, even, and homogeneous, and it receives a brilliant polish. Its specific gravity is 7,816, being greater than that of iron.

Test.—Break a few bars, taken at random; make tools of them, and try them in the severest manner.

HARDENING AND TEMPERING STEEL.—On these operations the quality of manufactured steel in a great measure depends.

Hardening is effected by heating the steel to a cherry red, or until the scales of oxyd are loosened on the surface, and plunging it into a liquid, or placing it in contact with some cooling substance; the degree of hardness depends on the heat and the rapidity of cooling. Steel is thus rendered so hard as to resist the hardest files, and it becomes at the same time extremely brittle. The degree of heat and the temperature and nature of the cooling medium must be chosen with reference to the quality of the steel and the purpose for which it is intended. Cold water, mercury, and acids give the greatest hardness; oils and fatty substances, sand, wet iron scales or cinders, &c., give an inferior degree of hardness, but prevent the cracks which are caused by too rapid cooling. The lower the heat at which the steel becomes hard, the better.

Tempering.—Steel in its hardest state being too brittle for most purposes, the requisite strength and elasticity are obtained by tempering, or *letting down the temper*, as the workmen term it, which is performed by heating the hardened steel to a certain degree, and letting it cool gradually. The requisite heat is usually ascertained by the color which the surface of the steel assumes from the film of oxyd thus formed. The degrees of heat to which these several colors correspond, are as follows:

At 430° Fahr.,	a very faint yellow.	Suitable for hard instruments; as hammer faces, drills for hard substances, &c.
At 450° "	a pale straw color.	
At 470° "	a full yellow.	For instruments requiring hard edges without elasticity; as shears, scissors, tools for turning iron and steel.
At 490° "	a brown color.	

English files are used exclusively at the arsenals and armories.

Files should be made of the best cast steel. The teeth are generally cut at an angle of 60° with the center line; at a smaller angle, the teeth are apt to choke, and at a greater angle, they do not cut.

In choosing files, they should be examined to see that they are straight, that they are free from cracks and flaws, and that they are cut regularly. The teeth should not be turned or broken by filing on iron or tempered steel. One out of each dozen may be tried on a piece of tempered steel, such as the tang of a file screwed in a vice; the file should "take" in its whole length, both on the flat and edge, and should not cut in drawing back; it should not make furrows, or show a tendency to deviate from the direction given to it by the hand.

The quality of the steel may be determined by breaking some of the files, and working the steel in the forge.

Case Hardening is the conversion of the surface of wrought iron into steel, for the purpose of adapting it to receive a polish, or to bear friction, &c.: this is effected by heating the iron to a cherry red, in a close vessel, in contact with carbonaceous materials, and then plunging it into cold water. Bones, leather, hoofs and horns of animals, are generally used for this purpose, after having been burnt or roasted, so that they can be pulverized. Soot is also frequently used.

SHEET IRON.

Sheet Iron is made by rolling. It should be soft and tough, its surface very smooth, without holes or thick scales; it is generally of a bluish color, sometimes clouded; the sheet should be of regular thickness, elastic, and crackling when bent in the hands. When bent at a right angle, there should be no appearance of fracture on the exterior.

Russia Sheet Iron has a planished, glossy and smooth surface of gray oxyd of iron; it should be free from rust or flaws, and be very soft and tough.

The severest test of sheet iron consists in hammering a part of the sheet into a concave form.

Sheet Steel should have the same qualities as sheet iron, with greater elasticity and hardness in a thinner sheet.

For the weight of sheet iron, see Tables, page 189.

SHEET TIN.

Sheet Tin is made by coating sheet iron with tin. The iron is first *scoured*, or thoroughly cleaned, by means of an acid, and then immersed in melted tin. There are two kinds, called *single tin* and *double tin*, differing in thickness and in the quantity of tin with which the iron is coated. The surface of the sheets should be bright and smooth, free from specks, beads, and blisters.

GOLD.

The double Chloride of Gold and Sodium is prepared by dissolving four parts of gold in nitro-muriatic acid, evaporating the solution to dryness, and dissolving the dry mass in *eight* times its weight of distilled water. To this solution one part of pure decrepitated common salt is added, previously dissolved in four parts of water. The mixed solution is then evaporated to dryness, being in the mean time constantly stirred with a glass rod. This salt is of a golden-yellow color, and when crystalized is in the form of long prismatic crystals, inalterable in the air.

The Cyanuret of Gold is best obtained as follows:—Prepare the chloride of gold as neutral as possible, by repeated solutions and crystalizations; and to the solution of this salt add, very cautiously avoiding any excess, a solution of *pure* cyanuret of potassium so long as any precipitate falls. The precipitate consisting of cyanuret of gold is to be washed in pure water, and dried in the dark.

A Receipt to deprive Oil of its Acid.

To four ounces of the best spermaceti oil add four grains of dried potash, in five ounces of distilled water; shake them well for a day or two, then pour the whole into a tumbler covered by another, and when exposed to the light for three or four weeks, the pure oil will float upon the top, to be skimmed off with a teaspoon. This oil neither dries nor turns green.

PASTE.

Flour paste is made best of rye flour. Sift the flour and mix it with $8\frac{1}{2}$ times its weight of water; heat it gently, stir it, and let it boil for $\frac{3}{4}$ of an hour; when it becomes ropy, pour it into bowls, and pass it through a sieve before it is quite cold. The flour yields 7 times its weight, of paste. Time required to make it, one hour and a half.

Given a square Sheet of Copper or Tin of any given number of equal Sides, to construct a Vessel that shall contain the greatest cubical Quantity.

Rule.—Cut out of each corner, at right angles to the side, $\frac{1}{6}$ part of the length of the side, and turn up the sides till the corners meet. This gives the greatest capacity.

Thus: A sheet 12 inches square will give the following: $\frac{1}{6}$ of 12 $=2-$, and 2 from each angle will leave 8 inches square, and 2 inches deep. $8^2 \times 2 = 128$ cubic inches, the greatest capacity that can be obtained from a 12-inch sheet.

The numbers 376 and 625, when squared, cubed, or quadrupled, always terminate with the same numbers. No other numbers do the same, and no reason can be given why these have this peculiarity of termination.

SCANTLING AND TIMBER,

Accurately reduced to Inch Board Measure.

EXPLANATION.

The length of any piece of scantling, or timber, will be found in the left-hand column, under the side dimensions. The breadth and depth (or side dimensions), in inches, will be found at the head or center of each column of computations. Thus, on page 226, a piece of scantling 2½ by 11 inches, side dimension, and 16 feet long, is shown to contain 36 feet and 8 inches, of board measure. On page 228, a piece of scantling 4 by 10 inches, side dimension, and 17 feet long, is shown to contain 56 feet 8 inches of board measure. The answer sought for, in all cases, will be found directly on the right of the length, and under the side dimensions. If a piece of scantling, or stick of timber, should exceed in length any provision which has been made in these tables, its contents would be shown by taking twice what is shown for half its length. Thus, a stick of timber, or piece of scantling, 46 feet long, would contain twice as many feet, board measure, as is shown in the table to be the contents of a stick 23 feet long. So also, one 39 feet long would contain as many feet, board measure, as these tables show opposite to 22 and 17 feet long, or 3 times the contents of one 13 feet long.

TABLES.

2 by 2.		2 by 3.		2 by 4.		2 by 5.		2 by 6.		2 by 7.	
Length 1	0·4	Length 1	0·6	Length 1	0·8	Length 1	0·10	Length 1	1·	Length 1	1·2
2	0·8	2	1·	2	1·4	2	1·8	2	2·	2	2·4
3	1·	3	1·6	3	2·	3	2·6	3	3·	3	3·6
4	1·4	4	2·	4	2·8	4	3·4	4	4·	4	4·8
5	1·8	5	2·6	5	3·4	5	4·2	5	5·	5	5·10
6	2·	6	3·	6	4·	6	5·	6	6·	6	7·
7	2·4	7	3·6	7	4·8	7	5·10	7	7·	7	8·2
8	2·8	8	4·	8	5·4	8	6·8	8	8·	8	9·4
9	3·	9	4·6	9	6·	9	7·6	9	9·	9	10·6
10	3·4	10	5·	10	6·8	10	8·4	10	10·	10	11·8
11	3·8	11	5·6	11	7·4	11	9·2	11	11·	11	12·10
12	4·	12	6·	12	8·	12	10·	12	12·	12	14·
13	4·4	13	6·6	13	8·8	13	10·10	13	13·	13	15·2
14	4·8	14	7·	14	9·4	14	11·8	14	14·	14	16·4
15	5·	15	7·6	15	10·	15	12·6	15	15·	15	17·6
16	5·4	16	8·	16	10·8	16	13·4	16	16·	16	18·8
17	5·8	17	8·6	17	11·4	17	14·2	17	17·	17	19·10
18	6·	18	9·	18	12·	18	15·	18	18·	18	21·
19	6·4	19	9·6	19	12·8	19	15·10	19	19·	19	22·2
20	6·8	20	10·	20	13·4	20	16·8	20	20·	20	23·4
21	7·	21	10·6	21	14·	21	17·6	21	21·	21	24·6
22	7·4	22	11·	22	14·8	22	18·4	22	22·	22	25·8
23	7·8	23	11·6	23	15·4	23	19·2	23	23·	23	26·10
24	8·	24	12·	24	16·	24	20·	24	24·	24	28·

2 by 8.		2 by 9.		2 by 10.		2 by 11.		2½ by 5.		2½ by 6.	
Length. 1	1·4	Length. 1	1·6	Length. 1	1·8	Length. 1	1·10	Length. 1	1·1	Length. 1	1·3
2	2·8	2	3·	2	3·4	2	3·8	2	2·1	2	2·6
3	4·	3	4·6	3	5·	3	5·6	3	3·2	3	3·9
4	5·4	4	6·	4	6·8	4	7·4	4	4·2	4	5·
5	6·8	5	7·6	5	8·4	5	9·2	5	5·3	5	6·3
6	8·	6	9·	6	10·	6	11·	6	6·3	6	7·6
7	9·4	7	10·6	7	11·8	7	12·10	7	7·4	7	8·9
8	10·8	8	12·	8	13·4	8	14·8	8	8·4	8	10·
9	12·	9	13·6	9	15·	9	16·6	9	9·5	9	11·3
10	13·4	10	15·	10	16·8	10	18·4	10	10·5	10	12·6
11	14·8	11	16·6	11	18·4	11	20·2	11	11·6	11	13·9
12	16·	12	18·	12	20·	12	22·	12	12·6	12	15·
13	17·4	13	19·6	13	21·8	13	23·10	13	13·7	13	16·3
14	18·8	14	21·	14	23·4	14	25·8	14	14·7	14	17·6
15	20·	15	22·6	15	25·	15	27·6	15	15·8	15	18·9
16	21·4	16	24·	16	26·8	16	29·4	16	16·8	16	20·
17	22·8	17	25·6	17	28·4	17	31·2	17	17·9	17	21·3
18	24·	18	27·	18	30·	18	33·	18	18·9	18	22·6
19	25·4	19	28·6	19	31·8	19	34·10	19	19·10	19	23·9
20	26·8	20	30·	20	33·4	20	36·8	20	20·10	20	25·
21	28·	21	31·6	21	35·	21	38·6	21	21·11	21	26·3
22	29·4	22	33·	22	36·8	22	40·4	22	22·11	22	27·6
23	30·8	23	34·6	23	38·4	23	42·2	23	24·	23	28·9
24	32·	24	36·	24	40·	24	44·	24	25·	24	30·

2½ by 7.		2½ by 8.		2½ by 9.		2½ by 10.		2½ by 11.		2½ by 12.	
Length. 1	1·6	Length. 1	1·8	Length. 1	1·11	Length. 1	2·1	Length. 1	2·4	Length. 1	2·6
2	2·11	2	3·4	2	3·9	2	4·2	2	4·7	2	5.
3	4·5	3	5·	3	5·8	3	6·3	3	7·	3	7·6
4	5·10	4	6·8	4	7.6	4	8·4	4	9·2	4	10·
5	7·4	5	8·4	5	9·5	5	10·5	5	11·6	5	12·6
6	8·9	6	10·	6	11·3	6	12·6	6	13·9	6	15·
7	10·3	7	11·8	7	13·2	7	14·7	7	16·	7	17·6
8	11·8	8	13·4	8	15·	8	16·8	8	18·4	8	20·
9	13·2	9	15·	9	16·11	9	18·9	9	20·8	9	22·6
10	14·7	10	16·8	10	18·9	10	20·10	10	23·	10	25·
11	16·1	11	18·4	11	20·8	11	22·11	11	25·3	11	27·6
12	17·6	12	20·	12	22·6	12	25·	12	27·6	12	30·
13	19·	13	21·8	13	24·5	13	27·1	13	29·10	13	32·6
14	20·5	14	23·4	14	26·3	14	29·2	14	32·1	14	35·
15	21·11	15	25·	15	28·2	15	31·3	15	34·4	15	37·6
16	23·4	16	26·8	16	30·	16	33·4	16	36·8	16	40·
17	24·10	17	28·4	17	31·11	17	35·5	17	39·	17	42·6
18	26·3	18	30·	18	33·9	18	37.6	18	41·3	18	45·
19	27·9	19	31·8	19	35·8	19	39·7	19	43·7	19	47·6
20	29·2	20	33·4	20	37·6	20	41·8	20	45·10	20	50·
21	30·8	21	35·	21	39·5	21	43·9	21	48·2	21	52·6
22	32·1	22	36·8	22	41·3	22	45·10	22	50·5	22	55·
23	33·7	23	38·4	23	43·2	23	47·11	23	52·9	23	57·6
24	35·	24	40·	24	45·	24	50·	24	55·	24	60·

3 by 3.		3 by 4.		3 by 5.		3 by 6.		3 by 7.		3 by 8.	
Length. 1	0·9	Length. 1	1·	Length. 1	1·3	Length. 1	1·6	Length. 1	1·9	Length. 1	2·
2	1·6	2	2·	2	2·6	2	3·	2	3·6	2	4·
3	2·3	3	3·	3	3·9	3	4·6	3	5·3	3	6·
4	3·	4	4·	4	5·	4	6·	4	7·	4	8·
5	3·9	5	5·	5	6·3	5	7·6	5	8·9	5	10·
6	4·6	6	6·	6	7·6	6	9·	6	10·6	6	12·
7	5·3	7	7·	7	8·9	7	10·6	7	12·3	7	14·
8	6·	8	8·	8	10·	8	12·	8	14·	8	16·
9	6·9	9	9·	9	11·3	9	13·6	9	15·9	9	18·
10	7·6	10	10·	10	12·6	10	15·	10	17·6	10	20·
11	8·3	11	11·	11	13·9	11	16·6	11	19·3	11	22·
12	9·	12	12·	12	15·	12	18·	12	21·	12	24·
13	9·9	13	13·	13	16·3	13	19·6	13	22·9	13	26·
14	10·6	14	14·	14	17·6	14	21·	14	24·6	14	28·
15	11·3	15	15·	15	18·9	15	22·6	15	26·3	15	30·
16	12·	16	16·	16	20·	16	24·	16	28·	16	32·
17	12·9	17	17·	17	21·3	17	25·6	17	29·9	17	34·
18	13·6	18	18·	18	22·6	18	27·	18	31·6	18	36·
19	14·3	19	19·	19	23·9	19	28·6	19	33·3	19	38·
20	15·	20	20·	20	25·	20	30·	20	35·	20	40·
21	15·9	21	21·	21	26·3	21	31·6	21	36·9	21	42·
22	16·6	22	22·	22	27·6	22	33·	22	38·6	22	44·
23	17·3	23	23·	23	28·9	23	34·6	23	40·3	23	46·
24	18·	24	24·	24	30·	24	36·	24	42.	24	48·

3 by 9.		3 by 10.		3 by 11.		3 by 12.		4 by 4.		4 by 5.	
Length. 1	2·3	Length. 1	2·6	Length. 1	2·9	Length. 1	3·	Length. 1	1·4	Length. 1	1·8
2	4·6	2	5·	2	5·6	2	6·	2	2·8	2	3·4
3	6·9	3	7·6	3	8·3	3	9·	3	4·	3	5·
4	9·	4	10·	4	11·	4	12·	4	5·4	4	6·8
5	11·3	5	12·6	5	13·9	5	15·	5	6·8	5	8·4
6	13·6	6	15·	6	16·6	6	18·	6	8·	6	10·
7	15·9	7	17·6	7	19·3	7	21·	7	9·4	7	11·8
8	18·	8	20·	8	22·	8	24·	8	10·8	8	13·4
9	20·3	9	22·6	9	24·9	9	27·	9	12·	9	15·
10	22·6	10	25·	10	27·6	10	30·	10	13·4	10	16·8
11	24·9	11	27·6	11	30·3	11	33·	11	14·8	11	18·4
12	27·	12	30·	12	33·	12	36·	12	16·	12	20·
13	29·3	13	32·6	13	35·9	13	39·	13	17·4	13	21·8
14	31·6	14	35·	14	38·6	14	42·	14	18·8	14	23·4
15	33 9	15	37·6	15	41·3	15	45·	15	20·	15	25·
16	36·	16	40·	16	44·	16	48·	16	21·4	16	26·8
17	38·3	17	42·6	17	46·9	17	51·	17	22·8	17	28·4
18	40·6	18	45·	18	49·6	18	54·	18	24·	18	30·
19	42·9	19	47·6	19	52·3	19	57·	19	25·4	19	31·8
20	45·	20	50·	20	55·	20	60·	20	26·8	20	33·4
21	47·3	21	52·6	21	57·9	21	63·	21	28·	21	35·
22	49·6	22	55·	22	60·6	22	66·	22	29·4	22	36·8
53	51·9	23	57·6	23	63·3	23	69·	23	30·8	23	38·4
24	54·	24	60·	24	66·	24	72·	24	32·	24	40·

4 by 6.		4 by 7.		4 by 8.		4 by 9.		4 by 10.		4 by 11.	
Length. 1	2·	Length. 1	2·4	Length. 1	2·8	Length. 1	3·	Length. 1	3·4	Length. 1	3 8
2	4·	2	4·8	2	5·4	2	6·	2	6·8	2	7·4
3	6·	3	7·	3	8·	3	9·	3	10·	3	11·
4	8·	4	9·4	4	10·8	4	12·	4	13·4	4	14·8
5	10·	5	11·8	5	13·4	5	15·	5	16·8	5	18·4
6	12·	6	14·	6	16·	6	18·	6	20·	6	22·
7	14·	7	16·4	7	18·8	7	21·	7	23·4	7	25·8
8	16·	8	18·8	8	21·4	8	24·	8	26·8	8	29·4
9	18·	9	21·	9	24·	9	27·	9	30·	9	33·
10	20·	10	23·4	10	26·8	10	30·	10	33·4	10	36·8
11	22·	11	25·8	11	29·4	11	33·	11	36·8	11	40·4
12	24·	12	28·	12	32·	12	36·	12	40·	12	44·
13	26·	13	30·4	13	34·8	13	39·	13	43·4	13	47·8
14	28·	14	32·8	14	37·4	14	42·	14	46·8	14	51·4
15	30·	15	35·	15	40	15	45·	15	50·	15	55·
16	32·	16	37·4	16	42·8	16	48·	16	53·4	16	58·8
17	34·	17	39·8	17	45·4	17	51·	17	56·8	17	62·4
18	36·	18	42·	18	48·	18	54·	18	60·	18	66·
19	38·	19	44·4	19	50·8	19	57·	19	63·4	19	69·8
20	40·	20	46·8	20	53·4	20	60·	20	66·8	20	73·4
21	42·	21	49·	21	56·	21	63·	21	70·	21	77·
22	44·	22	51·4	22	58·8	22	66·	22	73·4	22	80·8
23	46·	23	53·8	23	61·4	23	69·	23	76·8	23	84·4
24	48·	24	56·	24	64·	24	72·	24	80·	24	88·

4 by 12.		5 by 5.		5 by 6.		5 by 7.		5 by 8.		5 by 9.	
Length. 1	4·	Length. 1	2·1	Length. 1	2·6	Length. 1	2·11	Length. 1	3·4	Length. 1	3·9
2	8·	2	4·2	2	5·	2	5·10	2	6·8	2	7·6
3	12·	3	6·3	3	7·6	3	8·9	3	10·	3	11·3
4	16·	4	8·4	4	10·	4	11·8	4	13·4	4	15·
5	20·	5	10·5	5	12·6	5	14·7	5	16·8	5	18·9
6	24·	6	12·6	6	15·	6	17·6	6	20·	6	22·6
7	28·	7	14·7	7	17·6	7	20·5	7	23·4	7	26·3
8	32·	8	16·8	8	20·	8	23·4	8	26·8	8	30·
9	36·	9	18·9	9	22·6	9	26·3	9	30·	9	33·9
10	40·	10	20·10	10	25·	10	29·2	10	33·4	10	37·6
11	44·	11	22·11	11	27·6	11	32·1	11	36·8	11	41·3
12	48·	12	25·	12	30·	12	35·	12	40·	12	45·
13	52·	13	27·1	13	32·6	13	37·11	13	43·4	13	48·9
14	56·	14	29·2	14	35·	14	40·10	14	46 8	14	52·6
15	60·	15	31·3	15	37·6	15	43·9	15	50·	15	56·3
16	64·	16	33·4	16	40·	16	46·8	16	53·4	16	60·
17	68·	17	35·5	17	42·6	17	49·7	17	56·8	17	63.9
18	72·	18	37·6	18	45·	18	52·6	18	60·	18	67·6
19	76·	19	39·7	19	47·6	19	55·5	19	63·4	19	71·3
20	80·	20	41·8	20	50·	20	58·4	20	66·8	20	75·
21	84·	21	43·9	21	52·6	21	61·3	21	70·	21	78·9
22	88·	22	45·10	22	55·	22	64·2	22	63·4	22	82·6
23	92·	23	47·11	23	57·6	23	67·1	23	76·8	23	86·3
24	96·	24	50·	24	60.	24	70·	24	80·	24	90·

Length.	5 by 10.	6 by 6.	6 by 7.	6 by 8.	7 by 7.	7 by 8.
1	4·2	3·	3·6	4·	4·1	4·8
2	8·4	6·	7·	8·	8·2	9·4
3	12·6	9·	10·6	12·	12·3	14·
4	16·8	12·	14·	16·	16·4	18·8
5	20·10	15·	17·6	20·	20·5	24·4
6	25·	18·	21·	24·	24·6	28·
7	29·2	21·	24·6	28·	28·7	32·8
8	33·4	24·	28·	32·	32·8	37·4
9	37·6	27·	31·6	36·	36·9	42·
10	41·8	30·	35·	40·	40·10	46·8
11	45·10	33·	38·6	44·	44·11	51·4
12	50·	36·	42·	48·	49·	56·
13	54·2	39·	45·6	52·	53·1	60·8
14	58·4	42·	49·	56·	57·2	65·4
15	62·6	45·	52·6	60·	61·3	70·
16	66·8	48·	56·	64·	65·4	74·8
17	70·10	51·	59·6	68·	69·5	79·4
18	75·	54·	63·	72·	73·6	84·
19	79·2	57·	66·6	76·	77·7	88·8
20	83·4	60·	70·	80·	81·8	93·4
21	87·6	63·	73·6	84·	85·9	98·
22	91·8	66·	77·	88·	89·10	102·8
23	95·10	69·	80·6	92·	93·11	107·4
24	100·	72·	84·	96·	98·	112·

Length.	7 by 9.	8 by 8.	8 by 9.	8 by 10.	9 by 9.	9 by 10.
1	5·3	5·4	6·	6·8	6·9	7·6
2	10·6	10·8	12·	13·4	13·6	15·
3	15·9	16·	18·	20·	20·3	22·6
4	21·	21·4	24·	26·8	27·	30·
5	26·3	26·8	30·	33·4	33·9	37·6
6	31·6	32·	36·	40·	40·6	45·
7	36·9	37·4	42·	46·8	47·3	52·6
8	42·	42·8	48·	53·4	54·	60·
9	47·3	48·	54·	60·	60·9	67·6
10	52·6	53·4	60·	66·8	67·6	75·
11	57·9	58·8	66·	73·4	74·3	82·6
12	63·	64·	72·	80·	81·	90·
13	68·3	69·4	78·	86·8	87·9	97·6
14	73·6	74·8	84·	93·4	94·6	105·
15	78·9	80·	90·	100·	101·3	112·6
16	84·	85·4	96·	106·8	108·	120·
17	89·3	90·8	102·	113·4	114·9	127·6
18	94·6	96·	108·	120·	121·6	135·
19	99·9	101·4	114·	126·8	128·3	142·6
20	105·	106·8	120·	133·4	135·	150·
21	110·3	112·	126·	140·	141·9	157·6
22	115·6	117·4	132·	146·8	148·6	165·
23	120·9	122·8	138·	153·4	155·3	172·6
24	126·	128·	144·	160·	162·	180·

Weight of 1 ft - Cast shaft 6 in Dia

$6^2 = 36 \times \overset{2,478}{2,48} = \overset{89,208}{89,28}$

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3 Days @ $7,00 per Week

$$\begin{array}{r} 3 \\ 6)\overline{21\,00} \\ \hline \$3,50 \end{array}$$
